THE IMPACT OF THE ORDER OF NUMERICAL SCHEMES ON SLUG FLOWS MODELING

***hossein ZOLFAGHARY AZIZI* [1], *mohammad NAGHASHZADEGAN* [2*], *vahid SHOKRI* [3]**

[1,3] Department of Mechanical Engineering, Sari Branch, Islamic Azad University, Sari, Iran, P.O.B. 4816119318

[2] Department of Mechanical Engineering, University of Guilan, Rasht, Rasht, 3756, Iran

This paper aims to explore the impact of the order of numerical schemes on the simulation of two-phase slug flow with a two-fluid model initiation. The governing equations of the two-fluid model have been solved by a class of Riemann solver. The numerical schemes applied in this paper involve first-order (Lax-Friedrichs and Rusanov), second-order (Ritchmyer), and high-order (Flux-Corrected Transport or FCT and TVD). The results suggest that the TVD and FCT are able to predict the slug initiation with high accuracy compared with experimental results. Lax-Friedrichs and Rusanov are both first-order schemes and have second-order truncation error. This second-order truncation error caused numerical diffusion in the solution field and could not predict the slug initiation with high accuracy in contrast to TVD and FCT schemes. Ritchmyer is a second-order scheme and has third-order truncation error. This third -order truncation error caused dispersive results in the solution field and was not a proper scheme.

Key words: *Two-phase Flow, Two-fluid model, Slug flow*

1. Introduction

Prediction of two-phase flows behavior in pipeline is vital due to their application in transportation of crude oil from offshore to onshore and their steam and nuclear steam supply systems. Accordingly, three kinds of models for simulation of two-phase flows are presented in the literature: (a) homogeneous equilibrium model [1], (b) drift-flux model [2], and (c) two-fluid model [2, 3]. The two-fluid model is the best choice in this paper. With this in mind, there are two sets of conservation equations for each phase in the two-fluid model and this is the most detailed model among the models above.

The one-dimensional form of the two-fluid model is derived by area averaging the flow properties over the cross-section of the flow. The momentum transfer between the walls and the phases is calculated by source terms and must be formulated using empirical correlations [3]. The interaction between the phases in the interfaces is calculated by source terms, too. Therefore, this paper benefited from the slug capturing technique through which the slug flow is predicted as a mechanistic and automatic outcome of the growth of hydrodynamic instabilities [4].

* Corresponding author; E-mail: naghash@guilan.ac.ir

Issa and Kempf (2003) [5] using the two-fluid model for simulation of the slug flow with slug capturing technique. However, the two-fluid model was ill-posed and their results were limited to the flow conditions under which the governing equations was well-posed. Three-phase gas-liquid-liquid slug flow modeled by Bonizzi and Issa (2003 [6]. Gas bubbles entrance in slugs improved the closure relations for modeling of the bubble entrainment into slugs which have been done by Bonizzi and Issa (2003) [7]. Single pressure two-fluid model using a central numerical methods solved by Omgba Essama (2004) [1] was used for modeling of slug flow. The author used an adaptive mesh refinement to improve the effectiveness of the methods. More detailed constitutive relations for the modeling of bubble entrainment in slug body presented by Issa, et al. (2006)[8].

A technique for simulation of initiation and growth of slugs in the horizontal pipes are presented by Ansari and Shokri (2011) [9]. The researchers solved the governing equations of the transient compressible two-fluid model using groups of high-resolution shock-capturing methods. They found that high-order methods do not have dissipation and dispersion properties and can capture the slug properties with high accuracy [9]. An accurate simulation of the continuous slug flow with two-fluid model in oil and gas pipelines presented by Issa, et al. (2011) [10]. In their model hydrostatic pressure for two phases are assumed [10]. Simoes, et al. (2014) [11] modeled the slug frequency in a horizontal pipe with two-fluid model by finite-volume method. Bonzanini et al. (2017) proposed a numerical resolution of a one-dimensional (1D), transient, and simplified two-fluid model regularized with an artificial diffusion term for modeling stratified, wavy, and slug flow in horizontal and nearly horizontal pipes. They concluded that the artificial diffusion can prevent the unbounded growth of instabilities where the 1D two-fluid model is ill-posed [12]. Shokri and Esmaeili (2017) [13] presented a numerical study using single pressure transient two-fluid model in order to compare the effect of hydrodynamic and hydrostatic models for pressure correction term in two-fluid model in gas-liquid two-phase flow.

