FREE CONVECTION AROUND A SLENDER PARABOLOID OF NON-NEWTONIAN FLUID IN A POROUS MEDIUM

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This paper emphasizes the radiative heat transfer of non-Newtonian fluid on free convection around a slender paraboloid in a non-Darcy porous medium. The Ostwald–de Waele power-law representation is employed to express the non-Newtonian behaviour of fluid. Similarity analysis is applied to transform the set of non-dimensional PDEs into set of ODEs and then the resulting system of equations are solved by 4^{th} order Runge-Kutta scheme with Shooting technique. The control of pertinent parameters on velocity, temperature and non-dimensional heat transfer rates are analyzed through graphical representation and explored in detail. It is evident that as the radius of the slender body increases the heat transfer coefficient decreases but the role of radiation on heat transfer rate getting reduced for all feasible values of the power-law index parameter.

Keywords: Power-law fluid, Radiation, Free convection, Non-Darcy, Porous medium.

1. Introduction

From the past few years, due to broad area of appliance of non-Newtonian fluids in industries, numerous researches have been reported by many researchers. Several researchers, to mention a few [1-3, 8, 10] examined the non-Newtonian fluid flow over various geometries embedded in Darcy or non-Darcy porous medium. The Ostwald–de Waele power-law form is the most renowned and widely used model to describe the non-Newtonian fluid behavior. Convective heat transport accompanied with non-Newtonian fluid through porous media finds extensive applications in geophysical and thermal engineering related problems. In this direction, a similarity solution of unsteady flow over axisymmetric bodies of non-Newtonian fluids was presented by Mohanty [3]. Nakayama and Koyma [4] reported that any two dimensional/axisymmetric body offers similarity solution while surface temperature follows a particular class of distribution. Nakayama and Koyama [5] further extended their work to free convection with non-Newtonian fluids. Double diffusive convection of heat and mass from slender paraboloid and cylinder studied by Lai et. al [6]. While, Singh and Chandrika [7] obtained integral solution of the same problem reported by Lai et. al [6]. It is noticed that the integral solution also justified the results of Lai et. al [6] obtained numerically by RK method. Shenoy [8] discussed the heat transfer attributes of non-Newtonian power law fluids with/without yield stress embedded in porous media considering oil reservoir and geothermal engineering applications. The heat transfer phenomenon of power-law fluids over a non-isothermal stretching sheet was reported by Datti and Prasad [15]. Recently, Babu and Sandeep [16] examined cross-diffusion effects for power-law fluid in presence of MHD over a slendering stretching sheet. Whereas, Reddy et. al [17] studied the effect of frictional heating of ferrofluid over a slendering stretching sheet in presence of aligned magnetic field. They concluded that the fluid velocity diminished by rise in the aligned angle or surface thickness. In this direction, Reddy et. al [18] investigated convective heat transport of non-Newtonian Casson fluid over a heated paraboloid of revolution. It is established that Casson ferrofluid shows improved act on heat transfer with compared to Casson fluid.

On account of the surface characteristics and solid geometry of the heated body, radiative heat transport phenomena are comparable with that of the convective heat transport in many realistic uses. Natural convection considering radiation effect in porous medium using Rosseland approximation for the radiative heat flux was reported by Raptis [9]. While, the thermal radiation from a horizontal plate of power-law fluids reported in Mohammadein and El-Amin [10], considering power law variation in surface temperature.

From the existing literatures, radiative heat transport of non-Newtonian fluid on free convection around a slender paraboloid in porous medium has not been investigated so far. Motivated by the above studies, the main purpose of this work is to get a similarity solution for the abovementioned problem. The transformed equations are solved with the help of Runge-Kutta fourth order method. The role of different flow controlling parameters on flow, temperature and local Nusselt number is shown graphically.

