MHD FLOW AND HEAT TRANSFER OVER A MOVING CYLINDER IN A NANOFLUID UNDER CONVECTIVE BOUNDARY CONDITIONS AND HEAT GENERATION

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In this paper, the effect of convective boundary conditions, heat generation, Brownian motion, and thermophoresis on heat transfer characteristics of a moving cylinder embedded into cooling medium consists of water with Nanoparticles are studied. The governing boundary layer equations transformed to ordinary differential equations using similarity transformation method and then solved analytically using optimal homotopy asymptotic method for the general case. The velocity, temperature and concentration profiles within the boundary layer plotted and discussed in details for various values of the different parameters. Moreover, the effect of boundary layer behavior on the surface shear stress, rate of heat and mass transfer investigated.

Key words: Nanofluids, moving cylinder, heat generation, convective boundary conditions, OHAM

Introduction

The boundary layer flow caused by a moving surface or cylinder has drawn the attention of many researchers due to its important applications in the industry process such as paper production, wires extrusion… etc. also the study of boundary layer behavior over a moving cylinder during the process of cooling is a mathematical simulation to the process of heat treatment of metals. Ali Chamkha [1] studied the magnetohydrodynamic boundary layer over a moving cylinder under the influence of heat generation and chemical reaction. Swati *et al*. [2] analyzed the boundary layer over a stretching cylinder in porous medium. Elbashbeshy *et al*. [3] investigated the effect of heat source and suction/injection on the heat transfer characteristics of MHD boundary layer over a horizontal stretching cylinder. Rekha *et al*. [4] studied the behavior of the boundary layer over a stretching cylinder under variable thermal conductivity.

Because the main property of the cooling fluid is the heat absorption due to its high thermal conductivity, so the nanofluids became the appropriate for this process. Choi *et al*. [5] suggested these fluids to improve the thermal conductivity of the water by adding very fine particles that's in nanosized. From this date, many researchers focus their efforts to investigate the influence of these types of fluids on the heat transfer characteristics. Azizah *et al*. [6] studied the time-dependent motion of a shrinking sheet within nanofluid. Aminreza *et al*. [7] studied the effect of partial slip boundary condition on the flow of nanofluids over stretching sheet. Hamad [8] analyzed analytically the natural convection MHD flow of nanofluids over a stretching sheet. Alsaedi *et al*. [9] investigated the influence of heat generation on a stagnation point nanofluid boundary layer under convective boundary conditions. Elbashbeshy *et al*. [10] presented an exact solution for the MHD nanofluid boundary layer over a moving surface under the influence of suction/injection. Elbashbeshy *et al*. [11] studied the behavior of the boundary layer over a moving surface with variable thickness under the effect of thermal radiation. Abdel-wahed *et al*. [12] investigated the effect of the thermophoresis force and the Brownian motion of the nanofluids on the boundary layer over a moving surface with variable thickness. Abdel-wahed [13] studied the effect of the micro-rotation element on the nanofluid boundary layer over flat surface. Hayat *et* *al.* [14] investigated the influence of convective boundary conditions on MHD flow of nanofluid over a stretching sheet. Nazar *et al.* [15] studied the mixed convection boundary layer of nanofluid over horizontal cylinder. Anbuchezhian [16] investigated the effect of the thermophoresis force and the Brownian motion of nanofluid on a boundary layer subjected to thermal radiation. Ali *et al* [17] studied the transient natural convection flow of a nanofluid over vertical cylinder. Elbashbeshy *et al.* [18] investigated the effect of heat treatment process with nanofluids as new coolants on the mechanical properties of a moving cylinder. Elbashbeshy *et al.* [19] extended their previous work to study the effect of the thermal radiation and heat generation on the mechanical properties of a moving cylinder during the cooling process. Abdel-wahed *et al.* [20], [21] studied the effect of joule heating and hall current on the MHD nanofluid flow over a rotating disk in the presence of nonlinear thermal radiation. Nadeem *et al.* [22] studied the three-dimensional boundary layer of cassone nanofluid over a stretching sheet with convective boundary conditions.

The objective of the present work is to study the effect of convective boundary conditions, heat generation and hydromagnetic flow on the boundary layer behavior and the impact of these forces on the surface shear stress, heat and mass transfer over a moving cylinder in a Nanofluid.