The focus of this study is modeling of two-phase slug flow and investigation of the effect of the order of numerical schemes on simulation of slugs. Modeling of the slug flow is important to find the time and location of the slug flow initiation to prevent from its formation. Because when the slug formed the gas trapped behind it and its pressure increases and push the liquid slug forward with high velocity and consequently can damage the instruments. The literatures reviewed above indicate that the ability of the numerical schemes for simulation of the slug flow initiation with a class of Riemann solver has not been investigated. With this in mind, the current study investigates the ability of numerical methods in order to predict the slug flow initiation with two-fluid model.

2. Governing equation

In this paper, we use a simplified version of the one-dimensional two-fluid model for the derivation of the model. The two-fluid model used here consists of two partial differential equations (PDEs) and two algebraic relations. The first PDE is the total mass equation [14].

$$\frac{\partial}{\partial t}(\rho_L R_L + \rho_G R_G) + \frac{\partial}{\partial x}(\rho_L R_L V_L + \rho_G R_G V_G) = 0 \tag{1}$$

And the second PDE is the total momentum equation [14].

$$\frac{\partial}{\partial t}(\rho_L V_L + \rho_G V_G) + \frac{\partial}{\partial x}\left(\frac{1}{2}\rho_L V_L^2 - \frac{1}{2}\rho_G V_G^2 + (\rho_L - \rho_G) g \cos\beta \ h_L\right) = H \tag{2}$$

With the source terms given by [14].

$$H = -(\rho_L - \rho_G) g \sin\beta + \left(\frac{1}{R_L A} + \frac{1}{R_G A}\right)\tau_I S_I + \frac{\tau_G S_G}{R_G A} - \frac{\tau_L S_L}{R_L A} \tag{3}$$

k^{th} is phase ($k = G$ is gas and $k = L$ is liquid), R_k is volume fraction, V_k is velocity ρ_k is density, A is the pipe cross-section. A_G, A_L, and D are the gas phase and liquid phase cross-section area, and the pipe diameter, respectively. g is gravity, β is the pipe inclination angle, h_L is the height of the liquid phase and τ_G, τ_L and τ_I are gas-wall, liquid-wall, and interface shear stress, respectively. S_G, S_L, and S_I represent the wet gas perimeter, wet liquid wet , and interface , respectively. Cross-section and side views of two-phase flow pipe are shown in fig. 1(a). The geometric restriction for two phases is [15]:

$$R_L + R_G = 1 \tag{4}$$

In addition to the geometric constraint, another relation is required that is presented as following:

$$\frac{\partial}{\partial x}(R_L V_L + R_G V_G) = 0 \tag{5}$$

According to the eq. (5), it is concluded that there is only time functionality in this equation. This time function is indicated by $C(t)$ and considered as a function of the inlet flow parameters:

$$R_L V_L + R_G V_G = C(t) = (R_L V_L + R_G V_G)_{inlet} \tag{6}$$

The subscript "*inlet*" refers to the inlet of the pipe.

