A generalized Fourier and Fick's perspective for stretching flow of Burgers fluid with temperature-dependent thermal conductivity

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Abstract: This research addresses heat generation and mixed convection characteristics in Burgers fluid flow induced by moving surface considering temperature-dependent conductivity. The novel revised Fourier-Fick relations covering heat/mass paradoxes are introduced simultaneously. Boundary layer concept is implemented for simplification of mathematical model of considered physical problem. Compatible transformations are utilized to transform partial differential system into ordinary ones. The idea of homotopic scheme is employed to establish convergent series solutions. The mechanisms of heat/mass transportation are elaborated graphically by constructing graphs for distinct values of physical constraints. We noticed higher temperature and concentration for Fourier-Fick situations when compared with revised Fourier-Fick situations. Furthermore, an increment in variable conductivity factor yields higher temperature and related thickness of thermal layer. The obtained results are compared with available literature in a limiting manner and reasonable agreement is found.

Keywords: Revised Fourier-Fick relations; heat generation; Burgers fluid; Mixed convection; Temperature-dependent conductivity.

1 Introduction

Non-Newtonian liquids like toothpaste, paint, animal blood, milk, grease are naturally omnipresent and extensively utilized in oil exploration, food processing, medical, chemical and bio-chemical engineering [1-5]. The non-Newtonian liquids in general are categorized in rate, differential and integral types. Rate type models elaborates relaxation/retardation times characteristics. Burgers liquid among rate-type liquids is the generalization of Maxwell [6] and Oldroyd-B [7] models predominantly effective for polymers, asphalt concrete and reaction of asphalt [8]. Burgers model is considered and modeled utilizing distinct aspects. For illustration, Alsaedi et al. [9] formulated chemically reacted flow in the stagnation region of Burgers liquid. Thermal radiation impact in chemically reacted Burgers liquid flow towards heated surface is presented by Khan et al. [10]. Hayat et al. [11-14] established homotopic solutions for Burgers liquid considering various aspects. Recently, heat generation and gyrotactic microorganisms effects in magneto mixed convective Burgers liquid are addressed by Khan et al. [15].

The traditional heat conduction relation (Fourier relation [16]) communicates heat flux precisely to temperature gradient utilizing coefficient of thermal conductivity. Fourier relation is not effective for the problems which comprise high thermal gradient, absolute zero temperatures, small variations in temperature and nano/micro scales in space and time [17]. Several theories regarding improvement in Fourier heat conduction relation have been introduced. The situations in microelectronic devices like high frequency heating laser pulse, combined circuit chips, high flux for cutting and melting of objects and in few non-homogeneous objects, the heat conduction through revised Fourier relation is very consequential [18, 19]. Christov [20] revisited the analysis of Cattaneo [18] for material-invariant formulation by including the relaxation time contribution comprising upper-convected Oldroyd's derivatives. Afterwards, several researches in this direction have been reported (for detail see [21-30]).

Keeping aforestated analyses in mind, it is noticed that energy expression through revised Fourier relation is reported extensively. However concentration expression by revised Fick relation is not yet studied. Thus our objective here in this investigation is to venture further in this regime by considering mixed convective Burger liquid flow bounded by moving surface. Heat generation and variable conductivity aspects are retained in energy expression. Homotopy scheme [31-45] is opted for computations of nonlinear systems. The outcomes of presented analysis are displayed and discussed.

