# Analysis of heat transfer of hydromagnetic flow over a curved generalized stretching or shrinking surface with convective boundary condition

Muhammad Naveed[a*] , Zaheer Abbas[b], Qazi Muhammad Zaigham Zia[c]

and Muhammad Sajid[d]

[a]Department of Mathematics, Khwaja Fareed University of

Engineering & Information Technology Rahim Yar Khan 62400, Pakistan

[b]Department of Mathematics, The Islamia University of Bahawalpur,

Bahawalpur 63100, Pakistan

[c]Department of Mathematics, COMSATS Institute of Information Technology,
Islamabad 46000, Pakistan

[d]Department of Mathematics and Statistics, International Islamic University,

Islamabad 44000, Pakistan

**Abstract**: An investigation is carried out to discuss the heat transfer mechanism to an electrically conducting viscous fluid on a curved stretching/shrinking surface incorporated with convective boundary condition. The impact of uniform magnetic field is also considered. The mathematical formulation for the transport of heat and flow phenomena is developed by utilizing a curvilinear coordinates system. The obtained sets of partial differential equations are reconstructed into coupled nonlinear differential equations by incorporating similarity transformations. The numerical solution is attained by employing the shooting method. The obtained solutions are then used to discuss the impacts of various emerging parameters on the temperature and heat transfer across the surface. Dual nature of the solutions are obtained for definite range of convective, suction, magnetic, Prandtl and stretching or shrinking parameters. Comparison of the obtained results with the existing results for a flat sheet is found in acceptable agreement. It is noticed that with an increment in convective parameter increases the temperature of the fluid, while an increase in suction and magnetic parameters decreases the temperature of the fluid for both the solutions.

**Keywords:** Convective boundary condition, MHD flow, curved stretching/ shrinking sheet viscous fluid, numerical solution.

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*Corresponding Author. Tel.: +92 67 3001606 e-mail address: rana.m.naveed@gmail.com. (M. Naveed)

| NOMENCLATURE | | | |
|---|---|---|---|
| $B_0$ | constant magnetic field $\left[T\right]$ | $u$ | velocity component in the $s$-direction $\left[ms^{-1}\right]$ |
| $b$ | Constant $\left[-\right]$ | $v$ | velocity component in the $r$-direction $\left[ms^{-1}\right]$ |
| $f$ | dimensionless fluid velocity in $r$-direction $\left[ms^{-1}\right]$ | $T$ | Temperature $\left[K\right]$ |
| $f'$ | dimensionless fluid velocity in $s$-direction $\left[ms^{-1}\right]$ | $T_w$ | surface temperature $\left[K\right]$ |
| $v_w$ | mass suction/injection velocity $\left[ms^{-1}\right]$ | $T_\infty$ | ambient fluid temperature $\left[K\right]$ |
| $k$ | thermal conductivity of the fluid $\left[Wm^{-1}K^{-1}\right]$ | $T_f$ | final temperature $\left[K\right]$ |
| $K$ | dimensionless radius of curvature $\left[-\right]$ | **Greek symbols** | |
| $M$ | magnetic parameter $\left[-\right]$ | $c_p$ | specific heat at constant pressure $\left[Jkg^{-1}K^{-1}\right]$ |
| $p$ | Pressure $\left[kgm^{-1}s^{-2}\right]$ | $\rho$ | density of fluid $\left[kgm^{-3}\right]$ |
| $P$ | dimensionless pressure $\left[-\right]$ | $\mu$ | dynamic viscosity of the fluid $\left[m^2s^{-1}\right]$ |
| Pr | Prandtl number $\left[-\right]$ | $\nu$ | kinematics viscosity of the fluid $\left[m^2s^{-1}\right]$ |
| $S$ | Dimensionless mass transfer parameter $\left[-\right]$ | $\alpha$ | Constant $\left[-\right]$ |
| $R$ | radius of curvature $\left[m\right]$ | $\gamma$ | Biot number $\left[=h/k\sqrt{\nu/a}\right],[-]$ |
| $\mathrm{Re}_s$ | local Reynold number $\left[-\right]$ | $\theta$ | dimensionless fluid temperature $\theta(\eta)=\left[T-T_\infty/T_f-T_\infty\right][-]$ |
| $s$ | flow directional coordinate along the stretching surface $\left[m\right]$ | $\sigma$ | electrical conductivity $\left[sm^{-1}\right]$ |
| $r$ | distance normal to stretching surface $\left[m\right]$ | | |

## 1. Introduction

The analysis of heat transfer mechanism on a stretching or shrinking sheet is a significant fiels of research from last few decades. The boundary layer flow towards a stretching sheet have abundant practical applications including glass blowing, paper production, metal spinning, extrusion of plastic sheets and many more. The finishing of the ultimate product in these applications has a strong dependence on the amount of heat transfer across the surface and the fluid. This fact is explained by Karwe and Jaluria [1]. The heat transfer analysis in fluid flow by considering a constant surface temperature is rich and well established area of research and for more information readers are pointed to the book by Incropera [2].