3. Constitutive relations

The constitutive relations that we need to close the system of equations of the two-fluid model are wall-gas shear stress, wall-liquid shear stress, and interface shear stress. The following relations are used for calculating the wall-gas, wall-liquid, and interfacial shear stress, respectively [16]:

$$\tau_k = \frac{1}{2} f_k \rho_k V_k |V_k| \tag{7}$$

$$\tau_I = \frac{1}{2} f_I \rho_G (V_G - V_L)|V_G - V_L| \tag{8}$$

In the above equations, f_K and f_I are k^{th} phase and interface friction factor, respectively. In which a hydraulic diameter is used [16].

$$Re_k = \frac{\rho_k D_{hk} |V_k|}{\mu_k} \tag{9}$$

$$D_{hL} = \frac{4\ A_L}{S_L} \tag{10}$$

$$D_{hG} = \frac{4\ A_G}{S_G + S_L} \tag{11}$$

D_{hk} is hydraulic diameter of k^{th} phase and Re_k is Reynolds number of k^{th} phase. μ_k is viscosity of k^{th} phase. The surface roughness is assumed to $\varepsilon = 4.61 \times 10^{-5}$ [9]. The friction factors are calculated by following equations[1]:

$$f_G = max\left[\frac{16}{Re_G}, 0.001375\left[1 + \left[2 \times 10^4 \left(\frac{\varepsilon}{D_{hG}}\right) + \frac{10^6}{Re_G}\right]^{\frac{1}{3}}\right]\right] \tag{12}$$

$$\begin{cases} f_L = max\left[\frac{24}{Re_L}, \frac{0.263}{R_L}\sqrt{\frac{D_{hL}}{Re_L D}}\right] & if \quad j_G > 5\, m/s \\ f_L = max\left[\frac{16}{Re_L}, 0.001375\left[1 + \left[2 \times 10^4 \left(\frac{\varepsilon}{D_{hL}}\right) + \frac{10^6}{Re_L}\right]^{\frac{1}{3}}\right]\right] & otherwise \end{cases} \tag{13}$$

$$\begin{cases} f_I = f_G & for \quad j_G < 5\, m/s \\ f_I = f_G \left\{1 + max\left[0, 15\left(\frac{j_G}{5} - 1\right)\sqrt{\frac{h_L}{D}}\right]\right\} & for \quad j_G \geq 5\, m/s \end{cases} \tag{14}$$

In the above equation, D represents the pipe diameter and j_G is gas superficial velocity.

4. Numerical solution

Equations (1) and (2) can be written in conservative from as follows [14].

$$\frac{\partial Q}{\partial t} + \frac{\partial F}{\partial x} = H \tag{15}$$

In the above equation, Q is a vector field of conservative variables F and H are algebraic functions of Q only and are flux and source terms, respectively. In the current paper, a class of Riemann solver is used as follows:

$$Q_i^{n+1} = Q_i^n + \frac{\Delta t}{\Delta x}(\hat{F}_{i-1/2} - \hat{F}_{i+1/2}) + \Delta t\ H_i \tag{16}$$

In eq. (16), Δx is spatial step size, Δt is time step size, n and $n+1$ current and previous time, respectively. Q is a vector field of conservative variables, $\hat{F}$ is numerical flux and H is source term. Selection of the proper numerical scheme is important. Five following numerical methods have been studied in this paper:

4.1. Lax–Friedrichs

This is a first-order explicit method in space and time [17]. Flux term is calculated as follows:

$$\hat{F}^{LF}_{i+1/2} = \frac{1}{2}(F^n_{i+1} + F^n_i) - \frac{\Delta x}{2\Delta t}(Q^n_{i+1} - Q^n_i) \tag{17}$$

4.2. Rusanov

This is first-order explicit, too. However, it uses maximum value of characteristics derived by hyperbolic analysis [1]. Flux term is calculated as follows:

$$\hat{F}^{Rus}_{i+1/2} = \frac{1}{2}\Big((F(Q^n_i) + F(Q^n_{i+1}) - \lambda_{i+1/2}\,(Q^n_{i+1} - Q^n_i)\Big) \tag{18}$$

$$\lambda_{i+1/2} = max\big(max\left|\lambda^k_i\right|, max\left|\lambda^k_{i+1}\right|\big) \qquad k = 1, N_{eq} \tag{19}$$

Neq is the number of partial equations and $\lambda_{i+1/2}$ is the average wave velocity.

