

## OPTIMIZATION OF CONJUGATE HEAT TRANSFER IN THE ELECTROFUSION JOINT USING TAGUCHI METHOD

by

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*Applications of electrofusion joints for gas pipes are increasing and so study on temperature distribution in these joints have an important role in polyethylene welding. The objective of this study is assessment of optimum rate of the conjugate heat transfer in electrofusion joints. The simulations were planned based on Taguchi's L25 orthogonal array. The thermal lattice Boltzmann based on D2Q9 method was employed to simulate the flow and thermal fields. Signal-to-noise ratios analysis were carried out. The flow behaviors and the rate of heat transfer in terms of the Nusselt number have effectively changed by distance, height and radius of electrical wires inside the electrofusion joint. Finally, result analysis verified that Taguchi method achieved optimization of heat transfer rate with sufficient accuracy.*

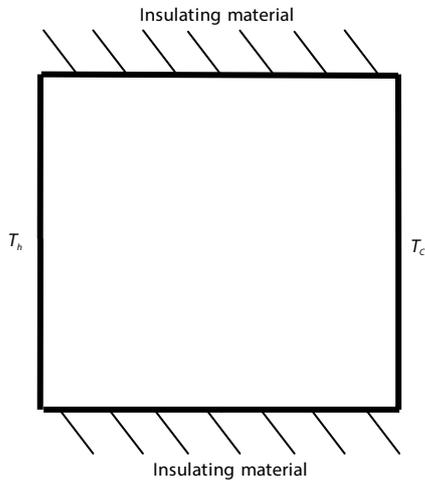
Key words: *Taguchi method, Lattice Boltzmann model, heat transfer, electrofusion joint*

### Introduction

Polyethylene (PE) has been widely used in gas distribution since it was initially introduced to transport natural gas in the 1960 [1]. Generally, it has lots of advantages such as excellent corrosion resistance, ease of installation, and cost-effectiveness. The temperature and the pressure usually affect the macromolecular diffusion at the interface [2, 3]. Therefore, it is essential to obtain an understanding of the electrofusion (EF) jointing of PE pipe systems. Research on predicting temperature profiles of the EF joint has been intensively performed in recent years. Shi *et al.* [4] established a 1-D axisymmetric heat transfer model. Nakashiba *et al.* [5], Nishimura *et al.* [6], and Fujikake *et al.* [7] investigated the temperature profiles with a finite element model. The main purpose of this study is simulation of natural-convection and heat conduction in EF joints.

Compared with the traditional CFD methods, the lattice Boltzmann method (LBM) is a meso-scale modelling method based on the particle kinematics. It has many advantages, such as simple coding, easy implementation of boundary conditions and fully parallelism. At present the applications of LBM have achieved great success in heat transfer and fluid convection problems. Esfahani and Alinejad [8-10] conducted the simulation for viscous-fluid-flow and conjugate heat transfer in a rectangular cavity by using LBM. D'Orazio *et al.* [11] and Shu *et al.* [12] performed the numerical calculations for the natural-convection in a cavity. Gao *et al.* [13] used LBM for conjugate heat transfer in porous media. Karani and Huber [14] used LBM for conjugate heat transfer in heterogeneous media. Wang *et al.* [15] performed LBM for

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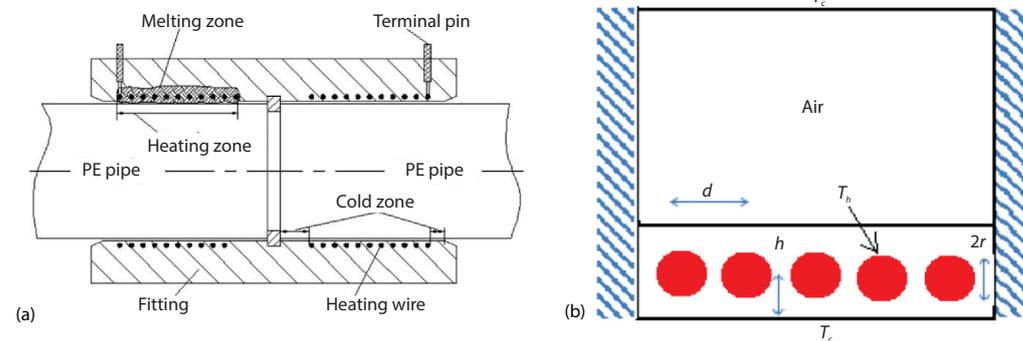
**Figure 1. Schematic diagram of the benchmark problem**

fluid-solid conjugate heat transfer. Zhihang [16] study the fan – assisted cooling using the Taguchi approach in open and closed data centers and Wang *et al.* [17] performed Taguchi method for optimization of H-type finned tube heat exchangers.