2. Mathematical Formulation

The two-dimensional, laminar, steady, boundary layer flow due to free/natural convection around slender paraboloid embedded in non-Darcy homogeneous and isotropic porous medium. The porous medium is saturated with non-Newtonian fluid. The power-law model is considered to characterize the non-Newtonian fluid behaviour. The fluid is assumed to be gray, absorbing-emitting radiation but non-scattering medium. The porous medium is considered to be clear and in thermal equilibrium with the fluid. The temperatures of the body and the ambient media are taken to be at constant temperatures T_w and T_∞, respectively. The fluid flow in the porous medium is moderate. The permeability of the medium is supposed to be small so that the Forchheimer flow model is appropriate and the boundary–drag effect is ignored. Assuming the linear Boussinesq approximations, the governing equations, of flow and energy may be written as:

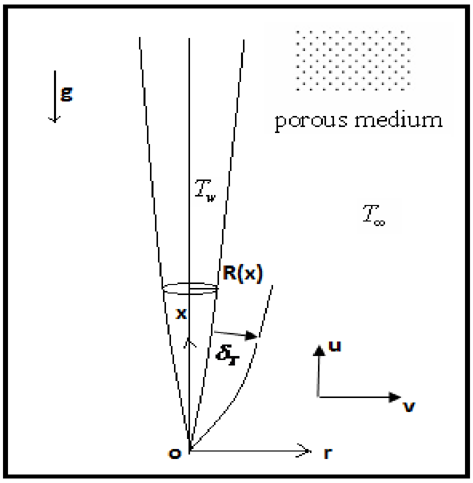

Fig. 1: Coordinate system and physical diagram

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0 \tag{1}$$

$$\frac{\partial u^n}{\partial r} + \frac{\rho_\infty b K^*}{\mu^*}\frac{\partial u^2}{\partial r} = \frac{K^* \rho_\infty g}{\mu^*}\beta_T \frac{\partial T}{\partial r} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{1}{\rho_\infty c_p}\frac{\partial}{\partial r}(rq_r) \tag{3}$$

The boundary conditions:

At the surface: $r = R(x): v = 0, T = T_w$ (4)

At the ambient medium (infinity): $r \to \infty: u \to 0, T \to T_\infty$ (5)

where, $R(x) = \left\{\frac{\alpha\mu^*}{\rho_\infty K^* g\beta_T(T_w - T_\infty)}\right\}^{\frac{1}{n}}(x\eta_0)^{\frac{1}{2}}$ describes the surface shape of the slender body, r and x radial and axial distance coordinates; u and v are velocity components along the x- and r- directions respectively, α indicates the effective thermal diffusivity, ρ_∞ is the reference density, T is the temperature, g represents the acceleration due to gravity, k is the effective thermal conductivity of the saturated porous medium, β_T thermal expansion coefficients, c_p is the specific heat at constant pressure, b indicates the empirical constant related to the Forchheimer porous inertia term, μ^* represents the consistency index of power-law fluid, n stand for the power-law index in which $n < 1$ refers to pseudo-plastics fluid, $n = 1$ represents Newtonian fluid and $n > 1$ indicates dilatants fluid. The modified permeability of the flow K^* as a function of the power-law index n is well defined in Christopher and Middleman [13] and Dharmadhikari and Kale [14].

The continuity equation is automatically satisfied by defining the stream function $\psi(x, r)$ such that

$u = \frac{1}{r}\frac{\partial\psi}{\partial r}$ and $v = -\frac{1}{r}\frac{\partial\psi}{\partial x}$.

The Rosseland approximation for radiation may be written as:

$$q_r = -\frac{4\sigma}{3\kappa^*}\frac{\partial T^4}{\partial r} \tag{6}$$

Using equation (6) into the equation (3):

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{4\sigma}{3\kappa^*}\frac{1}{\rho_\infty c_p}\frac{\partial}{\partial r}\left(r\frac{\partial T^4}{\partial r}\right) \tag{7}$$

where σ and κ^* are the Stefan-Boltzman constant and the mean absorption coefficient, respectively.

Expanding T^4 about T_∞ by Taylor series and ignoring higher order terms:

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4 \tag{8}$$

With the help of the approximation (8) into the equation (7):

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{1}{\rho_\infty c_p}\frac{16\sigma T_\infty^3}{3\kappa^*}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{9}$$

Making the use of following transformations:

$$\eta = \left(\frac{r}{x}\right)^2 Ra_x,\ \psi(\eta) = \alpha x f(\eta),\ \theta = \frac{T - T_\infty}{T_w - T_\infty}$$

where $Ra_x = \left(\frac{x}{\alpha}\right)\left[\frac{\rho_\infty g\beta_T K^*(T_w - T_\infty)}{\mu^*}\right]^{\frac{1}{n}}$.