Formulation of the problem

Consider a steady, incompressible, laminar, two-dimensional flow of a viscous electrically conduction Nanofluid over a continuous moving cylinder in the presence of heat generation Q_0. Assume the induced magnetic field produced by the motion of an electrically conducting fluid is negligible; this assumption is valid for small magnetic Reynolds number. Further, since there is no external electric field, the electric field due to polarization of charges is negligible. Moreover, we considered the bottom surface of the cylinder is heated by convection from a hot fluid at temperature T_f and concentration C_f which provide a heat transfer coefficient h and mass transfer coefficient k_m, respectively. Moreover, assume that both the fluid phase and nanoparticles are in thermal equilibrium state and no slip occurs between them. The physical model of the problem shown in figure (1)

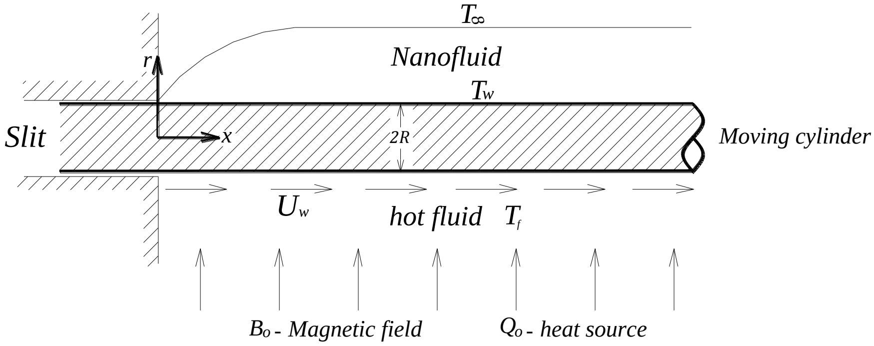

Figure (1): physical model and coordinate system.

The governing boundary layer equations for the steady two-dimensional laminar Nanofluid flows over a moving cylinder can be written as [14], [19]:

$$\frac{\partial}{\partial x}(r\,u)+\frac{\partial}{\partial r}(r\,v)=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\left(\frac{\upsilon}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)-\left(\frac{\sigma\beta_0^2}{\rho^*}\right)u \tag{2}$$

$$\left\{ \begin{array}{c} u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\left(\frac{\alpha}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\tau\left(D_B\frac{\partial C}{\partial r}\frac{\partial T}{\partial r}+\frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial r}\right)^2\right) \\ +\frac{Q_0}{\rho^* Cp}\left(T-T_\infty\right) \end{array} \right\} \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial r}=\left(\frac{D_B}{r}\right)\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)+\frac{D_T}{T_\infty}\frac{\partial}{r\,\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{4}$$

With boundary conditions

$$\begin{array}{l} u=U_w,\quad v=0,\quad -k\frac{\partial T}{\partial y}=h\left(T_f-T\right),\ -D_m\frac{\partial C}{\partial y}=k_m\left(C_f-C\right),\qquad at\quad r=R \\ u=0,\quad v=0,\quad T=T_\infty,\qquad as\,r\to\infty \end{array} \tag{5}$$

Where u and v are velocity components in the x and r directions, respectively, υ is the kinematic viscosity, ρ^* is the density of the base fluid, σ is the electrical conductivity, and α is the thermal diffusion, D_B is the Brownian diffusion coefficient, D_T is the Thermophoretic diffusion coefficient, D_m is the molecular diffusivity, k_m is the surface mass transfer coefficient, h is the conective heat transfer coefficient, and τ is the ratio between the effective heat capacity of the nanoparticle and heat capacity of the fliud.

The velocity of the cylinder is assumed in the form $U_w(x)=a\,x$, where a is constant (6)

Similarity Analysis and Numerical Procedure

Looking for similarity solution of equations (1-4) with the boundary conditions (5) using the following definitions:

$$\eta=\frac{r^2-R^2}{2R}\sqrt{\frac{a}{\upsilon}}\ ,\quad \psi=\sqrt{a\,\upsilon}\,x\,R\,f(\eta),\quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty},\quad \phi(\eta)=\frac{C-C_\infty}{C_f-C_\infty} \tag{7}$$

where η is the similarity variable, $\theta(\eta)$ is the dimensionless temperature, $\phi(\eta)$ is the dimensionless concentration, and ψ is the stream function which is defined as $u=\frac{\partial\psi}{r\,\partial r}$ and $v=-\frac{\partial\psi}{r\,\partial x}$ which satisfies equation (1), substituting equations (7) into eqs. (2-4), we obtain the following ordinary differential equations

$$(1+2\eta\rho)f'''+2\rho\, f''+ff''-f'^2-M\ f'=0 \tag{8}$$

$$(1+2\eta\rho)\left[\theta''+\Pr Nb\,\theta'\phi'+\Pr Nt\,\theta'^2\right]+(2\rho)\theta'+\Pr\, f\theta'+\Pr\delta\,\theta=0 \tag{9}$$

$$(1+2\eta\,\rho)\left[\phi''+\left(\frac{Nt}{Nb}\right)\theta''\right]+2\rho\phi'+\rho\left(\frac{Nt}{Nb}\right)\theta'+Le\ f\phi'=0 \tag{10}$$

with boundary conditions

$$f(0)=0,\quad f'(0)=1,\quad \theta'(0)=-\gamma_1\,[1-\theta(0)],\quad \phi'(0)=-\gamma_2\,[1-\phi(0)] \quad \text{and}$$

