LATTICE BOLTZMANN SIMULATION OF RAYLEIGH-BENARD CONVECTION IN ENCLOSURES FILLED WITH Al₂O₃-WATER NANOFLUID

by

Weihua CAI¹, Zhifeng ZHENG¹, Changye HUANG¹, Yue WANG¹, Xin ZHENG⁵, and Hongna ZHANG¹

¹School of Energy Science and Engineering, Harbin Institute of Technology, Harbin, China
²Key Laboratory of Efficient Utilization of Low and Medium Grade Energy, Tianjin University, Ministry of Education of China, Tianjin, China
⁵Univ Lyon, CNRS, INSA-Lyon, Universite Claude Bernard, Villeurbanne, France

Original scientific paper
https://doi.org/10.2298/TSCI171023038C

In order to clarify the controversies for the role of nanoparticles on heat transfer in natural convection, lattice Boltzmann method is used to investigate Rayleigh-Benard convection heat transfer in differentially-heated enclosures filled with Al₂O₃-water nanofluids. The results for streamline and isotherm contours, vertical velocity, and temperature profiles as well as the local and average Nusselt number are discussed for a wide range of Rayleigh numbers and nanoparticle volume fractions (0 ≤ φ ≤ 5%). The results show that with the increase of Rayleigh number and nanoparticles loading, Nuₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑₑἐ መFinite elements in the field of industrial applications. These techniques can provide a more accurate representation of the geometrical complexity and defect characteristics. However, the integration of high-resolution geometric features into the finite element framework requires a significant effort in terms of computational resources. The next step towards the fully coupled approach is the development of advanced material characterization methods that can provide more accurate and reliable data for the finite element models. The research efforts in this direction are expected to lead to a better understanding of the material behavior under high-stress conditions, which is crucial for the design and optimization of the advanced pressure vessel components. These components are required for the safe and efficient operation of high-pressure systems, and their failure can cause severe economic and environmental consequences. Therefore, the development of advanced material characterization methods is not only important for the prediction of the structural integrity but also for the reduction of the lifecycle costs and the improvement of the overall performance of the pressure vessel systems.
The earlier experimental studies in natural convection with nanofluids usually showed a decrease of heat transfer. For instance, Putra et al. [8] conducted an experiment on natural convection with nanofluids inside horizontal cylinder heated from one end and cooled from the other. A systematic and definite deterioration for heat transfer in natural convection was observed. Wen and Ding [9] reported that the presence of nanoparticles systematically reduced the heat transfer coefficient in natural convection for Rayleigh number less than $10^6$, and such deterioration increases with nanoparticle concentrations. However, Ho et al. [10] investigated the heat transfer in natural convection with Al$_2$O$_3$-water nanofluids in vertical square enclosures experimentally, and the results showed the heat transfer enhancement for nanofluids containing low nanoparticle volume fraction, $\phi$. Nnanna [11] experimentally found the same trend that heat transfer is enhanced at low nanoparticle volume fraction, while free convective heat transfer is reduced for larger nanoparticle volume fraction. Theoretically, Kim et al. [12] investigated the thermal characteristics of natural convection with nanofluids in enclosures heated from below, and found that the heat transfer coefficient of nanofluids was enhanced with respect to nanoparticle volume fraction. Khanafer et al. [13] also reported an augmentation in heat transfer by the addition of nanoparticles.

Meanwhile, some studies focus on the effects of variable thermal conductivity and variable viscosity of nanofluids on heat transfer. Ho et al. [14] reported that the uncertainties associated with different formulas adopted for the effective thermal conductivity and dynamic viscosity of nanofluids have a strong bearing on the heat transfer characteristics in natural convection. Significant difference in the effective dynamic viscosity enhancement, other than that in the thermal conductivity enhancement, was found to play as a major factor, leading to the contradictory heat transfer efficacy of nanofluids. Abu-Nada et al. [15] investigated the heat transfer enhancement in a differentially heated enclosure and found that the effects the viscosity models were predicted to be more predominant on the behavior of, average Nusselt number, $\text{Nu}_{\text{ave}}$, than the influence of thermal conductivity models. However, it is still unclear on the role of nanoparticles on the heat transfer enhancement in natural convection applications. In this paper, the conventional method for calculating $\text{Nu}_{\text{ave}}$ is considered unable to reasonably evaluate the efficacy of heat transfer enhancement, which may result in the previous contradiction. Detailed analysis will be mentioned, and another scheme will be presented to solve this problem.