The transformed coupled nonlinear differential equations are:

$$\left(nf'^{n-1} + 2^{3-n}Gr^* f'\right)f'' = \frac{1}{2^n}\theta' \tag{10}$$

$$\left(1 + \frac{4}{3}R_d\right)(2\eta\theta'' + 2\theta') + f\theta' = 0 \tag{11}$$

and the related boundary conditions become

$$f - \eta f' = 0, \theta = 1 \ \text{at}\ \eta = \eta_0 \qquad (12)$$

$$f' \to 0, \theta \to 0 \ \text{as}\ \eta \to \infty. \qquad (13)$$

In the above, $Gr^* = \left(\frac{\rho_\infty b K^*}{\mu^*}\right)\left(\frac{\alpha}{x}\right)^{2-n} Ra_x^{4-2n/3}$ is the modified Grashof number, $R_d = \frac{4\sigma T_\infty^3}{k\,\kappa^*}$ is the radiation parameter.

The Nusselt number is defined as $Nu_x = \frac{hx}{k(T_w - T_\infty)}$, where $h(T_w - T_\infty) = \left[\left(\frac{16\sigma T_\infty^3}{3\kappa^*} + k\right)\frac{\partial T}{\partial r}\right]$. The non-dimensional form of the Nusselt number may be written as:

$$\frac{Nu_x}{Ra_x^{\frac{1}{2}}} = -2\eta_0^{\frac{1}{2}}\theta'(\eta_0)\left[1 + \frac{4}{3}R_d\right]. \qquad (14)$$

3. Results and discussion

The system of differential equations (10) and (11) with the boundary conditions (12) and (13), is solved using 4th order RK-method by giving proper initial presume values for $f'(\eta_0)$ and $\theta(\eta_0)$ to match the values with the boundary conditions at $f'(\infty)$ and $\theta(\infty)$ respectively, these obtained values for $f'(\infty)$ and $\theta(\infty)$ are matched with the specified boundary conditions by making use of Newton-Raphson method with a shooting scheme. To satisfy the boundary condition at infinity, an integration length is chosen carefully. In the proposed study, $\eta_\infty = 60$ .In order to assess the correctness of the solution, the current results for the Nusselt number in case of pure thermal buoyancy driven flow of Newtonian fluid saturated in a Darcy Porous medium in absence of radiation are compared with those achieved by Lai et. al [6] and it is found that the results are in good agreement.

Our main focus is to explore heat transfer and associated changes in flow and temperature distributions caused by radiation from a slender paraboloid on natural convection. Interest has been sited to the important role played by the governing parameters; Power- law index, radiation parameter and modified Grashof number. Numerical computations were taken out for the various dimensionless parameters. Flow, temperature and heat transfer rates are plotted for some certain values of the flow influencing parameters.

In Figs. 2 and 3 the non-dimensional velocity profiles are plotted as a function the similarity variable η for $Gr^* = 0.0$ (Darcy porous media) and $Gr^* = 1.0$ (Non-Darcy Porous Media), respectively with various values of n and R_d. It is evident that for both in Darcy and non-Darcy porous media the velocity distribution increases inside the boundary layer as n and R_d increased. Also it is noted that the growth of the velocity profile due to radiation is relatively higher for dilatant fluids $(n > 1)$ than pseudoplastics $(n < 1)$. Moreover, Fig.2 depicts that in non-Darcy porous media the slip velocity $f'(0.01)$, decreases as the value of n increases while there is no changes in slip velocity in Darcy porous medium.

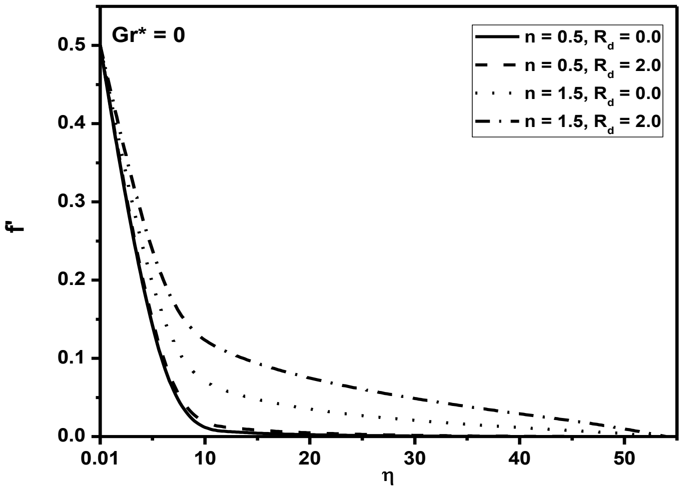

Fig. 2

Fig. 2: Velocity profiles vs. η (Darcy Porous media)