2 Formulation

We aim to formulate mixed convective laminar flow of incompressible Burgers liquid towards moving surface subject to revised Fourier-Fick relations. Heat generation and thermal dependence choice of conductivity aspects are considered for energy expression formulation. Whole analysis is addressed by ignoring viscous dissipation and thermal radiation contributions. Detailed flow assumptions can be understood through Fig. 1. We have the following governing expressions [1]:

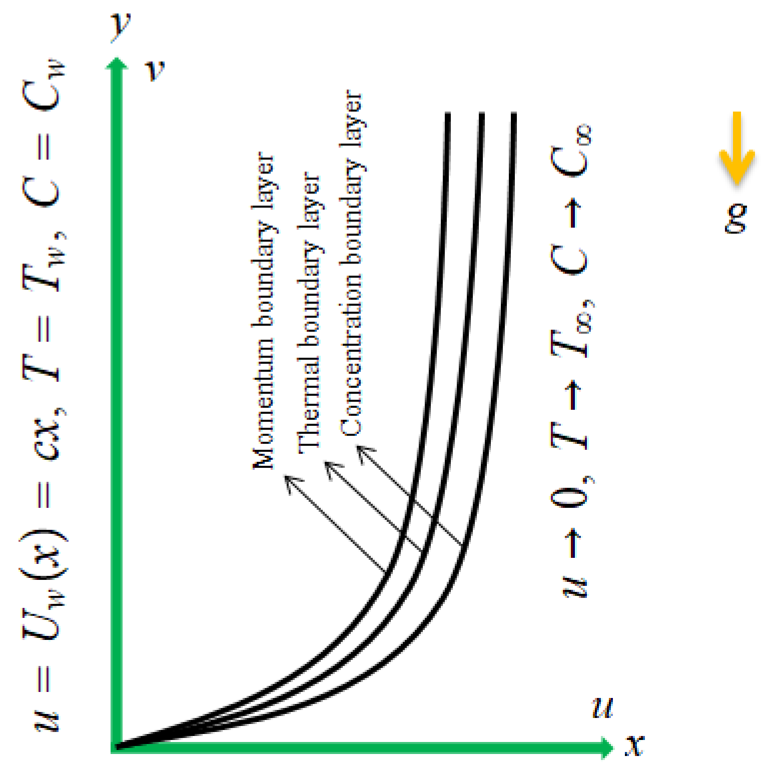

Fig. 1. Physical configuration.

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\begin{aligned}
&u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)\\
&+\lambda_2\left[\begin{matrix} u^3\frac{\partial^3 u}{\partial x^3}+v^3\frac{\partial^3 u}{\partial y^3}+u^2\left(\frac{\partial^2 u}{\partial x^2}\frac{\partial u}{\partial x}-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2}+2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}\right)\\ +3v^2\left(\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2}+\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)+3uv\left(u\frac{\partial^3 u}{\partial x^2\partial y}+v\frac{\partial^3 u}{\partial x\partial y^2}\right)\\ +2uv\left(\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2}+\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2}+\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial x\partial y}-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x\partial y}\right)\end{matrix}\right]\\
&=\nu\left[\frac{\partial^2 u}{\partial y^2}+\lambda_3\left(u\frac{\partial^3 u}{\partial x\partial y^2}+v\frac{\partial^3 u}{\partial y^3}-\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2}-\frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right)\right]\\
&+g\left(\Lambda_1(T-T_\infty)+\Lambda_2(C-C_\infty)\right),
\end{aligned} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_T\begin{pmatrix} u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x} \\ +2uv\frac{\partial^2 T}{\partial x\partial y}+u^2\frac{\partial^2 T}{\partial x^2} \\ +v^2\frac{\partial^2 T}{\partial y^2}-\frac{Q}{\rho c_p}\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)\end{pmatrix}$$

$$=\frac{1}{\rho c_p}\frac{\partial}{\partial y}\left(K\left(T\right)\frac{\partial T}{\partial y}\right)+\frac{Q}{\rho c_p}\left(T-T_\infty\right), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}+\lambda_C\begin{pmatrix} u\frac{\partial u}{\partial x}\frac{\partial C}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial C}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial C}{\partial x} \\ +2uv\frac{\partial^2 C}{\partial x\partial y}+u^2\frac{\partial^2 C}{\partial x^2}+v^2\frac{\partial^2 C}{\partial y^2}\end{pmatrix}=D\frac{\partial^2 C}{\partial y^2}, \tag{4}$$

with conditions [10]:

$$\begin{gathered} u=U_w(x)=cx,\ v=0,\ T=T_w,\ C=C_w \text{ at } y=0, \\ u\to 0,\ T\to T_\infty,\ C\to C_\infty \text{ when } y\to\infty. \end{gathered} \tag{5}$$