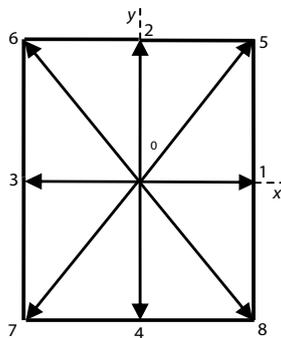
This study is rarely conform to the single-enclosure model, fig. 1, used in much of the natural-convection literature. This paper investigates the natural-convection heat transfer in a 2-D enclosure by means of Taguchi method in order to obtain significant process parameters and optimum factor combinations. For this reason the thermal LBM with the Boussinesq approximation is employed to simulate natural-convection and heat conduction. The schematic sketch of an EF coupler [18] and schematic sketch of an EF model are shown in fig. 2.

and analyzed in detail. Finally, result analysis verified that Taguchi method achieved optimization of heat transfer rate with sufficient accuracy.

The effects of distance, height and radius of electrical wires inside the EF joint has been observed



**Figure 2. The EF joint; (a) schematic sketch of an EF coupler, (b) schematic sketch of an EF model in this study**



**Figure 3. The 2-D with 9-velocities lattice (D2Q9) model**

**Lattice Boltzmann method**

The LBM is particularly successful as a numerical method for solving the different fluid dynamic problems [19]. The LBM is derived from lattice gas methods as an explicit discretization of the Boltzmann equation in the phase space is considered. The LBM is a vigorous numerical method, based on the kinetic theory to simulate fluid-flow and heat transfer.

Unlike the classical macroscopic approach (Navier-Stokes) the lattice Boltzmann is a mesoscopic model to simulate flow field. In this approach, the fluid domain is made discrete in uniform Cartesian cells, each one of which holds a fixed number of distribution functions (DF) that represent the number of fluid particles moving

in these discrete directions. Hence depending on the dimension and number of velocity directions, there are different models that can be used. The present study examined 3-D flow by using 2-D lattice with nine velocities (D2Q9 model). The velocities of the D2Q9 model are shown in fig. 3.

The DF are calculated by solving the lattice Boltzman equation, which is a special discretization of the kinetic Boltzmann equation. After introducing Bhatnagar-Gross-Krook approximation, the Boltzmann equation can be formulated [19]:

$$f_k(\mathbf{x} + \mathbf{c}_k \Delta t, t + \Delta t) = f_k(\mathbf{x}, t) + \frac{\Delta t}{\tau} [f_k^{eq}(\mathbf{x}, t) - f_k(\mathbf{x}, t)] + \Delta t \mathbf{c}_k F_k \quad (1)$$

where  $\Delta t$ ,  $\mathbf{c}_k$ ,  $\tau$ ,  $f_k^{eq}$ , and  $F_k$  denote lattice time step, the discrete lattice velocity in direction  $k$ , the lattice relaxation time, the equilibrium DF, and the external force in the direction of the lattice velocity, respectively. The equilibrium DF for the D2Q9 velocity model:

$$f_k^{eq} = \omega_k \rho \left[ 1 + \frac{\mathbf{c}_k \mathbf{u}}{c_s^2} + \frac{1}{2} \frac{(\mathbf{c}_k \mathbf{u})^2}{c_s^4} - \frac{1}{2} \frac{\mathbf{u}^2}{c_s^2} \right] \quad (2)$$

where the weights of  $\omega_k$  are  $\omega_k = 4/9$  for  $k = 0$ ,  $\omega_k = 1/9$  for  $k = 1, 2, 3, 4$ , and  $\omega_k = 1/36$  for  $k = 5, 6, 7, 8$ ,  $c_s = c_k/(3)^{1/2}$  and is the lattice speed of sound. In order to incorporate buoyancy force in the model, the force term in eq. (1) needs to be calculated, as below, in a vertical  $z$ -direction:

$$F = \rho \mathbf{g}_z \beta \Delta T \quad (3)$$

The macroscopic fluid variable densities and velocities are computed as the first two moments of the DF for each cell:

$$\rho = \sum_{k=0}^8 f_k \quad (4)$$

$$\mathbf{u} = \frac{1}{\rho} \sum_{k=0}^8 f_k \mathbf{c}_k \quad (5)$$

For the temperature field the  $g$  distribution:

$$\mathbf{g}_k(\mathbf{x} + \mathbf{c}_k \Delta t, t + \Delta t) = \mathbf{g}_k(\mathbf{x}, t) + \frac{\Delta t}{\tau_g} [\mathbf{g}_k^{eq}(\mathbf{x}, t) - \mathbf{g}_k(\mathbf{x}, t)] \quad (6)$$

where  $\tau_g$  denotes the lattice relaxation time for the temperature field and defined as  $\tau_g = (\alpha + 1/2)/c_s^2$ . For the D2Q9 model, the equilibrium energy DF:

$$\mathbf{g}_k^{eq} = \omega_k T \left[ 1 + \frac{\mathbf{c}_k \mathbf{u}}{c_s^2} \right] \quad (7)$$

$$\mathbf{g}_k^{eq} = \omega_k T \quad (8)$$

The temperature field:

$$T = \sum \mathbf{g}_k \quad (9)$$

Heat transfer between hot and cold walls was computed by local and mean Nusselt number:

$$\text{Nu}_l = \frac{-1}{\theta_m} \frac{\partial \theta}{\partial n} \Big|_{\text{wall}}, \quad \theta_m = \frac{T - T_c}{T_h - T_c} \quad (10)$$

$$\text{Nu}_m = \frac{1}{l} \int_0^l \text{Nu}_l dx \quad (11)$$

### Physical model

The physical geometry considered in this study is shown in fig. 2. We consider the natural-convection of a viscous incompressible fluid in a 2-D enclosure and heat conduction in EF joint. We suppose that the solid local energy source with constant temperature,  $T_h$ , the left and right walls of cavity are insulated and other walls have constant temperature,  $T_c$ . Physical and thermal properties of both air and PE at the base temperature 20 °C are listed in tab. 1 [20]. The distance of wires is changed from 25-43 (25, 30, 35, 40, and 43) and the height of wires from the bottom wall is changed from 10-25 (10, 13, 17, 22, and 25). The wires have five radius (5, 6, 7, 8, and 9) and the cavity is square ( $150 \times 150$ ). The change of variables has been tried to be selected based on the actual sample. In actual cases, the wires are close to the inner surface of the coupler. The distance between the wires cannot be too long or too short. The very short distance of the wires causes the flow of the melt. The excessive distance between the wires creates cold zones and this phenomena is not good for fusion.

**Table 1. Thermo physical properties**

Material	$\rho$ [kgm <sup>-3</sup> ]	$C_p$ [JKg <sup>-1</sup> K <sup>-1</sup> ]	$k$ [Wm <sup>-1</sup> K <sup>-1</sup> ]	$\beta \cdot 10^5$ [K <sup>-1</sup> ]
Air	1.205	1.005	0.0257	343
PE	950	2.2	0.4	20

### Grid independency

For grid independency, the average Nusselt number was calculated at high Rayleigh numbers for different grid points. As seen in tab. 2 for grid points passing from  $80 \times 80$  to  $100 \times 100$  for  $\text{Ra} = 10^8$ , no considerably change in the average Nusselt number is observed. According to the tab. 1, the  $100 \times 100$  grid points was used for  $\text{Ra} \leq 10^8$ .

**Table 2. Effect of spatial resolution on the mean Nusselt number**

Mesh size	$60 \times 60$	$80 \times 80$	$100 \times 100$
$\text{Ra} = 10^8$	26.104	28.542	29.590