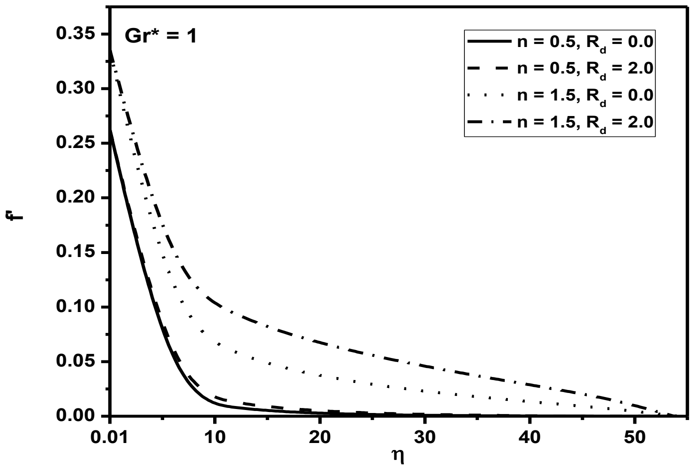

Fig. 3

Fig. 3: Velocity profiles vs. η (non-Darcy Porous media)

Figures 4 and 5 illustrate the non-dimensional temperature distribution θ with the similarity variable η for two different values of n and R_d with $Gr^* = 0.0$ (Darcy porous media) and $Gr^* = 1.0$ (Non-Darcy Porous Media), respectively. The temperature profile inside the boundary layer rises with increasing R_d for all n. On the other hand the thickness of the temperature boundary layer decreases with increasing n which results an increase in heat transfer coefficient with increasing n as indicated in the Figs. 6-9. As similar to the velocity distribution, the effect radiation parameter R_d on temperature distribution in dilatant fluids$(n > 1)$ is relatively higher than that of a pseudo-plastics $(n < 1)$.

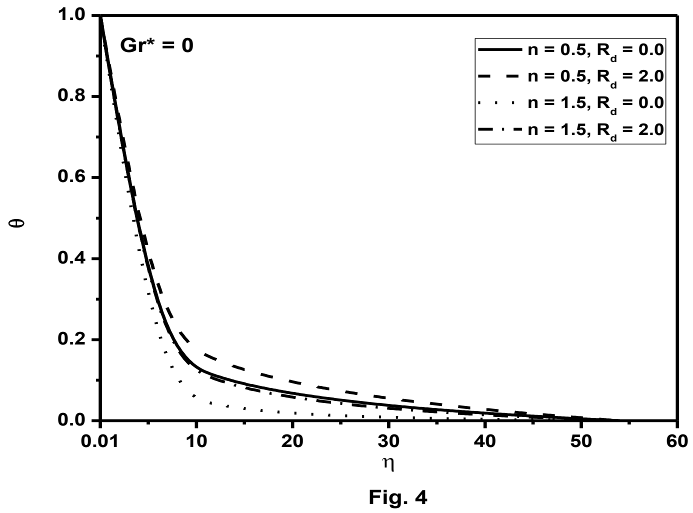

Fig. 4: Temperature profiles vs. η (Darcy Porous Media)

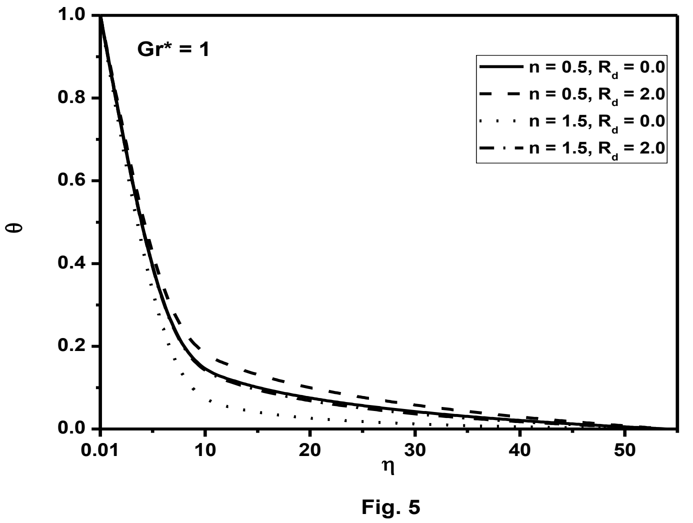

Fig. 5: Temperature profiles vs. η (non-Darcy Porous Media)