$$f'(\infty)=0,\quad \theta(\infty)=0,\quad \phi(\infty)=0 \tag{11}$$

here primes denote differentiation with respect to (η)

where $Pr=\dfrac{\upsilon}{\alpha}$, $Le=\dfrac{\upsilon}{D_B}$, $\rho=\sqrt{\dfrac{\upsilon}{a R^2}}$, $M=\dfrac{\beta_0^2\sigma}{a\rho}$, $Nb=\dfrac{\tau D_B}{\upsilon}(C_w-C_\infty)$, $Nt=\dfrac{\tau D_t}{\upsilon T_\infty}(T_w-T_\infty)$,

$$\delta=\frac{Q_0}{a\,\rho C_p},\ \gamma_1=\frac{h}{k}\sqrt{\tfrac{\upsilon}{a}},\ \gamma_2=\frac{k_m}{D_m}\sqrt{\tfrac{\upsilon}{a}}\ \text{ and }\ \tau=\frac{(\rho Cp)_p}{(\rho Cp)_f}$$

Here Pr, Le, ρ, M, Nb, Nt, δ, γ_1 and γ_2 donte the prandtl number, the lewis number, curvature parameter, magnetic field parameter, the brownian motion parameter, the thermophoresis parameter and the heat source parameter, thermal boit number, concentration boit number respectively.

OHAM Procedure

In this section, the Optimal Homotopy Asymptotic Method used to solve the system (8-10) with the boundary conditions (11) under the following assumptions

$$\mathcal{A}[f(\eta)] \equiv \mathcal{L}_f[f(\eta)] + \mathcal{N}_f[f(\eta)] \tag{12}$$

$$\mathcal{B}[\theta(\eta),\phi(\eta),f(\eta)] \equiv \mathcal{L}_\theta[\theta(\eta)] + \mathcal{N}_f[\theta(\eta),\phi(\eta),f(\eta)] \tag{13}$$

$$\mathcal{C}[\phi(\eta),\theta(\eta),f(\eta)] \equiv \mathcal{L}_\phi[\phi(\eta)] + \mathcal{N}_f[\phi(\eta),\theta(\eta),f(\eta)] \tag{14}$$

Such that $\mathcal{L}_f[f(\eta)], \mathcal{L}_\theta[\theta(\eta)], \mathcal{L}_\phi[\phi(\eta)]$ are the linear operators of the system and takes the following forms:

$$\mathcal{L}_f[f(\eta)] = f'''(\eta) + k\,f''(\eta) \tag{15}$$

$$\mathcal{L}_\theta[\theta(\eta)] = \theta''(\eta) + k\,\theta'(\eta) \tag{16}$$

$$\mathcal{L}_\phi[\phi(\eta)] = \phi''(\eta) + k\,\phi'(\eta) \tag{17}$$

And $\mathcal{N}_f[f(\eta)]$, $\mathcal{N}_f[\theta(\eta),\phi(\eta),f(\eta)]$, $\mathcal{N}_f[\phi(\eta),\theta(\eta),f(\eta)]$ are the non-linear operators of the system, k is an unkown constant to be obtained to control the asymptotic behaviour of the solution.

Let the general solution of the system (8-10) to be:

$$f(\eta) = f_0(\eta) + \sum_{i=1}^{\infty} f_i(\eta) \tag{18}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{i=1}^{\infty} \theta_i(\eta) \tag{19}$$

$$\phi(\eta) = \phi_0(\eta) + \sum_{i=1}^{\infty} \phi_i(\eta) \tag{20}$$

Such that the initial guesses taken in the following form:

$$f_0(\eta) = \frac{(1-e^{-k\eta})}{k}, \;\; \theta_0(\eta) = \left(\frac{1+\gamma_1}{\gamma_1}\right) e^{-k\,\eta} \;\text{ and }\; \phi_0(\eta) = \left(\frac{1+\gamma_2}{\gamma_2}\right) e^{-k\,\eta} \tag{21}$$

The optimal homotopy equations defined as:

$$(1-p)\,\mathcal{L}_f[F(\eta;p,a_n)] = p\,\mathcal{H}_f(\eta,a_n)\,\mathcal{A}[F(\eta;p,a_n)] \tag{22}$$

$$(1-p)\,\mathcal{L}_\theta[\Theta(\eta;p,b_n)] = p\,\mathcal{H}_\theta(\eta,b_n)\;\;\mathcal{B}[\Theta(\eta;p,b_n),\Phi(\eta;p,c_n),F(\eta;p,b_n)] \tag{23}$$

$$(1-p)\,\mathcal{L}_\phi[\Phi(\eta;p,c_n)] = p\,\mathcal{H}_\Phi(\eta,c_n)\;\;\mathcal{C}[\Phi(\eta;p,c_n),\Theta(\eta;p,b_n),F(\eta;p,b_n)] \tag{24}$$

Where $p \in [0,1]$ denotes an embedding parameter and varies from 0 to 1 and $\mathcal{H}_f(\eta,a_n),\mathcal{H}_\theta(\eta,b_n), and\ \mathcal{H}_\Phi(\eta,c_n)$ are arbitary auxiliary convergence control functions.