Crane [3] examined the boundary layer flow towards a stretching sheet and obtained an exact solution. Miklavcic and Wang [4] discussed for the first time the boundary layer flow of a viscous fluid on a shrinking wall. They also pointed out the existence and (non) singularity of both numerical and closed form solutions. Bhattacharyya [5] analyzed the effects MHD flow and heat transfer subjected to heat source or sink towards a shrinking wall with mass suction. Bejan [6] examined a similarity solution by taking the boundary condition of invariable heat flux at the surface. Aziz [7] studied the classical problem of thermal boundary layers and hydrodynamic flow towards a flat plate incorporated with convective surface boundary condition. The research work to study flow and heat transfer mechanism for different kind of flow geometries are discussed by many authors, the interesting readers see the articles [8-20] and references therein.

The application of an applied magnetic field to the flow has extensive application in cosmic fluid dynamics, polymer industry, metallurgy, geophysics and in the motion of earth's core. Jafar et al. [21] discussed the hydromagnetic flow and heat transfer towards stretching/shrinking sheet in with viscous dissipation and Joule effects. Kameswaran et al. [22] examined the impacts of hydromagnetic nanofluid flow caused by stretching/shrinking surface with chemical reaction and viscous dissipation effects. Rosca [23] examined the MHD flow past a permeable shrinking surface. Numerical solution of hydromagnetic flow with impacts of viscous dissipation was studied by Mishra and Jena [24]. For more detail regarding the flow phenomenon with magnetic field the readers are directed to the articles [25-31].

Generally, the flow problem on a stretching surface have been considered towards a flat plate. However, Sajid et al. [32] has given a unique concept of research by considering a curved stretching sheet having an invariable curvature and employed a curvilinear coordinate system to formulate the flow equations. The impacts of heat transfer on a MHD flow over a curved stretching surface was examined by Abbas et al. [33]. Naveed et al. [34] explored the hydromagnetic micropolar fluid flow due to a curved stretching sheet with thermal radiation. Recently, Abbas et al. [35] studied the influence of thermal radiation on MHD slip flow of a nanofluid towards a curved stretching surface.

The prime intention of this study is to extend the investigation carried out by Aziz [7] for convective boundary condition to the flow towards a curved generalized stretching/shrinking surface by considering constant magnetic field. The dual solution occurs for the present flow problem in case of curved shrinking surface. The formulation of the problem is given in section 2. Section 3 consists of discussion of the obtained numerical results. Some conclusions are summarized in section 4.

## 2. Formulation

Consider the steady, incompressible boundary layer flow of a viscous fluid on a curved stretching or shrinking sheet curled in a circle of radius $R$ with mass transfer in a static fluid. It is considered that the sheet is being stretched or shrinked in $s$- direction with a velocity $u_w = bs,$ where $b > 0$ indicates the stretching and $b < 0$ indicates the shrinking of the sheet. Furthermore, it is also considered that $v_w$ is the constant mass transfer velocity in the fluid with $v_w > 0$ for suction and $v_w < 0$ for injection. A uniform magnetic field of strength $B_0$ is applied in the $r$-direction. The impacts of induced magnetic field can be neglected by considering the low magnetic Reynolds number regime. The surface temperature of the sheet is maintained at constant value of $T_w$ by using convective heat transfer condition and the temperature of the ambient fluid is $T_\infty$. The boundary layer equations that governs the current flow situation are