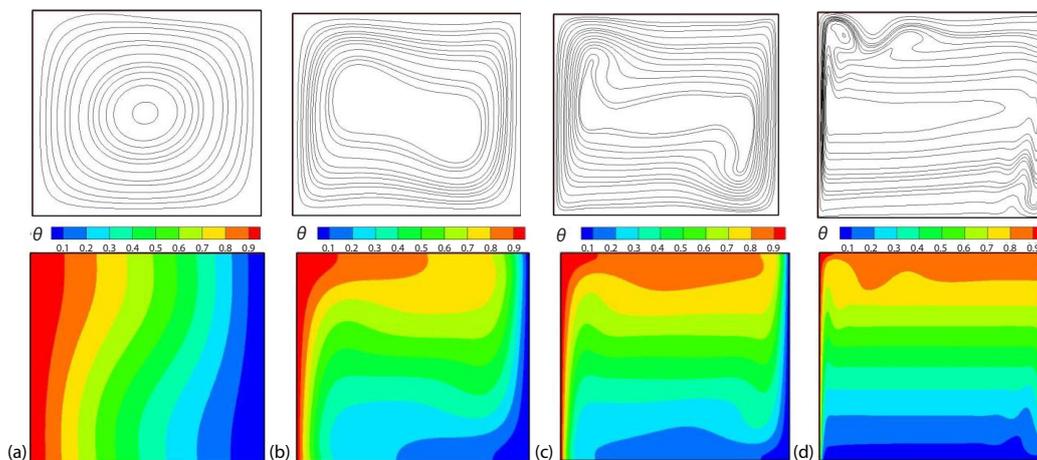
### Code validation

The numerical simulation was done by an in-house code written in FORTRAN, using LBM. Numerical investigations were carried out for the following values of dimensionless Rayleigh number,  $10^3 < \text{Ra} < 10^8$ . The influence of the main parameters characterizing the process was analyzed. The obtained results are compared with the previous simulations of natural-convection in a square cavity [21-23]. The comparison of streamlines, isotherms and mean Nusselt number at the interface between the solid wall and gaseous cavity with previous work at different Rayleigh numbers illustrates a fine agreement that has been obtained, fig. 4 and tab. 3. The isotherm lines vortex indicates a change in the dominant heat transfer mechanism with Rayleigh number. For low Rayleigh number, isotherms are aligned with the temperature constant walls and slightly deviated by the flow, and the heat is transferred mainly by heat conduction. As Rayleigh number increases, the controlling heat transfer mechanism changes from conduction convection, the shape of isotherms begins to bend in

the bulk region. The isotherm lines become flat in the central region of the cavity. These lines are vertical only in thin boundary-layers near the hot and cold walls and the fluid is thermally arranged in different layers. In other words, the isotherms become horizontal in the cavity. Observing the streamlines patterns reveals wavy disturbances occurrence close to the horizontal adiabatic boundary especially at upper-left and bottom right corners. These patterns intensify by increasing  $Ra = 10^8$  and finally eddies are developed. The temperature field becomes more and more stratified. The isotherms near the hot wall stretch upward as a result of the warm fluid wake. Finally, it should be noted that there is an excellent agreement between the present results and the benchmark LBM solution by Du and Lui [21] and Dixit and Babu [22] for all values of Rayleigh number (maximum difference is less than 2%), as well as with the CFD 2-D solutions by Barakos and Mitsoulis [23].

**Table 3. Comparison of mean Nusselt number for simulating 2-D natural-convection in a square cavity**

Ra	$10^3$	$10^4$	$10^5$	$10^6$	$10^7$	$10^8$
Present study	1.113	2.231	4.520	8.845	16.499	29.590
[21]	1.108	2.252	4.596	8.822	16.424	29.094
[22]	1.121	2.286	4.546	8.652	16.790	30.506
[23]	1.114	2.245	4.510	8.806	–	30.1



**Figure 4. Streamlines (top row) and isotherms (bottom row) for different Rayleigh number; (a)  $Ra = 10^5$ , (b)  $Ra = 10^6$ , (c)  $Ra = 10^7$ , and (d)  $Ra = 10^8$**

## Results and discussion

Figure 4 shows the streamlines and isotherms. By the exact view of the streamlines, it is determined that the fluid warming along the top wall of the EF and reduces its density and thus the fluid moves through the force of buoyancy to the upper wall of enclosure. When the flow reaches the upper wall, the streamlines incline to the sides wall. When the wires are near the walls, two hot spaces are made. The presence of hot spaces has a significant effect on the overall heat transfer rate between the top wall of EF and the air on it. As can be seen by increasing of wires' distance the Nusselt number is increase and then decrease, fig. 5.