Collecting the same powers of p, and equating each coefficient of p with zero, one can obtain the homotopy family equations. It is worth mentioning that, the solutions of the zero order equations p^0 are the initial guesses (21) which satisfied the boundary conditions (11).

The convergence of the solution depends on the choice of $\mathcal{H}_f(\eta,a_n),\mathcal{H}_\theta(\eta,\mathrm{b}_n)$, and $\mathcal{H}_\Phi(\eta,\mathrm{c}_n)$. whose assumed in the following form:

$$\mathcal{H}_f(\eta,a_n) = a_1\eta + (a_2 + a_3\eta + a_4\eta^2 + a_5 e^{-k\,\eta})\,e^{-k\,\eta} \tag{25}$$

$$\mathcal{H}_\theta(\eta,\mathrm{b}_n) = \mathrm{b}_1 + \mathrm{b}_2\eta + (\mathrm{b}_3 + \mathrm{b}_4\eta + \mathrm{b}_5\eta^2 + \mathrm{b}_6 e^{-k\,\eta})\,e^{-k\,\eta} \tag{26}$$

$$\mathcal{H}_\Phi(\eta,\mathrm{c}_n) = c_1 + c_2\eta + (c_3 + c_4\eta + c_5\eta^2 + c_6 e^{-k\,\eta})\,e^{-k\,\eta} \tag{27}$$

To get the solution of the system, there are auxiliary constants k, a_n, b_n, c_n still undetermined, so the Galerkin method is applied to minimization the three residuals obtained $\mathcal{R}_f(\eta,a_n),\mathcal{R}_\theta(\eta,\mathrm{b}_n)$, $\mathcal{R}_\Phi(\eta,\mathrm{c}_n)$ as follows:

$$\int_0^\infty \mathcal{R}_f(\eta,a_n)\,\ell_i(\eta)\;d\eta = 0 \tag{28}$$

$$\int_0^\infty \mathcal{R}_\theta(\eta,\mathrm{b}_n)\,\ell_i(\eta)\;d\eta = 0 \tag{29}$$

$$\int_0^\infty \mathcal{R}_\Phi(\eta,\mathrm{c}_n)\,\ell_i(\eta)\;d\eta = 0 \tag{30}$$

Where $\ell_i(\eta), i = 1{:}\,6$ are chosen functions taken as follows:

$$\ell_1 = 1, \ell_2 = e^{-k\,\eta}, \ell_3 = \eta\,e^{-k\,\eta}, \ell_4 = \eta^2\,e^{-k\,\eta}, \ell_5 = e^{-2k\,\eta}, \ell_5 = \eta e^{-2k\,\eta} \tag{31}$$

For example, the general solution of the momentum equation (8) at $\rho = 0.5, M = 0.5$ is

$$f(\eta) = 1.05677 + e^{-1.71872\eta}\,(9.10436 - 11.3328e^{-3.43744\,\eta} + 1.17169\,e^{-1.71872\,\eta}) +$$
$$\eta\,e^{-1.87955\,\eta}(-2.20775 - 16.0475\,e^{-3.43744\,\eta} + 1.98866e^{-1.71872\,\eta})\;+$$
$$\eta^2\,e^{-1.71872\,\eta}(-10.1921 - 1.28195e^{-3.43744\,\eta}) +$$
$$\eta^3\,e^{-1.71872\,\eta}(-0.381924 - 7.12911e^{-3.43744\,\eta})$$

Such that: k =1.71872, $a_1 = 63.0291, a_2 = -96.6372, a_3 = 49.0117, a_4 == -61.523,$
$a_5 = -1.14577$

To check the accuracy of the obtained results, a compression between the present results and the results of Hayat *et al.* [14] shown in table (1)

Table (1): compression between the present results and previous published results for $f''(0)$

	$\rho = 0, M = 0$	$\rho = 0, M = 0.4$	$\rho = 0, M = 10$
Present results OHAM	-1.0048	-1.45932	-3.3166
previous results	-1.0042 [22]	-1.45934 [14]	-3.3165 [22]

Physical aspect

Physically, the problem deals to investigate the skin friction coefficient, and Nusselt/ Sherwood numbers whose indicate to surface shear stress and rate of heat and mass transfer respectively.