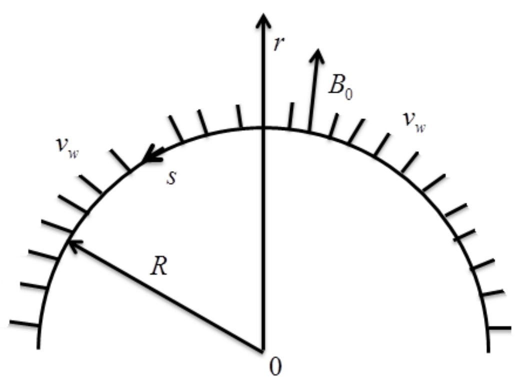



**Fig**. 1 Geometry of the flow problem

$$\frac{\partial}{\partial r}\{(r+R)v\} + R\frac{\partial u}{\partial s} = 0, \quad (1)$$

$$\frac{u^2}{r+R} = \frac{1}{\rho}\frac{\partial p}{\partial r}, \quad (2)$$

$$v\frac{\partial u}{\partial r} + \frac{Ru}{r+R}\frac{\partial u}{\partial s} + \frac{uv}{r+R} = -\frac{1}{\rho}\frac{R}{r+R}\frac{\partial p}{\partial s} + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r+R}\frac{\partial u}{\partial r} - \frac{u}{(r+R)^2}\right) - \frac{\sigma B_0^2}{\rho}u, \quad (3)$$

$$\rho c_p\left[v\frac{\partial T}{\partial r}+\frac{Ru}{r+R}\frac{\partial T}{\partial s}\right]=k\left(\frac{\partial^2 T}{\partial r^2}+\frac{1}{r+R}\frac{\partial T}{\partial r}\right), \tag{4}$$

where $v$ and $u$ indicates the velocity components in $r$ and $s$-directions, respectively, and $k, \nu, \sigma, p, \rho$ and $T$ represents the thermal conductivity, kinematic viscosity, electrical conductivity, pressure, density,and temperature of the fluid respectively. Viscous dissipation in the energy equation is also neglected in the present study.

The boundary conditions for the current study are

$$\begin{aligned} &u=-bs,\ v=-v_w, -k\frac{\partial T}{\partial r}=\mathrm{h}_f\left(T_f-T_w\right) \text{ at } r=0, \\ &u\to 0,\ \frac{\partial u}{\partial r}\to 0, T\to T_\infty \text{ as } r\to\infty, \end{aligned} \tag{5}$$

In above equation $h_f$ denotes the convective heat transfer coefficient, $b$ is constant having dimension (time)$^{-1}$ and $T_f$ is the final temperature of the fluid with $T_f > T_w > T_\infty$.

Our interest lies in reducing the problem given in equations (1)-(5) into differential equations. For this purpose we incorporate the similarity variables of the form

$$\begin{aligned} &u=asf'(\eta),\ v=\frac{-R}{r+R}\sqrt{a\nu}\, f(\eta),\ p=\rho a^2 s^2 P(\eta), \\ &\eta=\sqrt{\frac{a}{\nu}}r, \quad \theta(\eta)=\frac{T-T_\infty}{T_f-T_\infty}. \end{aligned} \tag{6}$$

The transformed equations in new variables are

$$\frac{\partial P}{\partial \eta}=\frac{f'^2}{\eta+K}, \tag{7}$$

$$\begin{aligned} \frac{2K}{\eta+K}P = &f'''+\frac{f''}{\eta+K}-\frac{f'}{(\eta+K)^2}-\frac{K}{\eta+K}f'^2 \\ &+\frac{K}{\eta+K}ff''+\frac{K}{(\eta+K)^2}ff'-Mf', \end{aligned} \tag{8}$$

$$\theta''+\frac{\theta'}{\eta+K}+\frac{\Pr K}{\eta+K}f\theta'=0, \tag{9}$$

$$\begin{aligned} &f(0)=S,\ f'(0)=b/a=\alpha,\ \theta'(0)=-\gamma\left(1-\theta(0)\right), \\ &f'(\infty)=0,\ f''(\infty)=0,\ \theta(\infty)=0, \end{aligned} \tag{10}$$

where $S=v_w/\sqrt{a\nu}$ is the mass transfer parameter such that $S>0$ indicates suction and $S<0$ is for injection and $\alpha$ represents the stretching/shrinking parameter with $\alpha>0$ for the stretching parameter and $\alpha<0$ for the shrinking parameter. Also

$M = \sigma B_0^2 / a\rho$, $\Pr = \mu c_p / k$, $K = R\sqrt{a/\nu}$ and $\gamma = h/k\sqrt{\nu/a}$, is the magnetic parameter, Prandtl number, dimensionless radius of curvature and Biot number respectively.