4.3. Ritchmyer

This is an explicit numerical method. It is a second-order method in time and space consisting of two steps [17]. Flux term should be calculated as following.

$$Q^n_{i+1/2} = \frac{1}{2}\,(Q^n_i + Q^n_{i+1}) \tag{20}$$

$$Q^{n+1/2}_{i+1/2} = Q^n_{i+1/2} - \frac{\Delta t}{2\Delta x}(F^n_i - F^n_{i+1}) + \frac{\Delta t}{2}H(Q^n_{i+1/2}) \tag{21}$$

$$\hat{F}^{RI}_{i+1/2} = F(Q^{n+1/2}_{i+1/2}) \tag{22}$$

4.4. FCT

This is a high-precision method proposed by Boric and Book [18]. This is a predictor/corrector method in which diffusion enters to the system in the prediction step in order to eliminate the extreme points in the flow field. FCT contains five steps [18]. $Q^n$is the solution at previous time step, and $\tilde{Q}$ is the new answer which has obtained with Ritchmyer scheme.

$$F^d_{i+1/2} = \upsilon_{i+1/2}(Q^n_{i+1} - Q^n_i) \tag{23}$$

$$Q_i^d = \tilde{Q}_i + \left(F_{i+1/2}^d - F_{i-1/2}^d\right) \tag{24}$$

$$F_{i+1/2}^{ad} = \gamma_{i+1/2}\left(\tilde{Q}_{i+1} - \tilde{Q}_i\right) \tag{25}$$

$$F_{i+1/2}^{cad} = s.\max\left(0, max\left(s.\left(\tilde{Q}_{i+1} - Q_i^d\right), \left|F_{i+\frac{1}{2}}^{ad}\right|, s.\left(\tilde{Q}_{i+2} - Q_{i+1}^d\right)\right)\right) \tag{26}$$

$$s = sign\left(F_{i+1/2}^{ad}\right) \tag{27}$$

$$\hat{F}_{i+1/2}^{FCT} = F_{i+1/2}^{cad} - F_{i+1/2}^d \tag{28}$$

The diffusion and anti-diffusion coefficients, i.e., υ and γ are calculated as follows [19]:

$$\upsilon = \frac{1}{6}\left(1 + 2(CFL)^2\right) \tag{29}$$

$$\gamma = \frac{1}{2}\left(1 + (CFL)^2\right) \tag{30}$$

4.5. TVD Lax-Friedrichs

This is a high-order method that does not include artificial diffusion. Different forms of this method have been described in the article by Toth and Odstrcil (1996) [20]. The flux term can be estimated by the following procedure:

$$\hat{F}_{i+1/2}^{TVDLF} = \frac{1}{2}\left[F\left(Q_{i+1/2}^L\right) + F(Q_{i+1/2}^R)\right] - \frac{\Delta x}{2\Delta t}\phi_{i+1/2}^{LR} \tag{31}$$

The left and right state vectors Q^L and Q^R are calculated by using an intermediate state vector $Q^{n+1/2}$and limited differences$\delta\tilde{Q}$.

$$Q_{i+1/2}^L = Q_i^{n+1/2} + \frac{1}{2}\delta\tilde{Q}_i^n \tag{32}$$

$$Q_{i+1/2}^R = Q_{i+1}^{n+1/2} - \frac{1}{2}\delta\bar{Q}_{i+1}^n \tag{33}$$

The intermediate vector, also called the Predicator step, is calculated by:

$$Q_i^{n+1/2} = Q_i^n - \frac{\Delta t}{2\Delta x}\left[\left(F\left(Q_i^n + \tfrac{1}{2}\delta\bar{Q}_i^n\right) - F\left(Q_i^n - \tfrac{1}{2}\delta\bar{Q}_i^n\right)\right] + \frac{\Delta t}{2}S(Q_i^n) \tag{34}$$