Rayleigh-Benard (RB) convection is a kind of natural convection heated from below and cooled from above, which has applications to many different areas, such as geophysics, astrophysics, meteorology, multilayer walls in buildings, among others. It also plays a crucial role in the development of stability theory in hydrodynamics and has been paradigmatic in pattern formation and in the study of spatial-temporal chaos [16]. For the enclosures filled with nanofluids heated from left and cooled from right, Kefayati et al. [17] used the traditional method to calculate $\text{Nu}_{\text{ave}}$ for enclosures with different aspect ratios, but this method did not reasonably consider the thermal conductivity. Abu-Nada et al. [15] proposed a new method to calculate $\text{Nu}_{\text{ave}}$ but did not consider the comparison among the results at different aspect ratios.

From the foregoing analysis, it is still controversial that how the nanofluids affect on the heat transfer in natural convection. While the puzzled results are obtained mainly for low Rayleigh number less than $10^6$ and low nanoparticle volume fractions, so RB cell filled with Al$_2$O$_3$-water nanofluid is studied for a wide range of $\text{Ra} = 10^4$-$10^6$ and nanoparticle volume fractions ($0 \leq \phi \leq 5\%$), using a single-phase lattice Boltzmann method (LBM), in the present study.
The governing equations/mathematical model

Problem statement

Figure 1 shows a schematic diagram of a differentially heated enclosure, i.e., RB cell. In the present problem, aspect ratio, $A_e$, is defined as the ratio of width to height, i.e., $A_e = W/H$. The bottom wall is heated and maintained at a constant temperature, $T_H$, while the top wall is cooled and kept at a low temperature, $T_C$. The remaining walls are adiabatic, i.e., $\partial T/\partial x = 0$. The fluid enclosed is water-based nanofluids containing $\text{Al}_2\text{O}_3$ nanoparticles. It assumes nanofluids as incompressible Newtonian fluid and the flow is 2-D and laminar. Moreover, it is idealized that both water and nanoparticles are in thermal equilibrium and they flow at the same velocity.

The lattice Boltzmann method

In the present study, a standard D2Q9 LBM is used to solve flow and temperature fields, instead of the conventional transport equations. The lattice Boltzmann model used here is the same as that adopted in [18]. The Bhatangar-Gross-Krook approximation lattice Boltzmann equation with external forces can be written, for the flow field:

$$f_i(x + e_i \Delta t, t + \Delta t) - f_i(x, t) = -\frac{1}{\tau_f} [f_i(x, t) - f_i^{eq}(x, t)] + f_g$$

For the temperature field:

$$T_i(x + e_i \Delta t, t + \Delta t) - T_i(x, t) = -\frac{1}{\tau_T} [T_i(x, t) - T_i^{eq}(x, t)]$$

For a standard D2Q9 model, the discrete velocity set for each component $i$ is:

$$e_i = \begin{cases} 
(0,0) & i = 0 \\
\sqrt{2}c \left( \cos \left(\frac{(i-1)\pi}{4}\right), \sin \left(\frac{(i-1)\pi}{4}\right) \right) & i = 1 \sim 4 \\
\sqrt{2}c \left( \cos \left(\frac{2(i-1)\pi}{4}\right), \sin \left(\frac{2(i-1)\pi}{4}\right) \right) & i = 5 \sim 8 
\end{cases}$$

where $c = 1.0$.

The density equilibrium distribution function is formulated:

$$f_i^{eq} = w_i \rho \left[ 1 + 3 \frac{(e_i \cdot u)}{c^2} + 45 \frac{(e_i \cdot u)^2}{c^4} - 15 \frac{uu}{c^2} \right]$$

The temperature equilibrium distribution function is formulated:

$$T_i^{eq} = w_i T \left[ 1 + 3 \frac{(e_i \cdot u)}{c^2} + 45 \frac{(e_i \cdot u)^2}{c^4} - 15 \frac{uu}{c^2} \right]$$
where \( w_i \) are the weight coefficients, given for D2Q9:

\[
\begin{aligned}
    w_i &= \begin{cases} 
        4/9 & \text{if } i = 0 \\
        1/9 & \text{if } i = 1 \sim 4 \\
        1/36 & \text{if } i = 5 \sim 8 
    \end{cases}
\end{aligned}
\]  

(6)

The corresponding kinematic viscosity, \( \nu \), and thermal diffusivity, \( \chi \), are defined in terms of their respective relaxation times by:

\[
\nu = \frac{1}{3} \left( \tau_f - \frac{1}{2} \right) c^2 \Delta t
\]  

(7)

\[
\chi = \frac{1}{3} \left( \tau_T - \frac{1}{2} \right) c^2 \Delta t
\]  

(8)

There are three important dimensionless parameters for natural convection, respectively defined:

\[
\text{Pr} = \frac{\nu}{\chi}
\]  

(9)

\[
\text{Ra} = \frac{g \beta \Delta T H^3 \text{Pr}}{\nu^2}
\]  

(10)

\[
\text{Ma} = \frac{\frac{u}{c_s}}{c_s}
\]  

(11)

To ensure that the flow is fully in the incompressible regime, Mach number is set to 0.1 in all the simulations.