Elimination of pressure between equations (7) and (8) yield

$$\begin{aligned} & f^{iv} + \frac{2f'''}{\eta+K} - \frac{f''}{(\eta+K)^2} + \frac{f'}{(\eta+K)^3} - \frac{K}{\eta+K}\left(f'f'' - ff'''\right) \\ & -\frac{K}{(\eta+K)^2}\left(f'^2 - ff''\right) - \frac{K}{(\eta+K)^3} ff' - Mf'' - M\frac{f'}{\eta+K} = 0, \end{aligned} \tag{11}$$

After obtaining the velocity profile $f(\eta)$, the pressure could be obtained from Eq. (8) as

$$P(\eta) = \frac{\eta+K}{2K}\begin{bmatrix} f''' + \dfrac{f''}{\eta+K} - \dfrac{f'}{(\eta+K)^2} - \dfrac{K}{\eta+K} f'^2 \\ + \dfrac{K}{\eta+K} ff'' + \dfrac{K}{(\eta+K)^2} ff' - Mf' \end{bmatrix}. \tag{12}$$

Physically the quantities of interest are the wall temperature, local Nusselt number and the skin friction coefficient at the surface, which are defined as

$$-k\frac{\partial T}{\partial r} = \mathrm{h}_f\left(T_f - T_w\right),\ \mathrm{N}_{u_s} = \frac{sq_w}{k\left(T_w - T_\infty\right)},\ C_f = \frac{\tau_{rs}}{\rho u_w^2}, \tag{13}$$

where $q_w$ and $\tau_{rs}$ are the heat flux and shear stress at the wall along the $s$ -direction which is given by

$$\mathrm{q}_w = -k\frac{\partial T}{\partial r}\bigg|_{r=0},\ \tau_{rs} = \mu\left(\frac{\partial u}{\partial r} - \frac{u}{(r+R)}\right)\bigg|_{r=0}. \tag{14}$$

Using Eq. (6), (14) with the help of Eq. (13) becomes

$$\begin{aligned} \mathrm{Re}_s^{1/2} C_f &= f''(0) - \frac{f'(0)}{K}, \\ \theta(0) &= 1 - \frac{\theta'(0)}{\gamma}, \\ \mathrm{Re}_s^{-1/2} N_{u_s} &= -\theta'(0). \end{aligned}$$

where $\mathrm{Re}_s = as^2/\nu$ represents the local Reynolds number.

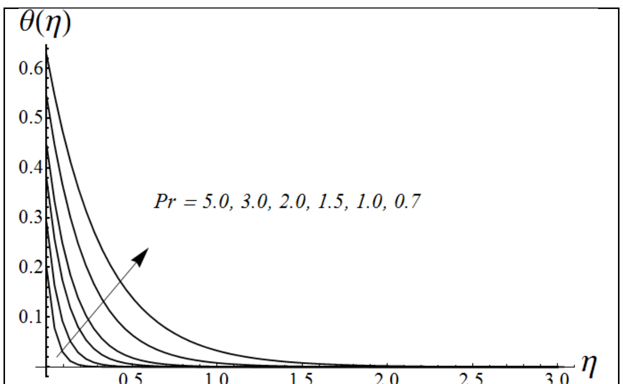
## 3. Result and discussions

Numerical solutions of the nonlinear boundary value problem composed of equation (9) and (11) subject to boundary conditions (10) are attained in terms of temperature and fluid velocity by using the shooting method with Runge-Kutta algorithm. The dual solutions in case of shrinking flow are attained by employing different initials values for $f''(0)$, $f'''(0)$ and $\theta'(0)$, where entire velocity and temperature fields fulfill the free stream boundary conditions asymptotically for entire values of the fluid parameters and choosing a suitable finite value of $\eta_\infty$ (where $\eta_\infty$ corresponds to $\eta \to \infty$). The impacts of radius of curvature on pressure distribution has already been explained in Abbas et al. [33]. The impacts of some physical parameters of interest on the fluid velocity and temperature field are plotted and given in Figs. 2-9. Tables 1-3 are made to give the comparison of current results with the available results in the literature [7, 19, 36, 37] as a special case of flat stretching or shrinking sheet.

