NUMERICAL STUDY OF TURBULENT MHD CONVECTION OF MOLTEN SODIUM WITH VARIABLE PROPERTIES IN A SQUARE CAVITY

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In this study, the finite volume method is used to simulate the turbulent natural convection in a square partitioned cavity. In this paper a fluid flow with Pr = 0.01 and Rayleigh numbers Ra = 10^6 and 10^7 in the presence of a magnetic field is investigated. The fluid properties are function of temperature. A parametric study is carried out using following parameters: non-dimensional different partition position from 0.2 to 0.6, non-dimensional different partition height from 0.1 to 0.4 and different Hartmann numbers from 0 to 200. It is found that Nusselt number is a decreasing function of partition height (H_p) and Hartmann number and the non-dimensional position of partitions (D_p) affects on streamlines and isotherms. It is observed that at Ra=10^6 and D_p=0.6 the Nusselt number is maximum and as Ha increases the Nusselt number tends to a constant number. Also at Ra=10^7 and D_p=0.4 the variation of mean Nusselt number for different partition heights is more different than the other cases. Also the Nusselt number at H_p=0.4 is nearly half for D_p=0.4 in comparison with the other cases.

Keywords: numerical study, MHD convection, molten sodium, square cavity.

1. Introduction

The natural convection of an electrically conducting fluid in an enclosure in the presence of a magnetic field has been thoroughly studied by several researchers in the laminar flow Ra < 10^6 [1–6]. Comparatively little attention has been paid to the turbulent flow, Ra ≥ 10^6, which is also of interest for many industrial processes.

A numerical two-dimensional study is carried out on the laminar and turbulent natural convection of sodium in a square enclosure heated from one vertical wall and cooled from an opposing vertical wall was studied by Jalil and Al-Tae’y [7]. They showed that the effect of a nonuniform magnetic field differs from that of a uniform magnetic field. The flow profile changes dramatically with external magnetic strength and direction, but this change in flow region depends on the distribution of magnetic field strength along magnetic pole faces.

Kakarantzas et al. [8] investigated laminar and turbulent regimes of a liquid metal flow in a vertical annulus under constant horizontal magnetic field numerically. They have considered that flow is driven by using the volumetric heating and temperature difference between the two cylindrical walls. Their results illustrated that when the magnetic field increases, the flow becomes laminar. Also, they observed that the magnetic field causes the loss of axisymmetry of flow.
Liu et al. [9] adopted the large-eddy-simulation (LES) technique with the dynamic sub-grid scale (SGS) model for body-fitted grids to investigate turbulent melt convection in an ellipsoidal crucible. Numerical comparisons were carried out for three configurations: without magnetic field, with a transverse magnetic field (TMF) and with a cusp-shaped magnetic field (CMF).

Direct numerical simulation results of magnetohydrodynamic liquid metal flow between two vertical coaxial cylinders under the effect of internal heating and a horizontal magnetic field was presented by Kakarantzas et al. They showed that the heat sources create bi-cellular flow patterns as the maximum temperature in inside the fluid bulk. The flow is azimuthally asymmetric due to the Hartmann and Robert layers formed on the walls normal and parallel to the magnetic field, respectively. [10]

Zhang et. al. [11] studied Two-dimensional turbulent convection in a toroidal duct of a liquid metal blanket of a fusion reactor. They found the turbulence results in stronger mixing and more uniform distribution of wall heat flux, indicating promising potential of this concept of the blanket.

Sajjadi and Kefayati [12] investigated MHD Turbulent and Laminar Natural Convection in a Square Cavity utilizing Lattice Boltzmann Method. The main aim of this study is to identify the ability of the LBM for solving turbulent MHD natural convection with a simple method. The MHD natural convection in a square cavity which was filled with a fluid with Pr=6.2 for different flow regimes with various Rayleigh numbers were studied.