So the dimensionless collision-relaxation time \( \tau_f \) and \( \tau_T \) can be, respectively, derived:

\[
\tau_f = 0.5 + \frac{\text{Ma} H \sqrt{3\text{Pr}}}{c \Delta t \sqrt{\text{Ra}}}
\]  

(12)

\[
\tau_T = 0.5 + \frac{3\nu}{\text{Pr} c^2 \Delta t}
\]  

(13)

In order to incorporate buoyancy force in the model, the force term adding in eq. (1) is calculated by Boussinesq approximation in vertical \( y \)-direction:

\[
f_g = 3w_i g_y \rho \beta (T - T_{ave}) e_y
\]  

(14)

The macroscopic density, velocity, and temperature are respectively calculated:

\[
\rho = \sum_{i=0}^{8} f_i
\]  

(15)

\[
u = \frac{1}{\rho} \sum_{i=0}^{8} f_i e_i
\]  

(16)

\[
T = \sum_{i=0}^{8} T_i
\]  

(17)
Grid testing and code validation

It assumes nanofluids to be a single-phase fluid and the thermophysical properties for water and Al\(_2\)O\(_3\) are listed in tab. 1 [18]. The density varies only in buoyancy force, so it is handled by Boussinesq approximation. The viscosity and thermal conductivity of nanofluids vary with nanoparticle volume fraction and temperature. However, the effect of temperature is considered to be insignificant, comparing with the effects of nanoparticle volume fraction. Hence, the physical parameters of nanofluids are expressed:

The effective density:

\[
\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_p
\]  
(18)

The effective heat capacity:

\[
\rho_{nf}C_{pf} = (1 - \phi)\rho_f C_{pf} + \phi\rho_p C_{pp}
\]  
(19)

The effective dynamic viscosity is given by Brinkman model:

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}
\]  
(20)

The effective thermal conductivity can be approximated by the Maxwell-Garnetts model:

\[
\frac{k_{nf}}{k_f} = \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f - \phi(k_f - k_p)}
\]  
(21)

The local Nusselt number, \(\text{Nu}_x\), (calculated by Kefayati's method) along the bottom wall in \(x\) direction is usually written as [17]:

\[
\text{Nu}_x = \left( \frac{\partial T}{\partial y} \right)_{T_H-T_C} \frac{H}{T_H-T_C}
\]  
(22)

Then \(\text{Nu}_{ave}\) is calculated:

\[
\text{Nu}_{ave} = \frac{1}{W} \int_0^W \text{Nu}_x \, dx
\]  
(23)

For convenience, a normalized average Nusselt number, *\(\text{Nu}_{ave}\), which is defined as the ratio of Nusselt number at any nanoparticle volume fraction to that for base fluid case, is often used as an indicator of heat transfer enhancement with nanoparticle volume fraction. It is expressed [17]:

\[
*\text{Nu}_{ave}(\phi) = \frac{\text{Nu}_{ave}(\phi)}{\text{Nu}_{ave}(\phi = 0)}
\]  
(24)

However, eq. (24) is not available to evaluate the effect of nanoparticles on the heat transfer characteristic, when Nusselt number is calculated by eqs. (22) and (23), because these equations do not consider the change of thermal conductivity with nanoparticle volume fraction.

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Fluid phase (water)</th>
<th>Nanoparticles (Al(_2)O(_3))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho) [kgm(^{-1})]</td>
<td>997.1</td>
<td>3970</td>
</tr>
<tr>
<td>(C_p) [Jkg(^{-1})K(^{-1})]</td>
<td>4179</td>
<td>765</td>
</tr>
<tr>
<td>(\mu) [kgm(^{-1})s(^{-1})]</td>
<td>1.004×10(^{-3})</td>
<td>–</td>
</tr>
<tr>
<td>(k) [Wm(^{-1})K(^{-1})]</td>
<td>0.613</td>
<td>25</td>
</tr>
<tr>
<td>(\beta) \times10(^5) [K(^{-1})]</td>
<td>21</td>
<td>0.85</td>
</tr>
<tr>
<td>(d_p) [nm]</td>
<td>0.384</td>
<td>47</td>
</tr>
</tbody>
</table>
Recently, Abu-Nada et al. [15] and Eslamian et al. [19] used a new equation, eq. (25) to define \( \text{Nu}_x \) so that it can be used to assess the effect of nanoparticle volume fraction, i.e.:

\[
\text{Nu}_x = \frac{k_{nf} \frac{\partial T}{\partial y}}{k_f \frac{\partial y}{T_H - T_C}}
\] (25)