Surface shear stress
$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{r=R} = \mu\, U_w \sqrt{\frac{a}{\upsilon}}\, f''(0) \tag{12}$$

since the skin friction coefficient is given by
$$C_f = \frac{2\tau_w}{\rho U_w^2} \qquad i.e. \qquad 2 f''(0) = \sqrt{\mathrm{Re}}\, C_f \tag{13}$$

Surface heat flux
$$q_w = -k \left(\frac{\partial T}{\partial r} \right)_{r=R} = -k\, (T_f - T_\infty) \sqrt{\frac{a}{\upsilon}}\, \theta'(0) \tag{14}$$

since the Nusselt number is given by
$$Nu = \frac{x\, q_w}{k (T_f - T_\infty)} \qquad i.e. \qquad \frac{Nu}{\sqrt{\mathrm{Re}}} = -\theta'(0) \tag{15}$$

Surface mass flux
$$q_m = -D_B \left(\frac{\partial C}{\partial r} \right)_{r=R} = -D_B (C_f - C_\infty) \sqrt{\frac{a}{\upsilon}}\, \phi'(0) \tag{16}$$

since the Sherwood number is given by
$$Sh = \frac{x\, q_m}{D_B (C_f - C_\infty)} \qquad i.e. \qquad \frac{Sh}{\sqrt{\mathrm{Re}}} = -\phi'(0) \tag{17}$$

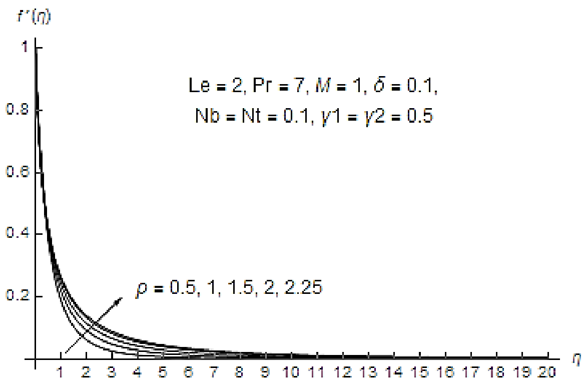

Figure (2): The velocity profiles with increasing of curvature parameter (ρ).

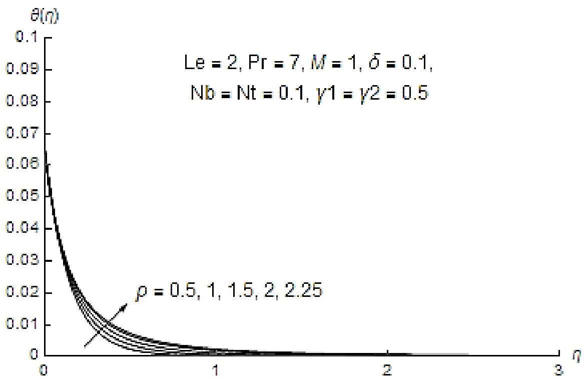

Figure (3): The temperature profiles with increasing of curvature parameter (ρ).

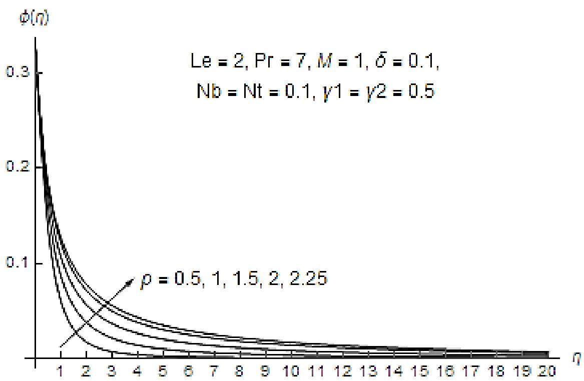

Figure (4): The concentration profiles with increasing of curvature parameter (ρ).

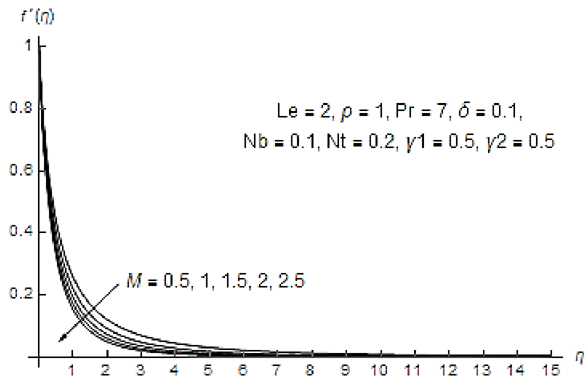

Figure (5): The velocity profiles with increasing of magnetic field parameter (M).

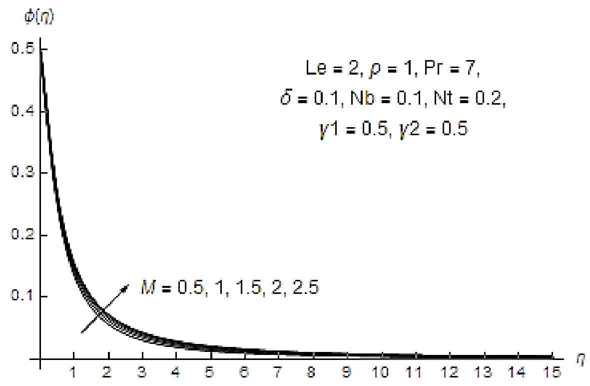

Figure (6): The concentration profiles with increasing of magnetic field parameter (*M*).