Fig. 2 shows the variation in the temperature field $\theta(\eta)$ for several values of the Prandtl number $\Pr$ in the case of stretching sheet ($\alpha = 2$) with $S = 4$, $-4$ respectively. From Fig. 2(a) it can be seen that both the temperature and also the thermal boundary layer thickness is decreased with raise in the parameter $\Pr$ in the case of suction $S = 4$. For the case of injection $S = -4$, it is found from Fig. 2(b) that the thermal boundary layer is blown away from the sheet and heat flux at the surface becomes very small. Further, it is noticed that the thermal boundary layer thickness is still thinner for higher values of $\Pr$, but the temperature distribution is quite interesting. Fig. 3 depicts the dual solutions of the temperature field $\theta(\eta)$ for different values of the Prandtl number $\Pr$ when ($\alpha = -2$) with $S = 4, \gamma = 5, \lambda = 0.2$ and $K = 10$. It can be seen from this Fig. that for both solutions the temperature of fluid and the thermal boundary layer thickness decrease with raise $\Pr$. Fig.4 elucidates the dual solutions of the temperature distribution $\theta(\eta)$ for different values of the mass suction parameter $S$ in the case of shrinking flow ($\alpha = -1.5$) by keeping other parameters fixed. From this Fig. it is evident that both the temperature and thermal boundary layer thickness decrease by increasing the mass suction parameter $S$. The influence of the mass suction/injection parameter $S$ on the temperature profile $\theta(\eta)$ is shown in Fig. 5, for stretching flow ($\alpha = 2$) with $\Pr = 0.7, \gamma = 2, \lambda = 0.2$ and $K = 10$. It is noticed from this Fig. that the rate of heat transfer to the fluid is decreased with an increase in mass suction/injection parameter $S$ i.e. $S$ changes from $-2$ to $2$. Fig. 6 illustrates the impacts of the convective parameter (or Biot number) $\gamma$ on the temperature distribution $\theta(\eta)$ for shrinking flow ($\alpha = -2$). It is observed from this Fig. that for both solutions the temperature of fluid decreases with an increase in convective parameter $\gamma$. Fig. 7 presents the dual solutions of temperature field $\theta(\eta)$ for divers values of the magnetic parameter $M$ in case of shrinking flow ( $\alpha=-1.5$ ) with other fluid parameters are fixed.. It can be seen that the temperature and the thermal boundary layer thickness decreases with raise in magnetic parameter $M$ .

Fig. 8 is plotted to show the variation of $\theta'(0)$ with the suction parameter $S$ for divers values of the magnetic parameter $M$ . It is clear from this Fig. that for $M=0.2$, dual solution exists for $S_c < S = 3.092451$ (where $S_c$ is the critical value of $S$ ) and no solution exists for $S < S_c$. For $M=0.19$, the duality of solution is obtained for $S \geq S_c$, and no dual solution is found for $S < S_c$ The dual nature of solution for $M=0.18$ is obtained for $S \geq S_c = 3.301900$ and the solution vanishes for $S < S_c = 3.301900$. Fig. 9 is plotted to present the variation of $\theta'(0)$ with the suction parameter $S$ for various values of the Prandtl number Pr. It can be seen from this Fig. that dual solutions exist for $S \geq S_c = 3.092451$, for different values of the Prandtl number while no dual solution exists for $S < S_c = 3.092451$ ..

Table 1 indicates the numerical values of the local Nusselt number $-\theta'(0)$ for several values of Pr when $\alpha=1, S=M=0$ and $\gamma=1000$ in the case of flat stretching sheet ( $K\to\infty$ ). From this table, the comparison of the present results with those reported by Makinde and Aziz [17], Wang [37] and Khan and Pop [36] and is given and found in good agreement. Tables 2 and 3 shows the absolute values of surface temperature $\theta(0)$ and Nusselt number $-\theta'(0)$, when $M=S=0$, for diverse values of Pr and $\gamma$ in case of flat stretching sheet (taking $K\to\infty$). It is worth mentioning from this table that the present numerical results are in acceptable agreement with the result reported by Aziz [7].




**(a)**

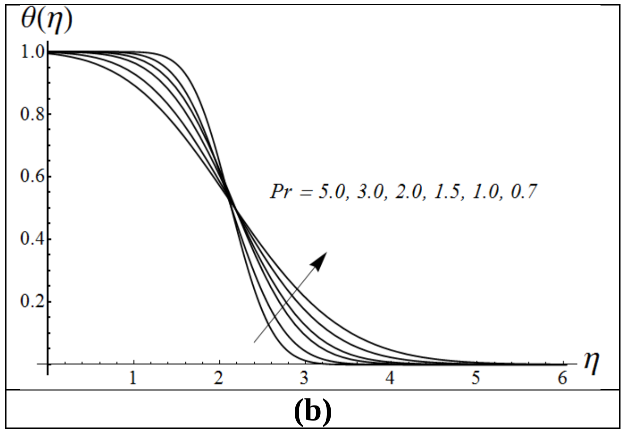



Fig. 2: Variation of $\Pr$ on $\theta(\eta)$ under suction $S=4$ (a) and injection $S=-4$ (b) with $\alpha=2, \gamma=5, K=10$ and $M=0.2$ fixed.