Natural convection in a differentially laterally heated vertical cylindrical reactor was numerically studied using ANSYS FLUENT in a 2-D axisymmetric configuration by Hooman Enayati. The main goal of this paper was to define and study the boundaries of the transitional flow regime leading to turbulent flow for this thermal configuration. Seven cases for a range of Rayleigh numbers from 750 to $8.8 \times 10^8$ were studied by using the FLUENT k–ω SST turbulent model. [13]

This paper is devoted to the study of turbulent convection heat transfer $Ra = 10^6$ and $Ra = 10^7$ in molten sodium under the influence of uniform magnetic field. The (k–ε) turbulence model was used. The effect of a magnetic field and partition height and its position on average Nusselt number, isotherms and streamlines is investigated.

2. Governing Equations

The geometry of the present problem is shown in Fig. 1. It consists of a two-dimensional cavity with height H. The temperatures of the two sidewalls of the cavity are maintained at $T_h$ and $T_c$, where $T_c$ has been considered as the reference condition. The horizontal walls are assumed to be insulated, non-conducting, and impermeable to mass transfer.

Magnetic field of strength $B_0$ is applied longitudinally. The top and bottom walls are insulated and the fluid is isothermally heated and cooled by the left and right walls at uniform temperatures of $T_h$ and $T_c$, respectively.
Dimensionless variables in the analysis are defined as:

\[
\begin{align*}
X &= \frac{x}{H} , \quad Y = \frac{y}{H} , \quad H_p = \frac{h_p}{H} , \quad D_p = \frac{d_p}{H} , \quad U = \frac{uH}{\alpha} , \quad V = \frac{vH}{\alpha} , \quad P = \frac{\rho H^2}{\rho \alpha^2} , \quad \theta = \frac{T - T_h}{T_h - T_c} \\
\varepsilon^* &= \frac{\varepsilon H^4}{\alpha^3} , \quad \sigma^* = \frac{\sigma}{\sigma_r} , \quad \mu_{\text{eff}}^* = \frac{\mu_{\text{eff}}}{\mu_r} , \quad \mu_j^* = \frac{\mu_j}{\mu_r} , \quad \Gamma_{\text{eff},x}^* = \frac{\Gamma_{\text{eff},x}}{\mu_r} , \quad \kappa^* = \frac{\kappa H^2}{\alpha^2} , \quad \Gamma_{\text{eff},k}^* = \frac{\Gamma_{\text{eff},k}}{\mu_r} , \quad \Gamma_{\text{eff},\ell}^* = \frac{\Gamma_{\text{eff},\ell}}{k_r}
\end{align*}
\]

Where, \(u\) and \(v\) are the velocity components, \(p\) is the pressure, \(T\) is the temperature, \(\alpha\) is the thermal diffusivity, \(\rho\) is the density, \(\mu_{\text{eff}} = \mu + \mu_j\), and \(\Gamma_{\text{eff}} = k_j\). \(\Gamma_{\text{eff}}\) is turbulent viscosity that related to \(k\) and \(\varepsilon\) [14].

According to the above dimensionless variables, the governing equations in this study are based on the conservation laws of mass, linear momentum, energy and magnetic induction are given in dimensionless form as:

\[
\begin{align*}
&\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \\
&U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + 2\Pr R \left(\frac{\partial \mu_{\text{eff}}}{\partial X} + \frac{\partial \mu_{\text{eff}}}{\partial Y}\right) + \Pr \left(\frac{\partial \mu_{\text{eff}}}{\partial X} + \frac{\partial \mu_{\text{eff}}}{\partial Y}\right) \\
&U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr \left(\frac{\partial \mu_{\text{eff}}}{\partial X} + \frac{\partial \mu_{\text{eff}}}{\partial Y}\right) + 2\Pr \left(\frac{\partial \mu_{\text{eff}}}{\partial X} + \frac{\partial \mu_{\text{eff}}}{\partial Y}\right) + \text{RaPr} \theta - \frac{\varepsilon H^4}{\alpha^3} \text{PrV} \\
&\frac{\partial (\varepsilon H^4 U \theta)}{\partial X} + \frac{\partial (\varepsilon H^4 V \theta)}{\partial Y} = \frac{\partial}{\partial X} \left(\Gamma_{\text{eff},X}^* \frac{\partial \theta}{\partial X}\right) + \frac{\partial}{\partial Y} \left(\Gamma_{\text{eff},Y}^* \frac{\partial \theta}{\partial Y}\right) + \frac{\partial P' \theta}{\partial X} + \frac{\partial P' \theta}{\partial Y} - \varepsilon \\
&\Pr \left(\frac{\partial \mu_{\text{eff}}}{\partial X} + \frac{\partial \mu_{\text{eff}}}{\partial Y}\right) + \frac{\partial}{\partial X} \left(\Gamma_{\text{eff},X}^* \frac{\partial \theta}{\partial X}\right) + \frac{\partial}{\partial Y} \left(\Gamma_{\text{eff},Y}^* \frac{\partial \theta}{\partial Y}\right) + \frac{\partial P' \theta}{\partial X} + \frac{\partial P' \theta}{\partial Y} - \varepsilon
\end{align*}
\]
\[ \frac{\partial}{\partial X}(U' \epsilon') + \frac{\partial}{\partial Y}(V' \epsilon') = \frac{\partial}{\partial X} \left( \Gamma^{*} \frac{\partial \epsilon'}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \Gamma^{*} \frac{\partial \epsilon'}{\partial Y} \right) + P' + G' + D' \]
\[ P' = C' \left[ 2 \mu' \left( \frac{\partial U}{\partial X} \right)^2 + \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)^2 \right] \frac{\partial \epsilon'}{\partial \epsilon}. \]
\[ G' = C'_C \rho_p \nu_R \left( \frac{\partial \theta}{\partial Y} \right)^2. \]

Pr, Ra, and Ha are the Prandtl, Rayleigh and Hartmann numbers, respectively are defined as follows:

\[ Pr = \frac{\nu}{\alpha}, \quad Ra = \frac{g \beta (T_h - T) H^3}{\alpha \nu}, \quad Ha = B_0 H \frac{\sigma}{\rho \nu} \]

(8)

Where, g is the gravitational acceleration, \( \nu \) is the kinematic viscosity, \( \beta \) is the coefficient of thermal expansion, \( B_0 \) is the magnitude of magnetic field and \( \sigma \) is the electrical conductivity. It is noted that \( Ha^2 prV \) in equation (4) is achieved by simplifying the Lorentz force term \( (J \times B) \) using constant magnetic field. The magnetic Reynolds number \( (Re_m) \) is very small in most of the engineering applications so that the magnetic field, \( B \), is unchanged by the flow [15,16].

Also the used constants in above equations are presented in Table 1 as follows:

### Table 1. Constants used in k - \( \varepsilon \) model

<table>
<thead>
<tr>
<th>C_1</th>
<th>C_2</th>
<th>C_3</th>
<th>C_( \mu )</th>
<th>( \sigma_k )</th>
<th>( \sigma_i )</th>
<th>( \sigma_\varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.44</td>
<td>1.92</td>
<td>( \tanh \left( \frac{\nu}{U} \right) )</td>
<td>0.09</td>
<td>1.0</td>
<td>1.0</td>
<td>1.3</td>
</tr>
</tbody>
</table>

In order to compare total heat transfer rate, Nusselt number is used. The local and average Nusselt numbers are defined as follows:

\[ Nu_y = \left. \frac{-\partial \theta}{\partial X} \right|_{X=0}, \quad \bar{Nu} = \int_0^1 Nu_y dY \]

(9)

Boundary conditions are:

\[ U \& V = 0 \text{ at all walls } (X = 0, X = 1, Y = 0, Y = 1) \]

(10)

\[ \theta(0, Y) = 1, \theta(1, Y) = 0, \left. \frac{\partial \theta}{\partial Y} \right|_{Y=0} = 0, \left. \frac{\partial \theta}{\partial Y} \right|_{Y=1} = 0 \]