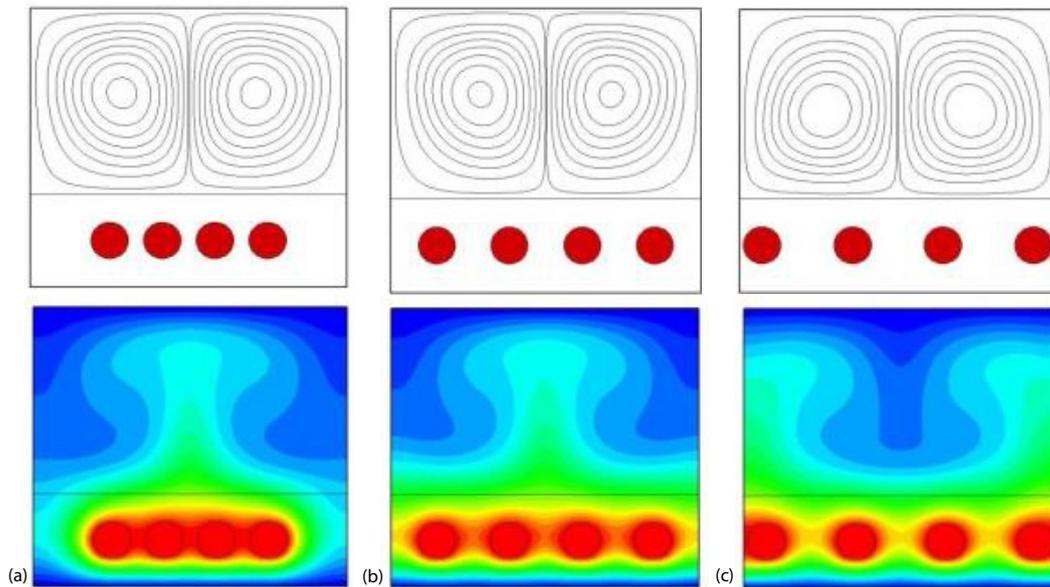


Figure 5. Streamlines (top row) and isotherms (bottom row) for different distance of wires at  $Ra = 10^5$ ,  $h = 25$  and  $r = 9$ ; (a)  $d = 25$ ,  $Nu_m = 2.31$ , (b)  $d = 35$ ,  $Nu_m = 2.52$ , and (c)  $d = 43$ ,  $Nu_m = 2.52$

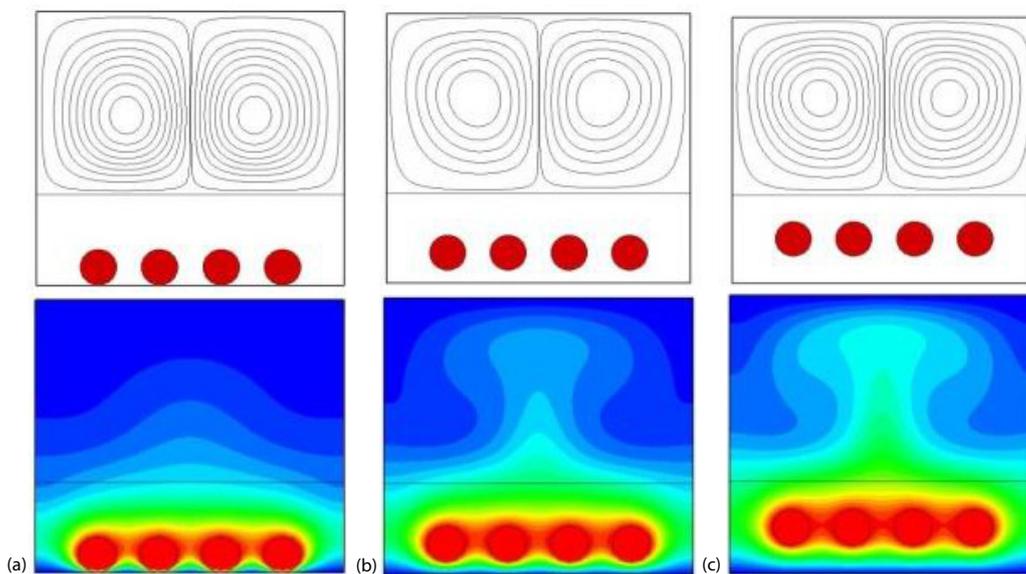


Figure 6. Streamlines (top row) and isotherms (bottom row) for different height of wires at  $Ra = 10^5$ ,  $d = 30$  and  $r = 9$ ; (a)  $h = 10$ ,  $Nu_m = 0.831$ , (b)  $h = 17$ ,  $Nu_m = 1.689$ , and (c)  $h = 25$ ,  $Nu_m = 2.502$

As can be seen in fig. 6 by increasing of wires' height the Nusselt number is increased. Figure 7 shows at same wires' height and distance the  $r = 9$  has a maximum Nusselt number and the  $r = 5$  has a minimum Nusselt number.