Figures 6 and 7 illustrate the deviation of non-dimensional heat transfer coefficient with $n$varying Gr^* and R_d keeping $\eta_0 = 0.01$ and $\eta_0 = 0.1$, respectively. It is observed that the non-dimensional heat transfer coefficient increases with n and R_d while decreases with an increasing Gr^*. The variation of dimensionless Nusselt number versus n for different η_0 and R_d with $Gr^* = 0.0$ and $Gr^* = 1.0$ are shown in figures 8 and 9, respectively. It is evident that as the radius of the slender body η_0 increases the heat transfer coefficient decreases but the role of radiation on heat transfer rate getting reduced for all feasible values of n. Moreover, it is observed that heat transfer coefficient decreases with an increase in the value of Gr^*.

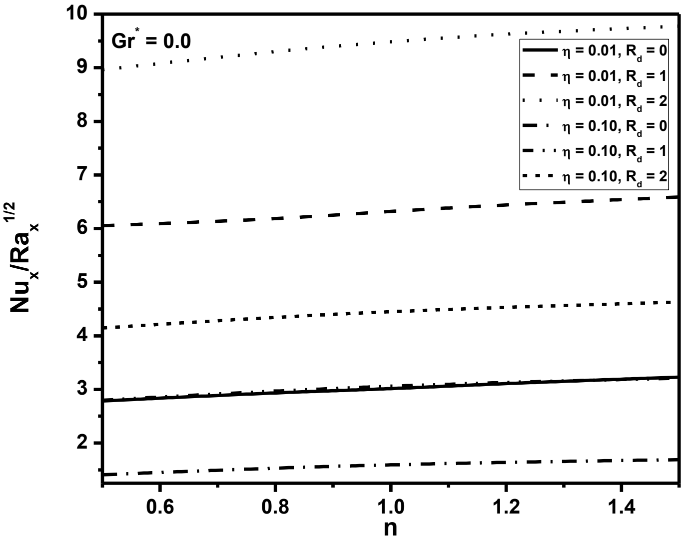

Fig. 6

Fig. 6: non-dimensional heat transfer coefficient vs. *n* (Darcy Porous Media)

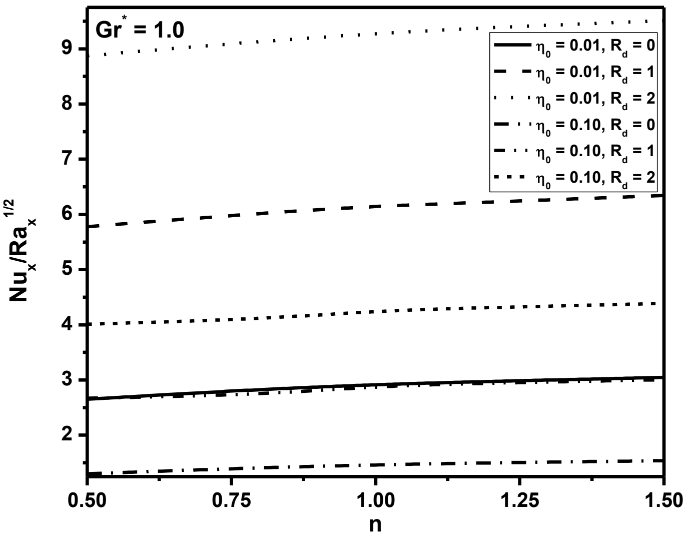

Fig. 7

Fig. 7: non-dimensional heat transfer coefficient vs. *n* (non-Darcy Porous Media)