(11)
On the partitions: 
\[
\frac{\partial \theta}{\partial X} = 0, \quad \frac{\partial \theta}{\partial Y} = 0
\] 

Also wall functions are used for turbulent parameters as follows [17]: 
\[
y^+ = \frac{C_{\mu}^{1/4} k^{1/2}}{\mu} y, \quad k' = 0.41
\]

\[
T^+ = -\frac{\rho \kappa^{1/2} (c_\mu)^{1/4}}{\sigma'_t \left( \frac{1}{k'} \ln(Ey^+) + CT \right)}, \quad CT = 9.24 \left[ 1 + 0.28 \exp \left( -0.007 \frac{Pr}{\sigma'_t} \right) \right] \left( \frac{Pr}{\sigma'_t} \right)^{1/4} - 1
\]

### 3. Numerical Method

The governing equations associated with the boundary conditions are solved numerically, employing a finite volume method. In order to couple the velocity field and pressure in the momentum equations, the well-known SIMPLER-algorithm [18] is adopted. The hybrid-scheme, which is a combination of the central difference scheme and the upwind scheme, is used to discretize the convection terms. A staggered grid system [18], in which the velocity components are stored midway between the scalar storage locations, is used. The solution of the fully coupled discretized equations is obtained iteratively using the TDMA method [18]. Special consideration for the number of grid points is given so that the narrow Hartmann boundary layers are adequately covered at the boundaries. To check the grid independency, the simulations were carried on for cell numbers ranging from 31×31 to 151×151 for Ra=10⁷ and Ha=100. It was found that using the 121×121 mesh leads to the grid independent result. In Table 2 the results of \( \psi_{\text{max}} \) and \( \overline{Nu} \) at the hot wall are presented. As it can be seen, changing the grid from 101×101 to 121×121 will make approximately 0.1 percent difference in the mentioned results. According to the grid dependency analysis, all results presented from now on, are generated by 121×121 grid (Fig. 2).

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>( \overline{Nu} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>51×51</td>
<td>102</td>
</tr>
<tr>
<td>61×61</td>
<td>3.227</td>
</tr>
<tr>
<td>91×91</td>
<td>3.199</td>
</tr>
<tr>
<td>101×101</td>
<td>3.187</td>
</tr>
<tr>
<td>121×121</td>
<td>3.183</td>
</tr>
</tbody>
</table>
4. Results and Discussion

4.1. Validations

In order to check the accuracy of the numerical technique employed for the solution of the problem considered in the present study, it was validated by performing simulation for magnetoconvection flow in a square enclosure with horizontal temperature gradient and in the presence of magnetic field which were reported by Jalal M. Jalil [7]. Fig. 3 plots the streamlines and isotherms for the present solution and the results published by Jalil for $Ra = 10^{10}$, $Ha=0$ and $Pr=0.01$.

![Streamlines and isotherms](image)

(a) Jalil's work  
(b) present work

Figure 3. Streamlines and isotherms of natural convection in a square enclosure for $Ra = 10^{10}$ and $Ha=0$ (the fluid properties are constant)

In this comparison the properties of fluid were assumed to be constant and the induced magnetic field due to the motion of the electrically conducting fluid was neglected. It is observed that results show good agreement with the Jalil's work.

Turbulent natural convection heat transfer of molten sodium inside a partitioned enclosure in the presence of magnetic field is investigated. The fluid properties are function of sodium temperature. The Rayleigh numbers $10^6$ and $10^7$, different Hartmann numbers, $H_p=0.1,0.2,0.3,0.4$ and $D_p=0.2,0.4,0.6$ are considered.
4.2. Flow field and heat transfer for $Ra=10^6$

For $Ra=10^6$ and $H=0.124$m, the difference between temperature of hot wall and cold wall will be 16 °C and this causes the variation of fluid properties to be rather low.