Note that $\left(u,v\right)$ illustrate liquid velocities in (horizontal, vertical) directions respectively, ν the kinematic viscosity, ρ the liquid density, $\left(\lambda_1,\lambda_3\right)$ the relaxation/retardation times, λ_2 the material variable of Burgers fluid, g the gravitational acceleration, $\left(\Lambda_1,\Lambda_2\right)$ the (thermal, solutal) expansion coefficients, $\left(T,C\right)$ the fluid (temperature, concentration), $\left(T_\infty,C_\infty\right)$ the ambient fluid (temperature, concentration), $\left(\lambda_T,\lambda_C\right)$ the (heat, mass) flux relaxation times, Q the heat absorption/generation coefficient, D the mass diffusion and c the stretching rate.
Variable conductivity in mathematical form is [7]:

$$K\left(T\right)=K_\infty\left(1+\varepsilon\frac{T-T_\infty}{\Delta T}\right), \tag{6}$$

in which $\Delta T=T_w-T_\infty$, K_∞ the ambient liquid conductivity and ε the small variable which identifies the characteristic of temperature for thermal dependence conductivity.
Letting [7]:

$$\begin{gathered} \eta=y\sqrt{\frac{c}{\nu}},\ u=cxf'(\eta),\ v=-\sqrt{c\nu}\,f(\eta), \\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty},\ \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}, \end{gathered} \tag{7}$$

equation (1) is justified automatically whereas Eqs. $(2)-(5)$ are reduced to the following forms:

$$\begin{aligned} & f''' + ff'' - f'^2 + \beta_1\left(2ff'f'' - f^2 f'''\right) + \beta_2\left(f^3 f^{iv} - 2ff'^2 f'' - 3f^2 f''^2\right) \\ & + \beta_3\left(f''^2 - ff^{iv}\right) + \lambda\left(\theta + N\phi\right) = 0, \end{aligned} \tag{8}$$

$$\begin{aligned} & \left(1+\varepsilon\theta\right)\theta'' + \varepsilon\theta'^2 + \Pr f\theta' + \Pr\delta\theta - \Pr\delta\gamma_1 f\theta' \\ & -\Pr\gamma_1\left(ff'\theta' + f^2\theta''\right) = 0, \end{aligned} \tag{9}$$

$$\phi'' + Scf\phi' - Sc\gamma_2\left(ff'\phi' + f^2\phi''\right) = 0, \tag{10}$$

$$f(0) = 0,\ f'(0) = 1,\ f'(\infty) \to 0, \tag{11}$$

$$\theta(0) = 1,\ \theta(\infty) \to 0, \tag{12}$$

$$\phi(0) = 1,\ \phi(\infty) \to 0, \tag{13}$$

where prime $(')$ designates differentiation with respect to η. The parameters occurring in Eqs. $(8)-(10)$ in non-dimensional forms can be described as

$$\begin{aligned} & \beta_1 = \lambda_1 c,\ \beta_2 = \lambda_2 c^2,\ \beta_3 = \lambda_3 c,\ \lambda = \frac{Gr_x}{\mathrm{Re}_x^2},\ N = \frac{Gr_x^*}{Gr_x},\ Gr_x = \frac{g\Lambda_1\left(T_w - T_\infty\right)x^3}{\nu^2}, \\ & Gr_x^* = \frac{g\Lambda_2\left(C_w - C_\infty\right)x^3}{\nu^2},\ \mathrm{Re}_x = \frac{xU_w(x)}{\nu},\ \Pr = \frac{\mu c_p}{K_\infty},\ \delta = \frac{Q}{c\rho c_p}, \\ & \gamma_1 = \lambda_T c,\ \gamma_2 = \lambda_C c,\ Sc = \frac{\nu}{D}. \end{aligned} \tag{14}$$