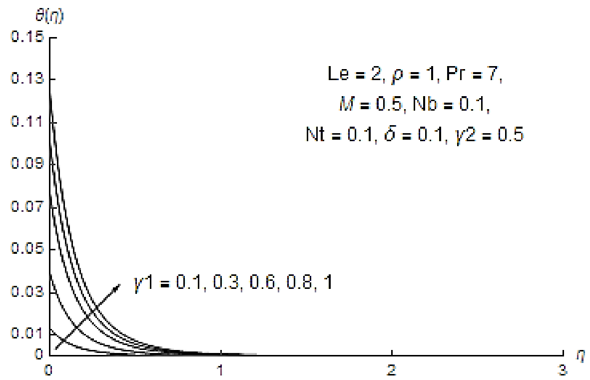

Figure (7): The temperature profiles with increasing of thermal boit number (γ_1).

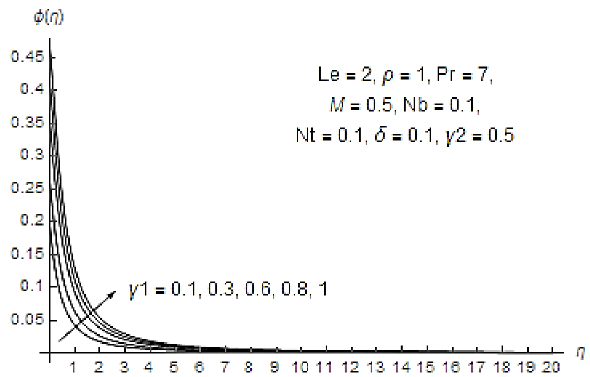

Figure (8): The concentration profiles with increasing of thermal boit number (γ_1).

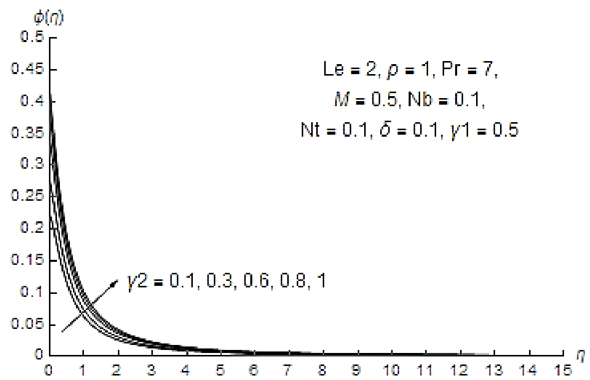

Figure (9): The concentration profiles with increasing of concentration boit number (γ_2).

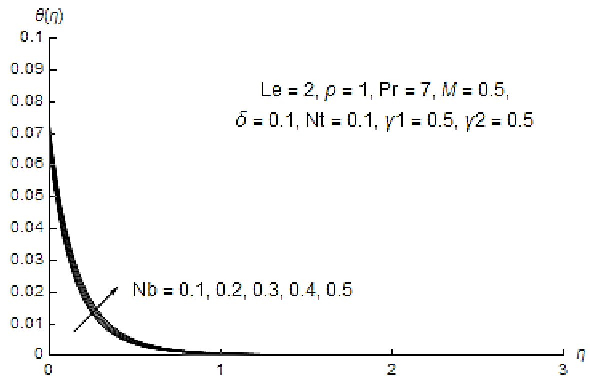

Figure (10): The temperature profiles with increasing of Brownian motion parameter (*Nb*).

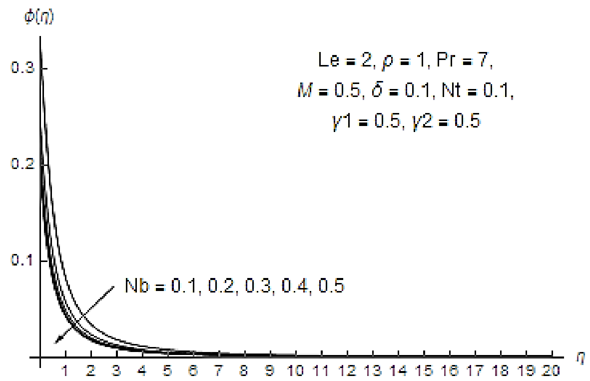

Figure (11): The concentration profiles with increasing of Brownian motion parameter (*Nb*).