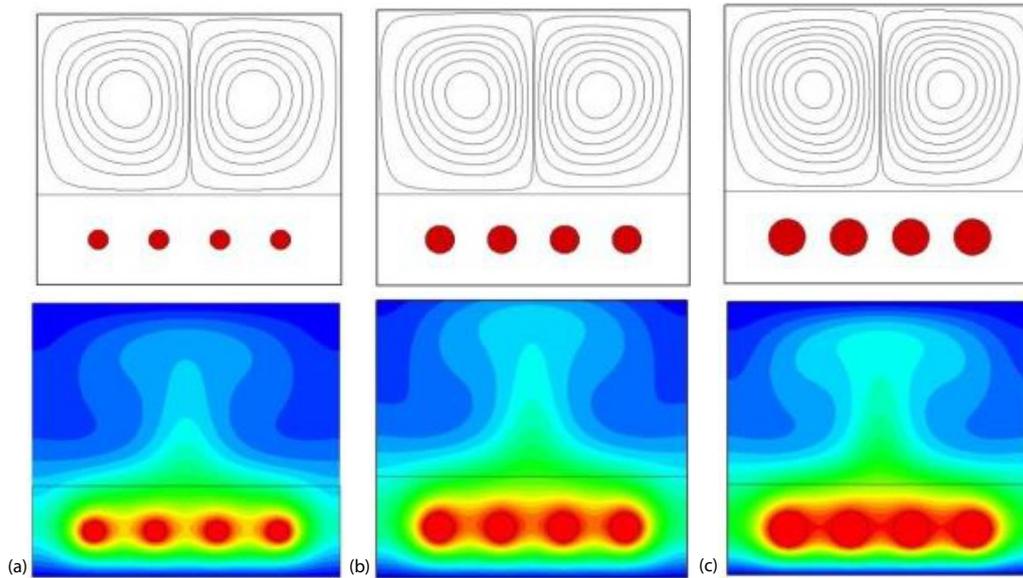


Figure 7. Streamlines (top row) and isotherms (bottom row) for different radius at  $Ra = 10^5$ ,  $d = 30$  and  $h = 25$ ; (a)  $r = 5$ ,  $Nu_m = 1.843$ , (b)  $r = 7$ ,  $Nu_m = 2.142$ , and (c)  $r = 9$ ,  $Nu_m = 2.502$

### Application of the Taguchi method

First proposed by Taguchi in 1960, the quality design is widely applied because of its proven success in improving industrial product quality greatly [24]. In addition this, low trial numbers, obtaining the effects of process parameters on quality characteristics and their optimum levels have easily increased its popularity. This method uses the special design of orthogonal arrays to learn the whole parameters space with small number of experiments. Taguchi method employs a signal-to-noise (S/N) ratio to measure the present variation. Also, in calculation procedure the effect of different control factors and their interaction was assumed. In Taguchi designs, a measure of robustness used to identify control factors that reduce variability in a product or process by minimizing the effects of uncontrollable factors (noise factors). The definition of S/N ratio differs according to an objective function, *i. e.*, a characteristic value. There are three kinds of characteristic value: nominal is best (NB:  $n = 10 \text{ Log}_{10}$  [square of mean/variance]), smaller is better (SB:  $n = 10 \text{ log}_{10}$  [mean of sum of squares of measured data]), and larger is better (LB:  $n = -10 \text{ log}_{10}$  [mean of sum squares of reciprocal of measured data]). In this study, the effects of different parameters, such as distance, height and arrangement of vertical obstacles inside the cavity on heat transfer rate (Nusselt number) have been determined and optimum factor levels have been obtained by analyzing Taguchi design. To get more accurate in terms of the heat transfer rate, the Nusselt numbers over the wall of the cavity for various designated trial have been calculated. The specified factors and their levels are depicted in and tab. 4.

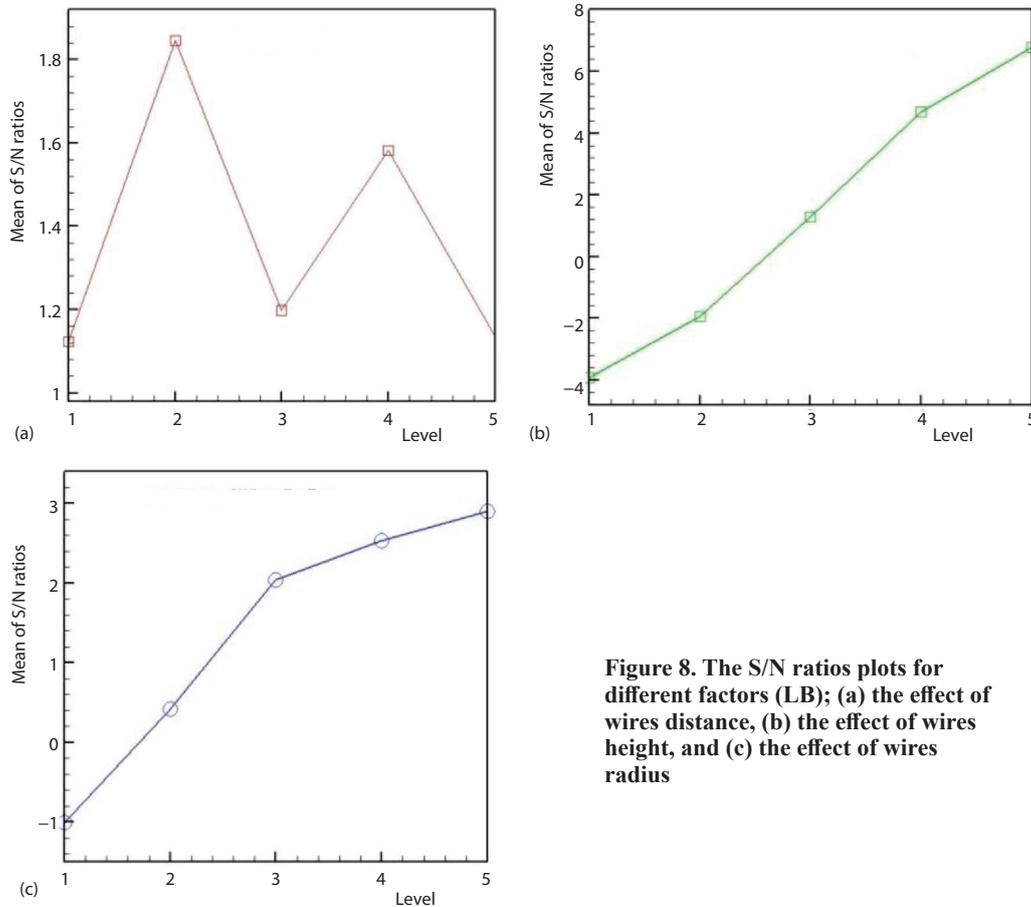
Table 4. Factors and their levels of simulation