4 Convergence analysis

We utilized homotopy scheme $[31-45]$ for the development of convergent solutions. No doubt h-curve (s) are crucial to ensure convergence of nonlinear differential systems. Therefore we portrayed h-curves in Fig. 1 at $16th$ -order approximation for such objective. Flat portions of these curves help to achieve admissible values of $\left(\mathrm{h}_f, \mathrm{h}_\theta, \mathrm{h}_\phi\right)$. From Fig. 2 we found $-1.25 \le \mathrm{h}_f \le -0.40$, $-1.48 \le \mathrm{h}_\theta \le -0.50$ and $-1.50 \le \mathrm{h}_\phi \le -0.50$ with $\beta_1 = 0.5,$ $\beta_2 = 0.2,$ $\beta_3 = 0.45,$ $\lambda = N = \delta = 0.1,$ $\varepsilon = \gamma_1 = \gamma_2 = 0.2,$ $Sc = 1.1$ and $\Pr = 1.2.$ Furthermore, convergence is also assured numerically (see Table 1). Clearly Eqs. $(8)-(10)$ converge at 20th order approximation respectively. Besides, presented analysis is also compared with [8] and reasonable agreement is found (see Table 2).

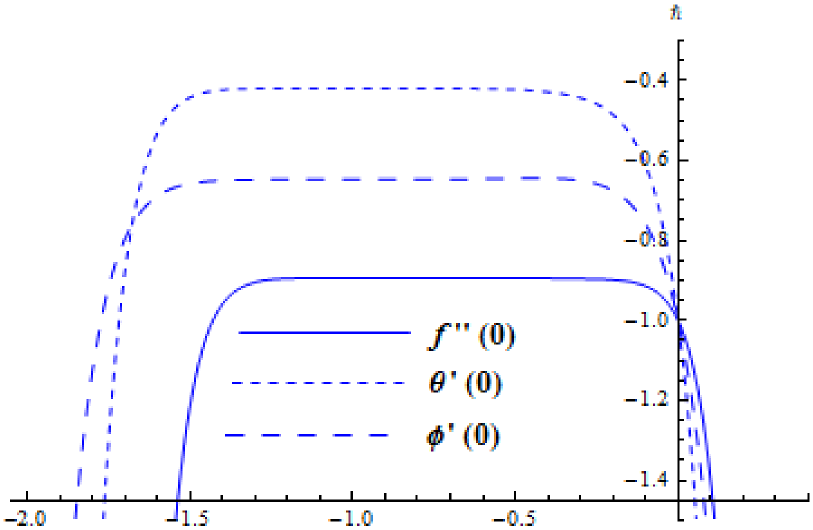

Fig. 2. h − curves for f, θ and ϕ.

Table 1: Convergence analysis of series solutions for distinct order approximations when $\beta_1 = 0.5,\ \beta_2 = 0.2,\ \beta_3 = 0.45,\ \lambda = N = \delta = 0.1,\ \varepsilon = \gamma_1 = \gamma_2 = 0.2,\ Sc = 1.1$ and $\Pr = 1.2$.

Order of approximations	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
1	0.8985	0.6239	0.7783
5	0.8953	0.4349	0.6428
10	0.8940	0.4201	0.6461
15	0.8940	0.4192	0.6476
20	0.8940	0.4192	0.6476
30	0.8940	0.4192	0.6476
40	0.8940	0.4192	0.6476