ϕ^{LR} is the dissipative Limiter and is evaluated as:

$$\phi^{LR} = \frac{\Delta t}{\Delta x}\lambda_{i+1/2}^{max}\left(Q^R - Q^L\right) \tag{35}$$

In this method, the wave velocity is calculated by:

$$\lambda_{i+1/2}^{max} = max\left|\lambda_k(\frac{Q^R + Q^L}{2})\right| \quad (36)$$

To find the Limited Differences value, the Min mod function is applied:

$$\delta\bar{Q}_i^n = \text{Min mod}(Q_i^n - Q_{i-1}^n, Q_{i+1}^n - Q_i^n) \quad (37)$$

4.6. Time step size

To calculate step size, Δx must be first determined. For explicit numerical methods there is a stability condition called Courant Friedrichs Levy Number and (*CFL*)$\leq$*1*. Using Δx and CFL number, time step size is calculated as follows:

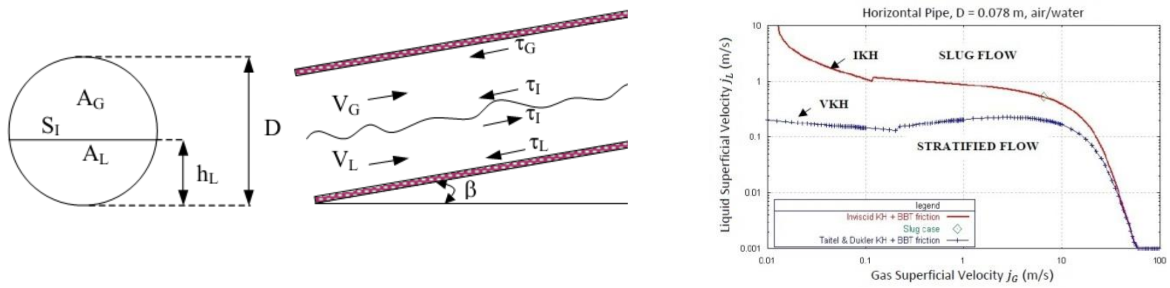
$$\Delta t = CFL\ \frac{\Delta x}{\lambda_{max}^n} \quad (38)$$

λ_{max}^n is the maximum value of wave velocity in the flow field at the previous time step. λ_{max}^n is selected at each time step and because of the variability of λ_{max}^n, the solution method has a variable time step size. The maximum value of wave velocity in the flow field is presented in Omgba Essama (2004)'s work [1].

5. Initial and boundary condition (Slug flow modeling)

Gas phase is considered as air with a density of 1.14 [kg m-3] and liquid phase is considered as water with a density of 1000 [kg m-3]. A horizontal pipe with the length of 5 [m] and the diameter of 0.078 [m] is selected with two phases in stratified regime initially. The inlet superficial gas velocity is 6.532 [m s-1] and inlet superficial liquid velocity is 0.532 [m s-1]. The volume fraction of liquid at the initial condition is 0.526. As shown in the fig. 1(b), the initial condition considered in slug flow regime is movable. In other words, the initial conditions located on the unstable inviscid Kelvin-Helmholtz line and are in transition from stratified pattern to slug pattern. The inviscid Kelvin-Helmholtz instability line is known as the well-posed region of two-fluid model [21].

The initial velocities of each phase are calculated by volume fraction and superficial velocity of each phase at the inlet of pipe. The boundary conditions are considered equal to the initial conditions at the entrance of the pipe. For outlet boundary conditions, a fully developed condition is considered.