Factors	The distance of wires, $d$					The height of wires from the bottom wall, $h$					The radius of wires, $r$				
	Levels	25	30	35	40	43	10	13	17	22	25	5	6	7	8
No.	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5

In the present study which takes three factors with five levels an L25 orthogonal array was chosen as shown in tab. 5. Taguchi method employs a S/N ratio to measure the present variation. As the objectives of this study are the maximization and minimization of heat transfer rate (Nusselt number), LB and SB are chosen. The S/N ratios plots for different factors are given in figs. 8 and 9.

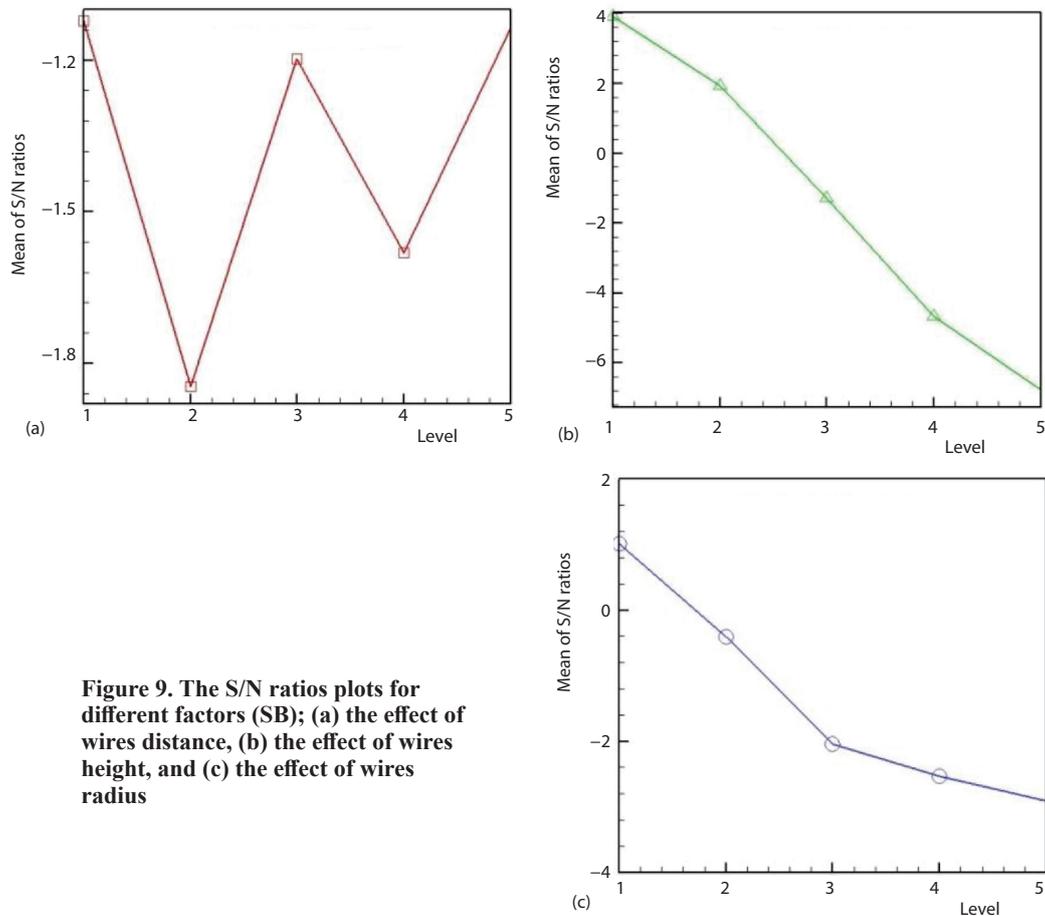
**Table 5. The L25 Orthogonal array with control factors**