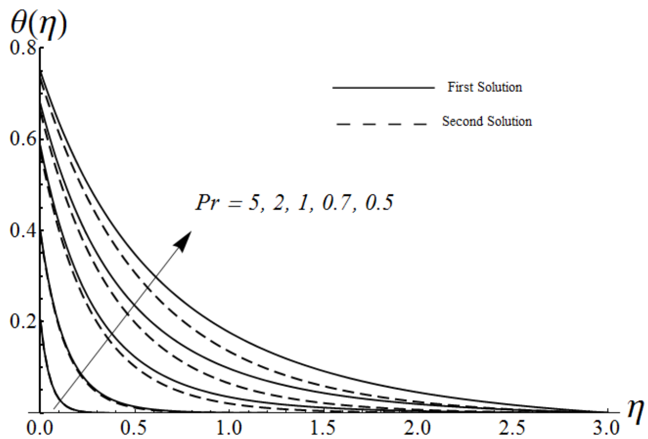



Fig. 3: Variation of $\Pr$ on the $\theta(\eta)$ with $\alpha=-2, \gamma=5, S=4, K=10$ and $M=0.2$ fixed.

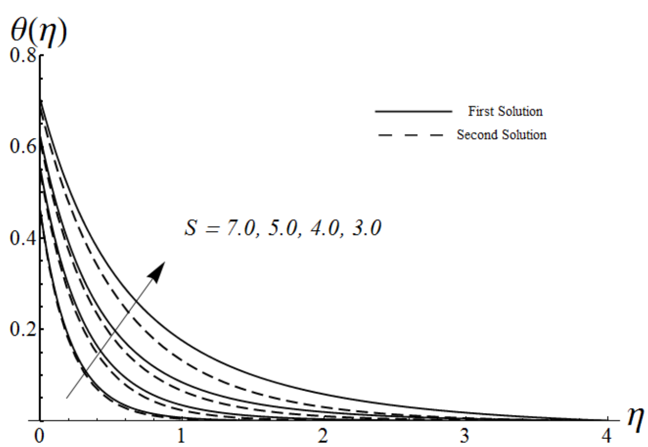



**Fig. 4:** Variation of $S$ on $\theta(\eta)$ with $\alpha=-1.5, \gamma=4, \Pr=0.7, K=10$ and $M=0.2$ fixed.

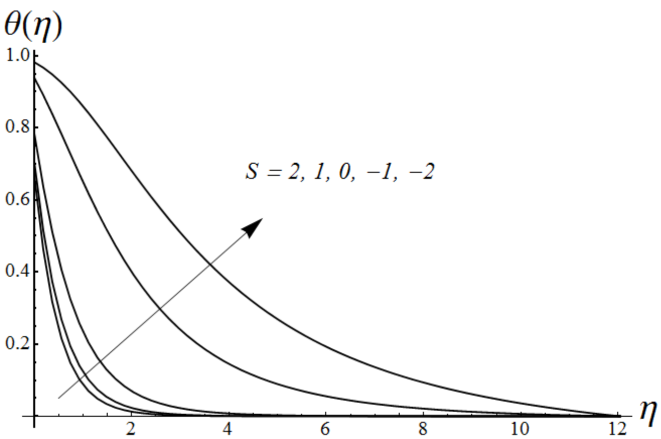



**Fig. 5:** Variation of $S$ on the temperature distribution $\theta(\eta)$ with $\alpha = 2, \gamma = 4, \Pr = 0.7, K = 10$ and $M = 0.2$ fixed.