(a) (b)

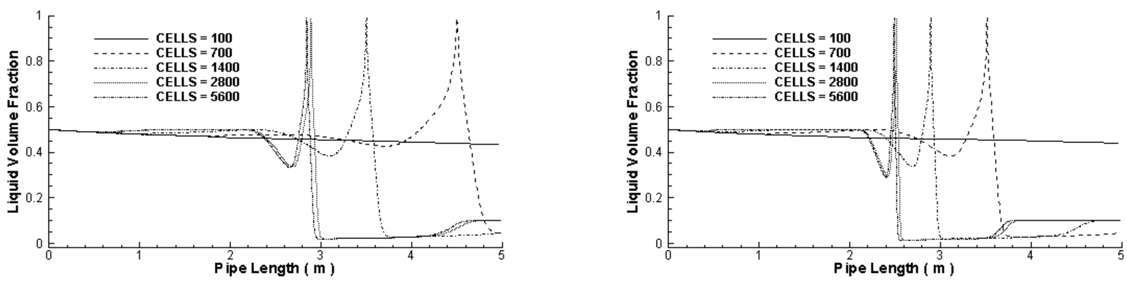
Figure 1. Cross-section and side views of two phase flow in pipe (a), Inviscid Kelvin-Helmholtz (IKH) transition lines from stratified flow (b)

5.1. Result and discussion

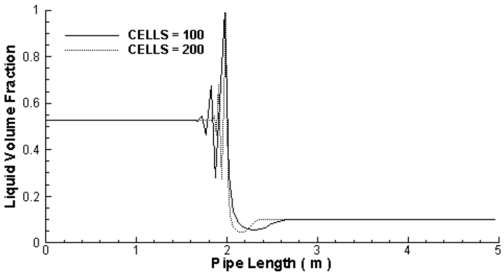
Figures fig. 2(a), fig. 2(b), fig. 2(c), fig. 2(d), and fig. 2(e) illustrate the mesh study of slug flow initiation point, with five different numerical methods: Lax-Friedrichs, Rusanov, Ritchmyer, and FCT and TVD Lax-Friedrichs, respectively. Time and CFL is considered 3 [s] and 0.3, respectively.

Figures fig. 2(a), fig. 2(b), fig. 2(d), and fig. 2(e) demonstrate the slug flow initiation point for 100, 700, 1400, 2800, and 5600 computational cells using Lax-Friedrichs, Rusanov, FCT and TVD Lax-Friedrichs methods. Results show that in 2800 the solution is independent from the cells number in these four methods.

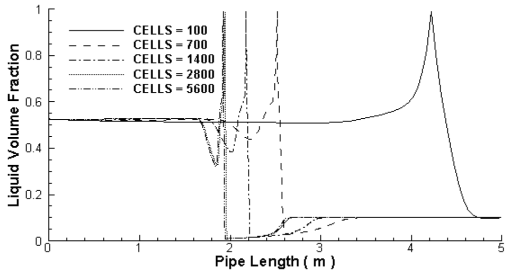
Figure 2(c) show the slug flow initiation point for 100 and 200 computational cell based on the Ritchmyer numerical method. Results show that the Ritchmyer numerical method has an oscillatory nature where reducing the size of the computational cell the oscillations become greater. This is the second-order method which has the third-order error and causes the solutions to be dispersed in the flow field. As a result, the Ritchmyer numerical method is not proper for modeling the slug flow regime. Ritchmyer numerical method is a second-order method which has third-order error. This type of error leads to numerical dispersion for solutions near the discontinuity due to formation of slug flow regime. Therefore, this method is omitted from the rest of this paper.



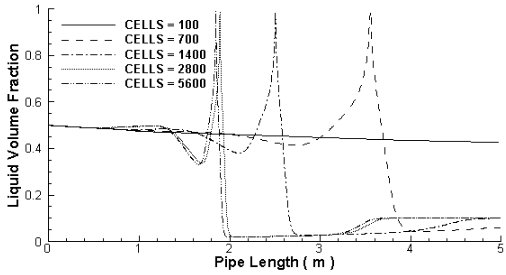
(a) (b)



(c)



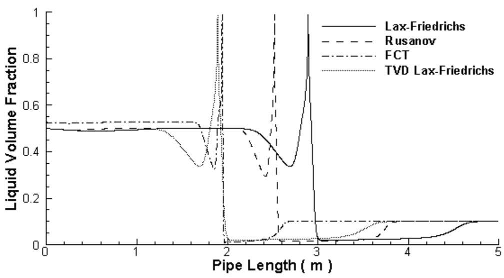
(d)