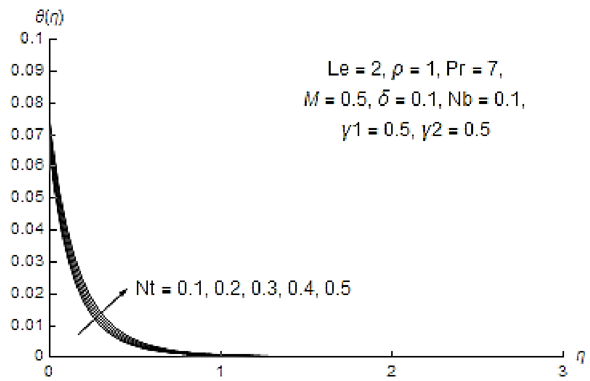

Figure (12): The temperature profiles with increasing of the thermophoresis parameter (*Nt*).

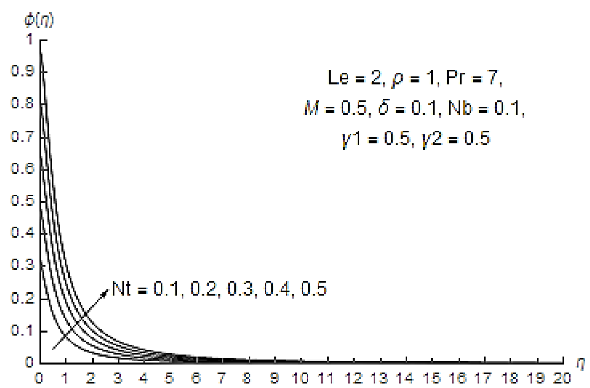

Figure (13): The concentration profiles with increasing of the thermophoresis parameter (*Nt*).

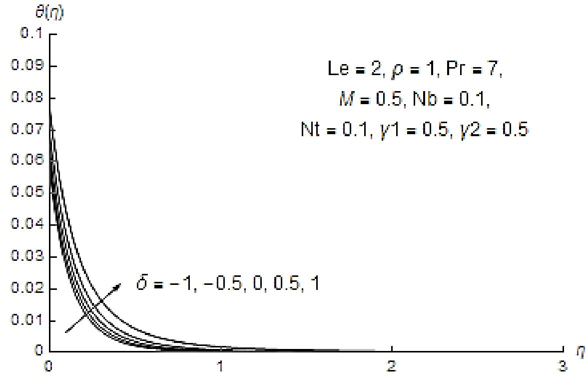

Figure (14): The temperature profiles with increasing of the heat source parameter (δ)

Table (2): values of velocity gradient, temperature gradient and concentration gradient at the surface at Nt=Nb=δ=0.1, M=1, Pr=7, γ_2=0.5

ρ	γ_1	$\frac{1}{2}\sqrt{R_e}C_f$	$\frac{Nu}{\sqrt{R_e}}$	$\frac{Sh}{\sqrt{R_e}}$
	0.10	-2.000000	-0.109886	-0.699383
0	0.20	-2.000000	-0.238933	-0.691576
	0.30	-2.000000	-0.386422	-0.683984
	0.10	-2.711306	-0.109868	-0.702305
2	0.20	-2.711306	-0.238954	-0.693641
	0.30	-2.711306	-0.386491	-0.685191

Table (3): values of velocity gradient, temperature gradient and concentration gradient at the surface at Nt=Nb=δ=0.1, M=0.5, Pr =7, γ_1=0.5.

ρ	γ_2	$\frac{1}{2}\sqrt{R_e}C_f$	$\frac{Nu}{\sqrt{R_e}}$	$\frac{Sh}{\sqrt{R_e}}$
	0.00	-2.368738	-0.734086	-0.107750
1	1.00	-2.368738	-0.734011	-0.229878
	2.00	-2.368738	-0.733940	-0.364874
	0.00	-2.711306	-0.734233	-0.107717
2	1.00	-2.711306	-0.734159	-0.229810
	2.00	-2.711306	-0.734090	-0.364846

Table (4): values temperature gradient and concentration gradient at the surface at ρ =1, M=0.5, δ=0.1, Pr=7 and Le=2.

Nt	$\frac{Nu}{\sqrt{R_e}}$	$\frac{Sh}{\sqrt{R_e}}$
0.1	-0.733847	-0.669420
0.2	-0.733316	-0.629311
0.3	-0.732752	-0.589217
Nb	$\frac{Nu}{\sqrt{R_e}}$	$\frac{Sh}{\sqrt{R_e}}$
0.1	-0.733847	-0.669420
0.2	-0.733391	-0.689485
0.3	-0.732913	-0.696174

Table (5): values of temperature gradient and concentration gradient at the surface at Nt=Nb=0.1,M=0.5,Pr =7, ρ=1, γ_1= γ_2=0.5.

δ	$\frac{Nu}{\sqrt{R_e}}$	$\frac{Sh}{\sqrt{R_e}}$
-1.00	-0.147133	-0.669395
-0.50	-0.146990	-0.669405
0.00	-0.146811	-0.669417
0.50	-0.146572	-0.669431
1.00	-0.146183	-0.669449

Discussions

The present work provides a mathematical model to the process of cooling of a moving cylinder in a Nanofluid and subjected to bottom hot fluid and normal magnetic field as well as heat source. The influence of all embedded parameters on the velocity, temperature and nanoparticles concentration within the boundary layer are shown in figures (2-14). The prandlt number of the base fluid (water) is kept constant at 7. Moreover, tables (2-5) show the effect of all embedded parameters on the surface skin friction, Nusselt number and Sherwood number.