5 Analysis

This portion highlights the significant features of various variables versus temperature (θ) and concentration (ϕ). For such interest, Figs. $3-8$ are constructed and described in detail. Fig. 3

addresses the characteristics of heat generation $(\delta > 0)$ and heat absorption $(\delta < 0)$ variables against θ. Here θ increments for $\delta > 0$ whereas it illustrates opposite impact when $\delta < 0$. Heat transfers promptly for $\delta > 0$ which yield θ enhancement. Less heat amount is transferred for $\delta < 0$ which corresponds to θ reduction. The contribution of Prandtl (Pr) number versus θ is analyzed through Fig. 4. Higher Pr estimations results in lower diffusivity which consequently diminishes θ. Fig. 5 describes change in θ for variable conductivity factor (ε). As expected, larger ε leads to higher θ and associated thickness layer. In fact liquid conductivity upsurges when ε is incremented. So extra heat amount is exchanged from surface to material and thus θ is enhanced. The role of γ_1 on θ is elaborated in Fig. 6. It is seen that larger γ_1 corresponds to non-conducting behavior due to which θ decays. Furthermore, temperature (θ) is higher for $\gamma_1 = 0$ in comparison to $\gamma_1 > 0$. Fig. 7 explores Sc impact on ϕ. Here ϕ dwindles for larger estimation of Sc. In fact, Brownian diffusivity arises in Sc expression which reduces via higher estimation of Sc. Consequently, concentration (ϕ) reduces. Analysis for the γ_2 characteristics is expressed through Fig. 8. Here concentration (ϕ) diminishes when γ_2 is increased.

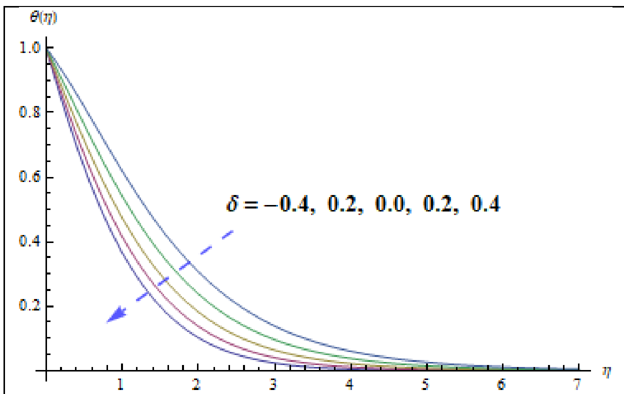

Fig. 3. θ via δ.

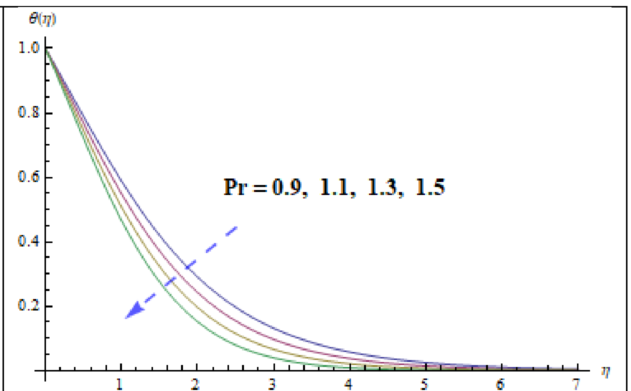

Fig. 4. θ via Pr.

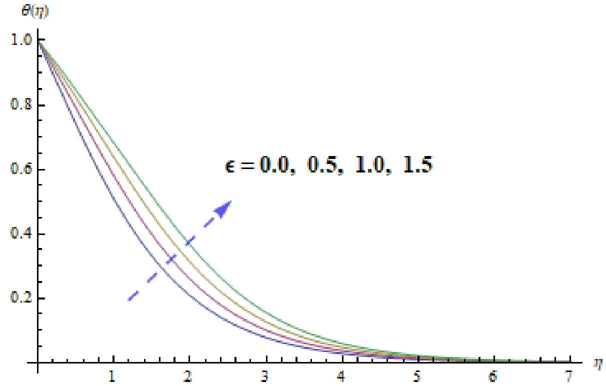

Fig. 5. θ via ε.