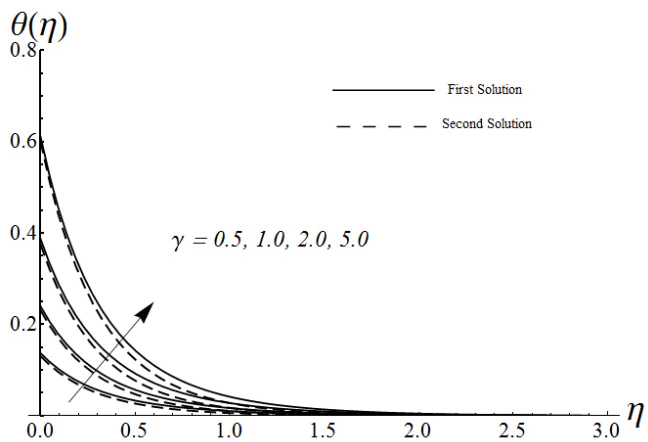



**Fig. 6:** Variation of $\gamma$ on $\theta(\eta)$ with $\alpha = -2, S = 5, \Pr = 0.7, K = 10$ and $M = 0.2$ fixed.

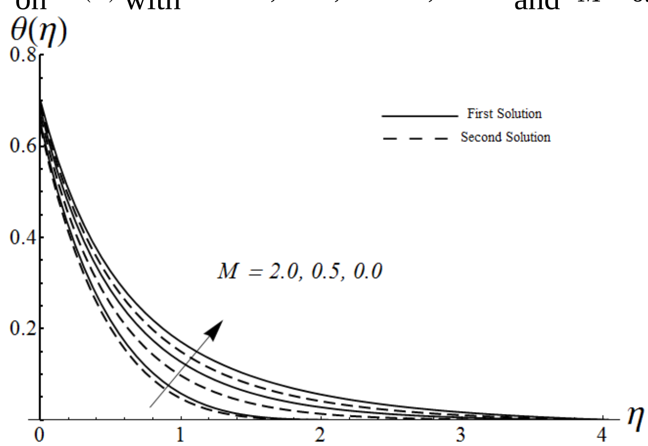



**Fig. 7:** Variation of $M$ on $\theta(\eta)$ with $\alpha = -1.5, \gamma = 4, \Pr = 0.7, K = 10$ and $S = 3$ fixed.

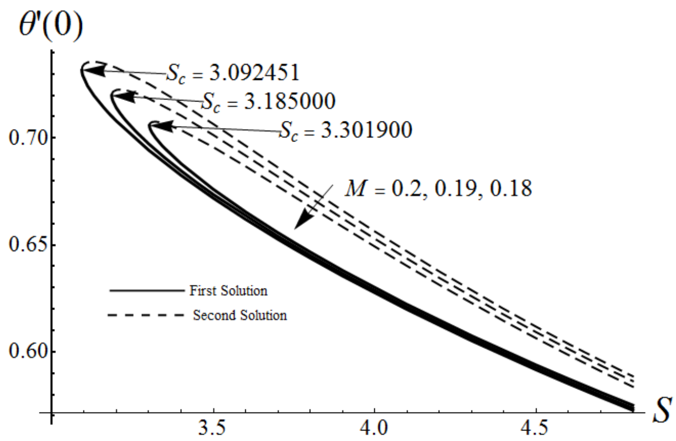



**Fig. 8:** Variation of $M$ on $\mathrm{Re}_s^{-1/2} Nu_s$ with $S$ by keeping $\alpha = -1.5, \mathrm{Pr} = 0.7, K = 10$ and $\gamma = 4$ fixed.

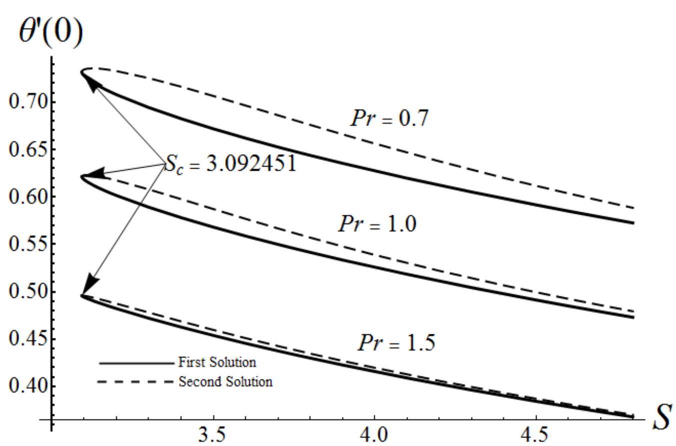



**Fig. 9:** Variation of $\mathrm{Pr}$ on $\mathrm{Re}_s^{-1/2} Nu_s$ with $S$ by keeping $\alpha = -1.5, M = 0.2, K = 10$ and $\gamma = 4$ fixed.