Fig. 4 presents the isotherms and streamlines for $Ra=10^6$, $H_p=0.2$, $D_p=0.2$ and Hartmann numbers between 0 and 400. In Fig. 4 (a) it is shown that at $Ha=0$ in addition to main flow two secondary vortices and also two counter clockwise vortexes on the partitions are formed. As $Ha$ increases the secondary vortices disappeared and concentration of streamlines near hot and cold wall disappeared but again at $Ha=400$ the weak secondary vortices at center of enclosure are observed.

![Streamlines and Isotherms](image)

**Figure 4. Streamlines (a) and isotherms (b) for $D_p=0.2$, $Ra=10^6$ and different Hartmann numbers**

In Fig. 4 (b) it is seen that there is a temperature stratification between two partitions in the vertical direction and the isotherms in the center of the enclosure are restricted to the partitions. By applying magnetic field the temperature stratification diminishes and as $Ha$ increases the temperature gradient becomes lower.

Fig. 5 presents the streamline and isotherm plots for Rayleigh number $10^6$, $H_p=0.2$, $D_p=0.4$, and Hartmann numbers between 0 and 400.

In Fig. 5 (a) it is shown that at $Ha=0$ two counterclockwise vortices are formed behind the partitions. As $Ha$ increases near the left and right walls the Lorentz force is induced in opposite direction of the buoyancy force and causes the centerline of the main vortex rotates in counterclockwise. At high Hartmann number ($Ha=400$) two secondary vortices are formed at the center of enclosure. It is evident from Fig. 5 (b) at $Ha=0$ the isotherms between the partitions are skewed that indicates the convection is dominant in this region but by applying magnetic field and increasing the Hartmann number the isotherms will be vertical and heat transfer between two side walls is occurred by conduction.
Figure 5. Streamlines (a) and isotherms (b) for $D_p=0.4$, $Ra=10^6$ and different Hartmann numbers.

Fig. 6 presents the streamline and isotherm plots for $D_p=0.6$.

In Fig. 6 (a) it is seen that when $Ha=0$, the centerline of primary vortex is horizontal and secondary vortices are not formed inside the main flow but two large vortices behind the partitions appeare. It is evident from Fig. 6 (b) at low Hartmann numbers the concentration of isotherms near hot and cold walls for $D_p=0.6$ is more than other cases and this means that the heat transfer in this case is the most.
Fig. 7 depicts the variation of the mean Nusselt number with Hartmann number versus several partition heights for $D_p=0.2$, $D_p=0.4$, $D_p=0.6$ and $Ra=10^6$. Generally it is observed that as Hartmann number and partitions height increase the mean Nusselt number decreases. In low Hartmann numbers deviation between Nusselt numbers for different $H_p$ is high and with increasing the magnetic field strength this deviation decreases. Also it is seen that at $D_p=0.6$ the Nusselt number is maximum and as $Ha$ increases the Nusselt number tends to a constant number.

**Figure 7.** Variation of the mean Nusselt number with Hartmann number for $Ra=10^6$ and different non-dimensional partition heights and positions: (a) $D_p=0.2$, (b) $D_p=0.4$, (c) $D_p=0.6$

4.3. Flow field and heat transfer for $Ra=10^7$

Fig. 8 presents the isotherms and streamlines for $Ra=10^7$, $H_p=0.2$, $D_p=0.2$ and Hartmann numbers between 0 and 750.
In Fig. 8 (a) it is shown that at Ha=0 in addition to secondary vortices that are formed inside the primary vortex, two counterclockwise vortices above the partitions are appeared. As Ha increases these secondary vortices become small so that at high Hartmann numbers they are disappeared. It is evident from Fig. 8 (b) the thermal boundary layer is formed near the hot and cold walls and the convection is dominant mechanism of heat transfer. As Ha increases the stratification of isotherms at center of enclosure becomes lower and at high Hartmann numbers the isotherms will be vertical.

Fig. 9 presents the isotherms and streamlines for Ra=10^7, H_p=0.2, D_p=0.4 and Hartmann numbers between 0 and 750.