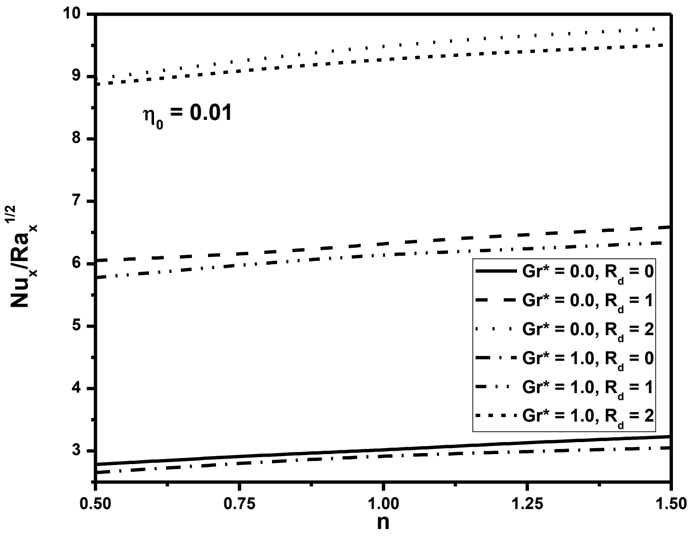

Fig. 8

Fig. 8: non-dimensional heat transfer coefficient vs. *n* ($\eta_0 = 0.01$)

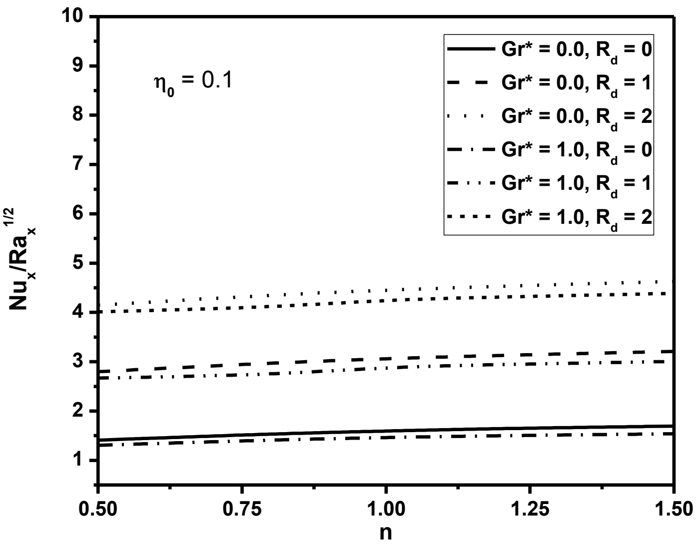

Fig. 9

Fig.9: non-dimensional heat transfer coefficient vs. *n* ($\eta_0 = 0.1$)

Conclusion

Radiative heat transport of non-Newtonian fluid on free convection around a slender paraboloid in a porous media is investigated. The similarity transformations used to reduce the governing equations. The numerical solutions are obtained by using 4th order Runge Kutta method. The consequence of various flow controlling parameters on flow, temperature and heat transfer rates are shown graphically. It is found that the non-dimension heat transfer coefficient raises with power law index and radiation parameter while decreases with increasing non-Darcy parameter. The most imperative conclusion is that, as the radius of the slender body increases the heat transfer rate decreases but the influence of radiation on the same reduces for all three types of power law fluids ($n > 1, n = 1, n < 1$).

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Nomenclature

g	Acceleration due to gravity [$\mathrm{m/s^2}$]
x, r	Axial and radial co-ordinates [m]
b	Coefficient in the Forchheimer term
f	Dimensionless stream function
k	Effective thermal conductivity
K^*	Modified permeability of the porous medium of power law fluid [$\mathrm{m^2}$]
Ra_x	Modified Rayleigh number
Gr^*	Non-Darcian (inertia) parameter or Grashof number based on permeability for power law fluid
Nu	Nusselt number
n	Power law index
h	Local heat transfer coefficient
q_r	Radiative heat flux
R_d	Radiation parameter
c_p	Specific heat at constant pressure [J / kg K]
$R(x)$	Surface shape of the slender body
u, v	Velocity components in x and r directions [m/s]
T	Temperature [K]

Greek Symbols

β_T	Coefficient of thermal expansion [$\mathrm{1/K}$]
$\mu*$	Consistency index of power-law fluid [$\mathrm{kg/(ms)}$]
η_0	Dimensionless radius of the slender body
η	Dimensionless similarity variable
ψ	Dimensionless stream function
θ	Dimensionless temperature
α	Effective thermal diffusivity [$\mathrm{m^2/s}$]

κ^* Mean absorption coefficient

ρ_∞ Reference density [$\mathrm{kg/m^3}$]

σ Stefan-Boltzman constant

Subscripts

w,∞ Conditions at the surface of the slender body and the ambient medium

References

[1] Chen, H. T. and Chen, C. K.: Natural convection of a non-Newtonian fluid about a horizontal cylinder and sphere in a porous medium, *International Communications in Heat and Mass Transfer* 15 (1988), pp. 605–614.