Corcione [20] used the Churchill-Ozoe heat transfer correlation to evaluate the effect of nanoparticle volume fraction on the heat transfer enhancement, basing on eqs. (22) and (23). It is given:

\[
E = \frac{h_{nf}}{h_f} - 1 = \frac{\text{Nu}_\text{ave} (\phi)}{\text{Nu}_\text{ave} (\phi = 0)} \frac{k_{nf}}{k_f} - 1
\] (26)

In the present study, eq. (25) is used to calculate \( \text{Nu}_x \) below, and then \( \text{Nu}_{\text{ave}} \) and \( \text{Nu}_{\text{ave}}^* \) are calculated, and the differences between these three calculation methods will be discussed in section The effect of different Nusselt number calculation methods.

Grid testing and code validation

In order to guarantee a grid independent solution for the given configuration, a grid sensitivity test was explored with different mesh sizes of 128 \( \times \) 128, 192 \( \times \) 192, 256 \( \times \) 256, and 320 \( \times \) 320 for the case of Ra = 10^5, \( A = 1 \), and \( \phi = 0 \), as shown in tab. 2. It can be found that the mesh size of 256 \( \times \) 256 ensures a grid independent solution, where there is little change in \( \text{Nu}_{\text{ave}} \) on the hot wall when using more grids, so this mesh size is used in all the following calculations.

To validate the developed codes for in this work, the present numerical code results for air-flow (Pr = 0.71) in RB convection was compared with the numerical results of Turan et al. [21] and Ouertatani et al. [22], as shown in fig. 2. It can be seen that there is a good and consistent agreement for simulation results.

Results and discussion

In this paper, it mainly focuses to investigate the influence of various physical parameters on the heat transfer characteristics of RB convection in enclosures filled with Al_2O_3-water nanofluids. While at Ra = 10^3, no flow was observed regardless of adding nanoparticles, and the temperature profile was linear (not shown), this condition will not be introduced in detail below.

The effect of different Nusselt number calculation methods

Figure 3 illustrates a comparison between three Nusselt number calculation methods (mentioned in section Grid testing and code validation) on the heat transfer enhancement. Actu-
ally, for evaluating the effect of nanoparticle volume fraction on the heat transfer enhancement, Corcione's method is equal to Abu-Nada's. But there is a remarkable difference between Kefayati's and Corcione's calculation method. When using the traditional Nusselt number calculation methods, it is contradicting to the previous analysis for the effect of nanoparticle volume fraction on the heat transfer enhancement. Therefore, Corcione's calculation method is adopted in this paper to analyze the effects of various parameters on Nusselt number.

The effect of different nanoparticle volume fractions and Rayleigh numbers

Figure 4 presents the variation of $\text{Nu}_x$ along the bottom wall varying nanoparticle volume fractions for $A = 1$. Overall observation indicates that for all Rayleigh number cases, the increase of nanoparticle volume fraction results in the increase of $\text{Nu}_x$. At $Ra = 10^4$ and $Ra = 10^5$, it can be also observed that the increase of $\text{Nu}_x$ is taking place in the whole range $0 < x^* < 1$, $()^*$ represents the dimensionless parameter, similarly hereinafter, and from $Ra = 10^4$ to $Ra = 10^5$, the maximum Nusselt number shifts from the side to the middle, while only one peak occurs. However, at $Ra = 10^6$, there are different performances for different volume fractions, and two peaks come into being on both sides. This behavior will be clearly demonstrated next.