In Fig. 9 (a) it is seen that at Ha=0 as well as the secondary vortices inside the main flow, two vortices behind and above the partitions are formed. At Ha=150 the vortices above the partitions become smaller and the other become bigger and the secondary vortices inside the primary vortex are disappeared. Fig. 9 (b) indicates that effect of the partitions in blocking the fluid flow and the reducing heat transfer is more than the other cases and the stratification of the isotherms at low Hartmann numbers is less so that at high Hartmann numbers the conduction mode is dominant in enclosure.

Fig. 10 presents the isotherms and streamlines for Ra=10^7, H_p=0.2, D_p=0.6 and Hartmann numbers between 0 and 750.

In Fig. 10 (a) it is depicted that when Ha=0 in addition to primary flow two vortices behind the partitions are formed. At Ha=150 these vortices become larger and weaker but as Ha increases they are disappeared. At Ha=900 two secondary vortices are seen in the primary vortex. It is evident from Fig. 10 (b) at Ha=0 the thermal boundary layer is formed near the hot and cold walls and as Ha increases the stratification of isotherms and temperature gradient near the vertical walls reduces and at high Hartmann numbers the heat transfer occurs by conduction.
Figure 9. Streamlines (a) and isotherms (b) for $D_p=0.4$, $Ra=10^7$ and different Hartmann numbers

Figure 10. Streamlines (a) and isotherms (b) for $D_p=0.6$, $Ra=10^7$ and different Hartmann numbers

Fig. 11 depicts the variation of the mean Nusselt number with Hartmann number versus several partition heights for $D_p=0.2$, $D_p=0.4$, $D_p=0.6$ and $Ra=10^7$. 
It is observed that for $D_p=0.4$ the variation of mean Nusselt number for different partition height is more different than the other cases. Also it is observed that the Nusselt number at $H_p=0.4$ is nearly half for $D_p=0.4$ in comparison with the other cases. Because at $Ra=10^7$ the advection is large and when $H_p=0.4$ the flow blockage is the most and the enclosure is divided to two hot and cold zones, so the heat transfer is less than the other cases.

5. Conclusions
- Generally it is observed that as Hartmann number and partitions height increase the mean Nusselt number decreases.
- For $Ra=10^6$ in low Hartmann numbers deviation between Nusselt numbers for different $H_p$ is high and with increasing the magnetic field strength this deviation decreases. Also it is seen that at
D_φ=0.6 the Nusselt number is maximum and as Ha increases the Nusselt number tends to a constant number.
- For Ra=10^7 it is observed that at D_φ=0.4 the variation of mean Nusselt number for different partition height is more different than the other cases. Also it is observed that the Nusselt number at H_p=0.4 is nearly half for D_φ=0.4 in comparison with the other cases.

**Nomenclature**

- \( g \): acceleration due to gravity
- \( H \): height of the cavity
- \( P \): pressure
- \( Pr \): prandtl number
- \( Ra \): Rayleigh number
- \( T \): temperature
- \( u, v \): velocity components
- \( k \): thermal conductivity
- \( K \): permeability of the porous medium
- \( Nu \): local Nusselt number
- \( \bar{Nu} \): mean Nusselt number
- \( H_p \): partition height
- \( D_p \): partition position
- \( y_p \): distance of a point from wall.
- \( T^+ \): wall function for temperature

**Greek Symbols**

- \( \beta \): coefficient of thermal expansion
- \( \alpha \): thermal diffusivity
- \( \rho \): density
- \( \theta \): dimensionless temperature
- \( \mu \): dynamic viscosity
- \( \nu \): kinematic viscosity
- \( \mu_t \): turbulent viscosity
- \( \varepsilon \): dissipation rate
- \( K \): turbulent energy

**Subscripts**

- \( c \): cold
- \( h \): hot
- \( f \): fluid
- \( s \): solid

**Reference**


Submitted: 01.11.2017
Revised: 04.02.2018
Accepted: 31.02.2018