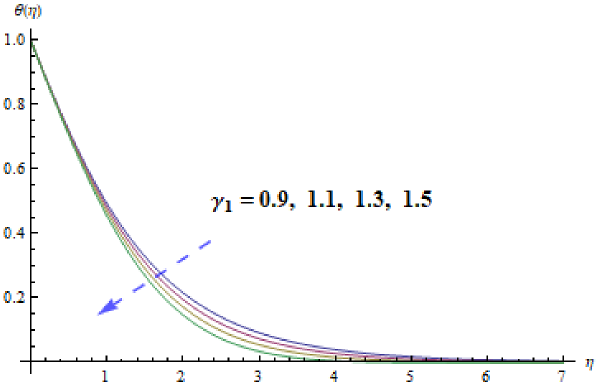

Fig. 6. θ via γ_1.

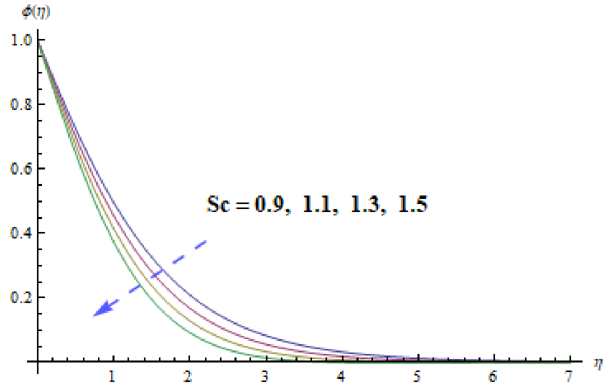

Fig. 7. ϕ via Sc.

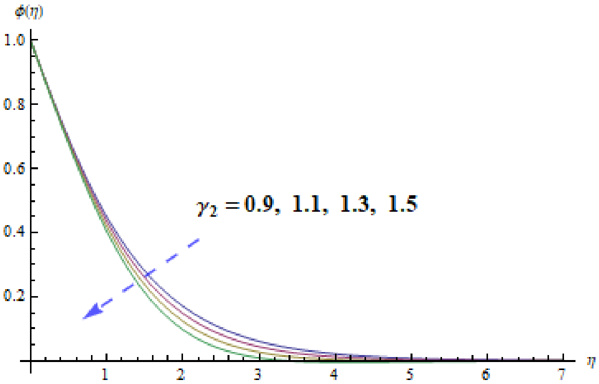

Fig. 8. ϕ via γ_2.

Table 2: Comparative outcomes of $f''(0)$ with [8] for several values of β_1 when $\beta_2 = 0 = \beta_3 = \lambda = N$.

β_1	Ref. [8]	Present
0.0	1.000000	1.000000
0.2	1.051948	1.051889
0.4	1.101850	1.101903
0.6	1.150163	1.150137

0.8	1.196692	1.196711
1.2	1.285257	1.285363
1.6	1.368641	1.368758
2.0	1.447617	1.447651

6 Final remarks

This research describes heat generation, variable conductivity and mixed convection characteristics in non-Newtonian (Burgers) fluid flow towards moving surface. Revised Fourier-Fick relations are considered for modeling energy and concentration expressions. We obtained following significant points from the aforestated analysis:

- Temperature (θ) rises when heat generation $(\delta > 0)$ and variable conductivity (ε) factors are enhanced.
- Larger Prandtl number $(\Pr)$ and thermal relaxation variable (γ_1) correspond to θ reduction.
- An increment in Schmidt number (Sc) and solutal relaxation variable (γ_2) yield lower concentration (ϕ).
- The situations regarding traditional Fourier-Fick relations can be retrieved by setting $\gamma_1 = \gamma_2 = 0$ in Eqs. (9) and (10).
- Burgers fluid model corresponds to Oldroyd-B fluid model $(\beta_3 = 0)$, Maxwell fluid model $(\beta_2 = \beta_3 = 0)$ and viscous fluid model $(\beta_1 = \beta_2 = \beta_3 = 0)$.

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