The curvature parameter is the parameter which controls the surfaces shape such that for $\rho = 0$ the problem transformed to flat surface. The effect of this parameter on the velocity, temperature, and nanoparticle concentration appears through figures 2-4. One can observe that the increasing of surface curvature increases the velocity, temperature, and nanoparticle concentration. Also figure 3 depict that the temperature at the surface has no change with the increasing or decreasing of curvature parameter. On the other hand, the effect of curvature parameter on the surface skin friction, surface heat flux, and surface mass flux presented in table (2). One can observe that the surface skin friction, heat flux, and mass flux are higher in the case of cylinder than the case of flat surface.

The effect of magnetic field on the velocity and nanoparticles concentration showed in figures 5 and 6. One can observe that the increasing of the magnetic parameter decreases the velocity, temperature, and increase nanoparticle concentration.

On the other hand, tables 2 and 3 confirm that using magneto-hydrodynamic flow as a cooling medium increases the skin friction and decrease Nusselt number as well as the Sherwood number by limited values. Consequently, the surface shear stress increases and rate of heat transfer and rate of mass transfer decrease by increasing of magnetic parameter M.

The effect of thermal and concentration boit numbers on the temperature and nanoparticles concentration showed in figures 7, 8, and 9. It is clear that the temperature and nanoparticle concentration both increase by increasing of thermal boit number and concentration boit number.

Moreover, the effect of these parameters on the surface skin friction, Nusselt number and Sherwood number presented in tables 2 and 3. By investigation, one can observe that Nusselt number increases by increase of thermal and concentration boit numbers but the Sherwood number increases by concentration boit numbers and opposite effect appears with thermal boit number which consequently affected on the rate of heat and mass transfer.

Brownian motion is the random moving of particles suspended in a fluid (nanoparticles) resulting from their bombardment by the fast-moving atoms or molecules in the fluid. This motion controls the temperature and the concentration of the particles within the boundary layer over the surface. The Brownian motion parameter Nb is the key of this mechanism such that the increasing of Nb leads to increasing of the boundary layer temperature and decreasing of the nanoparticles concentration as shown in figures 10 and 11. On the other hand, the effect of Brownian motion on the temperature gradient, concentration gradient and the corresponding values of Nusselt number, Sherwood number presented in table 4. It is clear that this motion decreases the rate of heat and mass transfer by decreasing the Nusselt number, Sherwood number.

Thermophoresis is a phenomenon observed in mixtures of mobile particles where the different particle types exhibit different responses to the force of a temperature gradient. Explain this phenomenon appears in this study through the thermophoresis parameter Nt such that increasing this parameter leads to increasing of boundary layer temperature and nanoparticles concentration as shown in figures 12 and 13.

It is worth mentioning that the thermophoresis parameter is possible to be a positive or negative signal such that the negative value of Nt indicates to hot surface while positive to cold surface. Moreover, for hot surfaces, thermophoresis tends to blow the nanoparticles concentration boundary layer away from the surface since a hot surface repels the sub-micron sized particles from it, thereby forming a relatively particle free layer near the surface.

On the other hand, the effect of thermophoresis parameter on the temperature gradient, concentration gradient and the corresponding values of Nusselt number, Sherwood number shown in table 4. it is clear that the increasing of thermophoresis parameter decreases the rate of heat transfer and increase mass transfer.

The effect of heat source parameter (δ) on the temperature is shown in figure 14. As expected, the increasing of δ leads to increasing of boundary layer. Moreover, table 5 shows the effect of this parameter on the temperature gradient and the corresponding values of Nusselt number. The results obtained in this table indicate that the increasing of heat source parameter leads to decreasing of Nusselt number and heat transfer from the surface.

Conclusion

This study provides a mathematical model of a continuous moving cylinder embedded into a magneto-hydrodynamic Nanofluid under the effect of convective boundary conditions, Brownian motion, thermophoresis force. The following results are obtained:

- Boundary layer velocity increases in the presence of curvature parameter and the absence of magnetic parameter.
- Boundary layer temperature increases in the presence curvature parameter, thermal boit parameter, thermophoresis parameter, Brownian motion parameter, and heat source parameter
- Nanoparticles concentration increases with increase of magnetic parameter, thermal and concentration boit parameters, and thermophoresis parameter and the opposite is true with Brownian motion parameter.
- The rate of heat and mass transfer over the surface increase in the presence of thermal and concentration boit parameters by increasing the values of Nusselt and Sherwood number
- Generally, the rate of heat and mass transfer over the cylindrical surface is large than that in the case of flat surface [4], but this study indicates that this increase may be reduced in the presence of convictive boundary conditions.

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