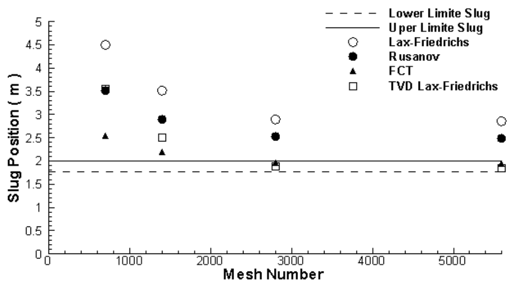
(e)

Figure 2. The initiation point of the slug flow map for different computational cells using different methods: (a) Lax–Friedrichs method, (b) Rusanov method, (c) Ritchmyer method, (d) FCT method, and (e) TVD Lax–Friedrichs method

Figure 3(a) indicates the comparison of different methods of predicting the slug initiation point. The calculations are in 2800 computational cell. Time and CFL are 3 [s] and 0.3, respectively. Figure 3(b) indicates the slug initiation point compared with the Ansari's experimental result [22].



(a)



(b)

Figure 3. Comparison of various numerical accuracy in predicting the beginnings of slug flow regime (a), Comparison of obtained initiation point of slug flow with experimental results (b)

Figure 3(b) shows that the Lax-Friedrichs has the lowest accuracy predicting the slug regime and TVD-Lax-Friedrichs is the most accurate method. The numerical diffusion embedded in the first-order Lax- Friedrichs predicts the flow field discontinuities in dispersion form. Thus, Lax-Friedrichs cannot predict the slug initiation with reasonable accuracy.

In comparison with the Lax-Friedrichs, Rusanov predicts the slug initiation more accurately. Rusanov is a first-order method and has second-order error, too. This second-order error contributes to a numerical diffusion cause to dispersion of discontinuity in flow field. Rusanov numerical method is a characteristic-based method which has more information about the flow characteristics.Hence, Rusanov generates less numerical diffusion in contrast with the Lax-Friedrichs and can predict the slug flow initiation with higher accuracy.

In addition, fig. 3(b) indicated that FCT method predicts the slug flow initiation more accurate compared to the Lax-Friedrich and Rusanov and TVD- Lax-Friedrichs is the most accurate scheme in the prediction of the slug initiation among other methods introduced in this paper. FCT is a high-precision and TVD- Lax-Friedrichs is a high-order method and they do not suffer from diffusive error (second-order truncation error) and dispersive error (third-order truncation error). However, the TVD- Lax-Friedrichs is more accurate because it is a high-order method.

6. Conclusions

This paper aimed to present the impact of the order of numerical schemes on simulation of slug initiation. The governing equations of two-fluid model have been solved by a class of Riemann solver. The numerical schemes used in this paper include Lax-Friedrichs, Rusanov, Ritchmyer, FCT, and TVD Lax-Friedrichs. The results show that the TVD Lax-Friedrichs and FCT can predict the slug initiation with reasonable accuracy compared with experimental results. However the TVD- Lax-Friedrichs is more accurate because it is a high-order method. Lax-Friedrichs and Rusanov are both first-order schemes and both of them have second-order truncation error which causes numerical diffusion in the solution field. Consequently, they cannot predict the slug initiation with reasonable accuracy in contrast to TVD and FCT schemes. Ritchmyer is a second-order scheme and has third-order truncation error. This causes dispersive results in the solution field and meaning that it is not a proper scheme.