Figure 4. The distribution of $\text{Nu}_x$ along the bottom wall, $A = 1$; (a) $Ra = 10^4$, (b) $Ra = 10^5$, (c) $Ra = 10^6$
Figure 5 shows the effect of Rayleigh number on isotherms (on the left) and streamlines (on the right) for $\phi = 0$ and $\phi = 5\%$. It also presents a comparison for flow field and temperature distribution of nanofluids (dashed lines) and base fluid (smooth lines) at $Ra = 10^4$ and $Ra = 10^5$. As shown from these isotherms, the nanofluids isotherms seem to be closer to the horizontal walls, while in the core region of the enclosure are more uniformly distributed. Nevertheless, the change is not obvious for the isotherms contours close to the hot and cold walls when adding low nanoparticle volume fraction, which demonstrates that the thickness of thermal boundary-layer next to the hot wall is hardly influenced by the addition of nanoparticles. Also, it is observed that at $Ra = 10^4$, the induced flow is weak and the isotherms concentrate leftward on the hot side, resulting in a larger temperature gradient than that in the rightward. While at $Ra = 10^5$, the induced flow is stronger, but the isotherms concentrate on the center region of hot wall. This behavior causes the aforementioned shift of the maximum Nusselt number. Moreover, it is evident from the streamline contours that the maximum absolute value of stream function enhances and the streamlines expand towards the enclosure wall with the increase of nanoparticle volume frac-
tion. These enhancements are as a result of an increase in the energy transport through the flow deemed to be associated with the irregular motion of the ultrafine particles [17]. Therefore, the addition of nanoparticles can prompt the motion of the fluid, eventually enhance the heat transfer. Besides, the effect of Rayleigh number on streamlines is shown from the behavior that a singular RB cell is formed at Ra = 10^4 while there are two RB cells at Ra = 10^5, which means an increase of the flow strength as Rayleigh number increases.

For Ra = 10^6 case, fig. 6 shows the comparison of isotherms contours between nanofluids at φ = 5% and base fluid. It is noted that the isotherms do not follow a specific pattern due to the presence of a stronger induced flow and the flow becomes asymmetric, not like the flow at lower Rayleigh number. This is similar to the observation by Eslamian et al. [19], using the two-phase LBM. However, it is clearly seen that the isotherms spacing between the isotherms close to the hot wall decreases as Ra increases, which is indicative of an enhancement in the heat transfer rate.

Conclusions

In this paper, the heat transfer characteristics of RB convection in differentially-heated enclosures filled with Al_2O_3-water nanofluids have been investigated using LBM. The effects of Rayleigh number, and nanoparticle volume fraction on RB convection heat transfer behavior are analyzed. Finally, some important conclusions are summarized as follows.

- When Nusselt number is used to evaluate the effect of φ on heat transfer enhancement, it is proved to consider the change of thermal conductivity coefficient at different φ.
- At low Rayleigh number (Ra = 10^4–10^6), the augment of nanofluids convection heat transfer mainly comes from the increase of thermal conductivity coefficient, only partially due to the enhancement of flow intensity.

Acknowledgment

This study was supported by National Natural Science Foundation of China (No. 51576051) and Key Laboratory of Efficient Utilization of Low and Medium Grade Energy (Tianjin University), Ministry of Education of China (201604-501).
Nomenclature

\( A \) – aspect ratio
\( C_p \) – special heat, [Jkg\(^{-1}\)K\(^{-1}\)]
\( c \) – lattice speed
\( c_s \) – lattice sound velocity, \((c / \sqrt{3})\)
\( d_p \) – particle diameter, [nm]
\( e_i \) – discrete particle velocity vectors in direction \( i \)
\( f \) – distribution function
\( f_{eq} \) – local equilibrium distribution function for flow field
\( f_b \) – buoyant body force
\( g \) – gravitational acceleration, [ms\(^{-2}\)]
\( H \) – height of RB cell, [m]
\( h \) – heat transfer coefficient, [Wm\(^{-2}\)K\(^{-1}\)]
\( k \) – thermal conductivity
\( Ma \) – Mach number
\( Nu \) – Nusselt number
\( Pr \) – Prandtl number
\( Ra \) – Rayleigh number
\( T \) – temperature, [K]
\( T_{eq} \) – local equilibrium distribution function for temperature field
\( \Delta T \) – temperature difference between, [K]
\( \Delta t \) – lattice time step
\( u \) – macroscopic velocity, [ms\(^{-1}\)]
\( u_c \) – characteristic velocity for natural convection, \((\sqrt{g \alpha T H})\), [ms\(^{-1}\)]
\( W \) – width of RB cell, [m]
\( \omega_i \) – weight coefficients

Greek symbols

\( \beta \) – thermal expansivity, [K\(^{-1}\)]
\( \mu \) – dynamical viscosity, [kgm\(^{-1}\)s\(^{-1}\)]
\( \nu \) – kinetic viscosity, [m\(^{2}\)s\(^{-1}\)]
\( \rho \) – density, [kgm\(^{-3}\)]
\( \tau_f \) – the relaxation time for flow field
\( \tau_T \) – the relaxation time for temperature field
\( \tau \) – nanoparticle volume fraction
\( \chi \) – thermal diffusivity, [m\(^{2}\)s\(^{-1}\)]

Subscripts

\( f \) – base fluid
\( nf \) – nanofluids
\( p \) – nanoparticles

References