Table 1: Numerical values of $-\theta'(0)$ for various values of $\mathrm{Pr}$ by keeping $\alpha = 1, S = M = 0, \gamma = 1000$ and $K = 1000$.

| Pr | Khan and Pop [36] | Wang [37] | Makinde and Aziz [17] | Present results |
|---|---|---|---|---|
| 0.07 | 0.0663 | 0.0656 | 0.0656 | 0.0656 |
| 0.20 | 0.1691 | 0.1691 | 0.1691 | 0.1691 |
| 0.70 | 0.4539 | 0.4539 | 0.4539 | 0.4539 |
| 2.00 | 0.9113 | 0.9114 | 0.9114 | 0.9114 |
| 7.00 | 1.8954 | 1.8954 | 1.8954 | 1.8954 |
| 20.0 | 3.3539 | 3.3539 | 3.3539 | 3.3539 |
| 70.0 | 6.4621 | 6.4622 | 6.4622 | 6.4622 |

Table 2: Numerical values of $-\theta'(0)$ for various values of $\gamma$ and $\Pr$ by keeping $K = 1000$.

| $\gamma$ | $\Pr = 0.1$ | | $\Pr = 0.72$ | | $\Pr = 10$ | |
|---|---|---|---|---|---|---|
| | Aziz [7] | Present results | Aziz [7] | Present results | Aziz [7] | Present results |
| 0.05 | 0.0373 | 0.0373 | 0.0428 | 0.0428 | 0.0468 | 0.0468 |
| 0.10 | 0.0594 | 0.0594 | 0.0747 | 0.0747 | 0.0879 | 0.0879 |
| 0.20 | 0.848 | 0.848 | 0.1193 | 0.1193 | 0.1569 | 0.1569 |
| 0.40 | 0.1076 | 0.1076 | 0.1700 | 0.1700 | 0.2582 | 0.2582 |
| 0.60 | 0.1182 | 0.1182 | 0.1981 | 0.1981 | 0.3289 | 0.3289 |
| 0.80 | 0.1243 | 0.1243 | 0.2159 | 0.2159 | 0.3812 | 0.3812 |
| 1.0 | 0.1283 | 0.1283 | 0.2282 | 0.2282 | 0.4213 | 0.4213 |
| 5.0 | 0.1430 | 0.1430 | 0.2791 | 0.2791 | 0.6356 | 0.6356 |
| 10.0 | 0.1450 | 0.1450 | 0.2871 | 0.2871 | 0.6787 | 0.6787 |
| 20.0 | 0.1461 | 0.1461 | 0.2913 | 0.2913 | 0.7026 | 0.7026 |

Table 3: Numerical values of the surface temperature $\theta(0)$ for different values of $\gamma$ and $\Pr$ by keeping $K = 1000$.

| | $\Pr = 0.1$ | | $\Pr = 0.72$ | | $\Pr = 10$ | |
|---|---|---|---|---|---|---|
| $\gamma$ | Aziz [7] | Present results | Aziz [7] | Present results | Aziz [7] | Present results |
| 0.05 | 0.2536 | 0.2536 | 0.1447 | 0.1447 | 0.0643 | 0.0643 |
| 0.10 | 0.4046 | 0.4046 | 0.2528 | 0.2528 | 0.1208 | 0.1208 |
| 0.20 | 0.5761 | 0.5761 | 0.4035 | 0.4035 | 0.2155 | 0.2155 |
| 0.40 | 0.7310 | 0.7310 | 0.5751 | 0.5751 | 0.3546 | 0.3546 |
| 0.60 | 0.8030 | 0.8030 | 0.6699 | 0.6699 | 0.4518 | 0.4518 |
| 0.80 | 0.8446 | 0.8446 | 0.7302 | 0.7302 | 0.5235 | 0.5235 |
| 1.0 | 0.8717 | 0.8717 | 0.7718 | 0.7718 | 0.5787 | 0.5787 |
| 5.0 | 0.9714 | 0.9714 | 0.9441 | 0.9441 | 0.8729 | 0.8729 |
| 10.0 | 0.9855 | 0.9855 | 0.9713 | 0.9713 | 0.9321 | 0.9321 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 20.0 | 0.9927 | 0.9927 | 0.9854 | 0.9854 | 0.9649 | 0.9649 |

## 4. Concluding remarks

The boundary layer flow and heat transfer in a viscous fluid over a curved generalized stretching or shrinking sheet with convective boundary condition is carried out. This study is more general and novel by considering convective heating boundary condition instead of a constant heat flux or a constant temperature of the wall. The transformed similarity equations are solved numerically and the influence of various pertinent parameters on the temperature distribution is analyzed, presented and discussed graphically. Dual solutions occur for shrinking flow for certain values of generalized shrinking parameter $\alpha$. An increment in the surface convective parameter $\gamma$ yields to increase in the temperature of the fluid which results in an increase in the heat transfer rate. The enhancement in the magnetic parameter $M$ leads to decrease the temperature of the fluid.

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