References

[1] Omgba-Essama, C., *Numerical modelling of transient gas-liquid flows (application to stratified & slug flow regimes)*, Ph. D. Thesis, Cranfield University, England, 2004

[2] Ishii, M., *Thermo-Fluid Dynamic Theory of Two-Phase Flow, Eyrolles, Paris, France, 1975*

[3] Ishii, M., Mishima, K., Two-fluid model and hydrodynamic constitutive relations, *Nuclear Engineering and design*, 82(1984), 2, pp. 107-126

[4] Woodburn, P., Issa, R., Well-posedness of one-dimensional transient, two-fluid models of two-phase flows, Third International Conditioning of Multiphase Flow, Lyon, France, 1998, pp. 8-12

[5] Issa, R., Kempf, M., Simulation of slug flow in horizontal and nearly horizontal pipes with the two-fluid model, *International journal of multiphase flow*, 29(2003), 1, pp. 69-95

[6] Bonizzi, M., Issa, R., On the simulation of three-phase slug flow in nearly horizontal pipes using the multi-fluid model, *International journal of multiphase flow*, 29(2003), 11, pp. 1719-1747

[7] Bonizzi, M., Issa, R., A model for simulating gas bubble entrainment in two-phase horizontal slug flow, *International journal of multiphase flow*, 29(2003), 11, pp. 1685-1717

[8] Issa, R., *et al.*, Improved closure models for gas entrainment and interfacial shear for slug flow modelling in horizontal pipes, *International journal of multiphase flow*, 32(2006), 10, pp. 1287-1293

[9] Ansari, M., Shokri, V., Numerical modeling of slug flow initiation in a horizontal channels using a two-fluid model, *International Journal of Heat and Fluid Flow,* 32(2011), 1, pp. 145-155

[10] Issa, R., *et al*., Accurate simulation of intermittent/slug flow in oil and gas pipelines, 15th International Conditioning on Multiphase Production Technology, Cannes, France, 2011, pp. 15-17

[11] Simões, E. F., *et al*., Numerical prediction of non-boiling heat transfer in horizontal stratified and slug flow by the Two-Fluid Model, *International Journal of Heat and Fluid Flow,* 47(2014), pp. 135-145

[12] Bonzanini, A., *et al*., Simplified 1D Incompressible Two-Fluid Model with Artificial Diffusion for Slug Flow Capturing in Horizontal and Nearly Horizontal Pipes, *Energies,* 10(2017), 9, pp. 1372, 2017

[13] Shokri, V., Esmaeili, K., Comparison of the effect of hydrodynamic and hydrostatic models for pressure correction term in two-fluid model in gas-liquid two-phase flow modeling, *Journal of Molecular Liquids*, 237(2017), pp. 334-346

[14] Watson, M., Non-linear waves in pipeline two-phase flows, *in Third International Conference on Hyperbolic Problems*. 1990.

[15] Li, G., *et al*., Gas reservoir evaluation for underbalanced horizontal drilling, *Thermal Science,* 18(2014), 5, pp. 1691-1694

[16] Montini, M., *Closure relations of the one-dimensional two-fluid model for the simulation of slug flows*, Ph. D. Thesis, Imperial College London, London, 2011.

[17] Hirsch, H., Numerical computation of internal and external flows, *Computational methods for inviscid and viscous flows,* Wiley InterScience, Brussels, Belgium, 1990

[18] Boris, J. P., Book, D. L., Flux-corrected transport. I. SHASTA, a fluid transport algorithm that works, *Journal of computational physics,* 11(1973), 1, pp. 38-69

[19] Hoffmann, K. A., Chiang, S. T., Computational Fluid Dynamics Volume I, *Engineering Education System, Wichita, Kan, USA*, 2000

[20] Tóth, G., Odstrčil, D., Comparison of some flux corrected transport and total variation diminishing numerical schemes for hydrodynamic and magnetohydrodynamic problems, *Journal of Computational Physics,* 128(1996), 1, pp. 82-100

[21] Louaked, M., *et al*., Well-posedness of incompressible models of two-and three-phase flow, *IMA journal of applied mathematics,* 68(2003), 6, pp. 595-620

[22] Ansari, M. R., Experimental investigation on wave initiation and slugging of air-water stratified flow in horizontal duct, *Journal of Nuclear Science and Technology,* 26(1989), 7, pp. 681-688