Trial No.	1	2	3	4	5	6	7	8	9	10	11	12	13
d	1	1	1	1	1	2	2	2	2	2	3	3	3
h	1	2	3	4	5	1	2	3	4	5	1	2	3
r	1	2	3	4	5	2	3	4	5	1	3	4	5
Nu	0.482	0.721	1.290	1.843	2.310	0.580	0.837	1.506	2.147	1.843	0.656	0.889	1.381
Trial No.	14	15	16	17	18	19	20	21	22	23	24	25	
d	3	3	4	4	4	4	4	5	5	5	5	5	
h	4	5	1	2	3	4	5	1	2	3	4	5	
r	1	2	4	5	1	2	3	5	1	2	3	4	
Nu	1.198	2.065	0.739	1.003	0.863	1.626	2.389	0.775	0.609	0.903	1.912	2.358	



**Figure 8. The S/N ratios plots for different factors (LB); (a) the effect of wires distance, (b) the effect of wires height, and (c) the effect of wires radius**

The final step in verifying the results based on Taguchi design is the confirmation test. Figures 8 and 9 show the numerical simulation results for optimal factor settings. It can be concluded that Taguchi method achieves the statistical assessment of maximum and minimum heat transfer rate of natural-convection in an enclosure. As shown in fig. 8, the optimum settings of control factors that maximize the Nusselt number are  $d = 30$ ,  $h = 25$ , and  $r = 9$ . By the view of the fig. 9, the optimum settings of control factors that minimize the Nusselt number are  $d = 25$ ,  $h = 10$ , and  $r = 5$ .



**Figure 9. The S/N ratios plots for different factors (SB); (a) the effect of wires distance, (b) the effect of wires height, and (c) the effect of wires radius**

## Conclusions

In this article, effect and optimization of different parameters of natural-convection in a 2-D cavity with EF joint on maximum and minimum Nusselt number were investigated through Taguchi method. The practical benefit of this study is that the use of obtained optimum condition increases the maximum heat transfer rate and decreases the minimum heat transfer rate. The simulation is numerically predicted by using LBM. The experiments were planned as per Taguchi's L25 orthogonal array with each trial performed under different conditions of distance of wires,  $d$ , height of wires,  $h$ , and radius of them,  $r$ . In conclusion, some of the main points are briefly remarked:

- In comparison with conventional CFD method, using LBM to simulate natural-convection in the present study having a simple calculation procedure.
- Comparisons of the results with previous work at low and high Rayleigh numbers show that a very good agreement has been obtained.
- Based on the S/N ratio plots, all control factors have significant effect on the quality characteristic statistically.
- The maximum Nusselt number was obtained on the second levels of wires' distance, ( $d = 30$ ), top height of obstacles ( $h = 25$ ), and last radius of wires ( $r = 9$ ).
- The minimum Nusselt number was obtained on the first levels of wires' distance ( $d = 25$ ), down height of wires ( $h = 10$ ) and first radius of wires ( $r = 5$ ).
- Result analysis indicates that Taguchi method can be used in the optimization of natural-convection cooling successfully.

### Nomenclature

$g$  – gravitational acceleration, [ $\text{ms}^{-2}$ ]  
 $T$  – temperature, [K]  
 $u, v$  – velocities, [ $\text{ms}^{-2}$ ]  
 $x, y$  – co-ordinates, [m]  
 $\text{Nu}_l$  – local Nusselt number  
 $\text{Nu}_m$  – mean Nusselt number  
 $\text{Ra}$  – Rayleigh number, [ $=(g \beta \Delta T H^3)/(a \nu)$ ]

#### Greek Letters

$\alpha$  – thermal diffusivity, [ $\text{m}^2\text{s}^{-1}$ ]  
 $\theta$  – dimensionless temperature

#### Acronyms

DF – distribution function  
 EF – electrofusion  
 PE – polyethylene

#### Subscript

$c$  – cold  
 $h$  – hot

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