[2] Chen, H. T. and Chen, C. K.: Natural convection of non-Newtonian fluids along a vertical plate embedded in a porous medium, *ASME Journal of Heat* Transfer, 110 (1988), pp. 257 -260.

[3] Mohanty, H. K.: Similarity solutions of unsteady laminar, incompressible boundary layer equations for flow, heat and mass transfer in non-Newtonian fluids around axisymmetric bodies, *Rheological Acta*, 17 (1978), 4, pp. 342-352.

[4] Nakayama, A .and Koyama, H.: Buoyancy induced flow of non-Newtonian fluids over a non-isothermal body of arbitrary shape in a fluid-saturated porous medium, *Applied Scientific Research*, 48 (1991), pp. 55–70.

[5] Nakayama, A. and Koyama, H.: Free convective heat transfer over a non-isothermal body of arbitrary shape embedded in a fluid saturated porous medium, *ASME Journal of Heat Transfer*, 109 (1987), pp. 125 -130

[6] Lai, F. C., Choi, C. Y. and Kulacki, F. A.: Coupled Heat and mass transfer by natural convection from a slender bodies of revolution in porous media, *International Communications in Heat and Mass Transfer*, 17 (1990), pp. 609-620.

[7] Singh, B. B. and Chandarki, I. M.: Integral treatment of coupled heat and mass transfer by natural convection from a cylinder in porous media, *International Communications in Heat and Mass Transfer*, 36 (2009), pp. 269-273.

[8] Shenoy, A. V.: Darcy-Forchheimer natural, forced and mixed convection heat transfer in non-Newtonian power-law fluid –saturated porous media, *Transport in Porous Media*, 11(1993), pp. 219-241.

[9] Raptis, A.: Radiation and free convection flow through a porous medium, *International Communications in Heat and Mass Transfer*, 25 (1998), pp. 289–295.

[10] Mohammadein, A. A. and El-Amin, M. F.: Thermal radiation effect on power law fluid over a Horizontal plate embedded in a porous medium, *International Communications in Heat and Mass Transfer*, 27 (2000), pp. 1025–1035.

[11] Shenoy, A.V.: Non-Newtonian fluid heat transfer in porous media, *Advances in Heat Transfer*, 24(1994), pp. 102-184.

[12] Bejan, A.: *Convective Heat Transfer*, Wiley, New York, 1984, pp.343-416.

[13] Christopher, R. H. and Middleman, S.: Power-law fluid flow through a packed tube, *Ind. Eng. Chem. Fundamentals*, 4(1965), pp. 422-426.

[14] Dharmadhikari, R. V. and Kale, D. D.: The flow of non-Newtonian fluids through porous media, *Chemical Engineering Science*, 40 (1985), pp. 527-529.

[15] Datti, P. S. and Prasad, K. V.: Non-Newtonian Power-law fluid flow and heat transfer in a porous medium over a nonisothermal stretching sheet, *International Journal of Fluid Mechanics Research*, 35 (2008), 5, pp. 417-433.

[16] Babu, M. J. and Sandeep, N.: MHD non-Newtonian fluid flow over a slendering stretching sheet in the presence of cross-diffusion effects, *Alexandria Engineering Journal*, 55 (2016), 3, pp. 2193-2201.

[17] Reddy, J. R., Sugunamma, V. and Sandeep, N.: Effect of frictional heating on radiative ferrofluid flow over a slendering stretching sheet with aligned magnetic field, *The European Physical Journal Plus*, 132, (2017), 7. (https://doi.org/10.1140/epjp/i2017-11287-1)

[18] Reddy, J. R., Sugunamma, V. and Sandeep, N.: Enhanced heat transfer in the flow of dissipative non-Newtonian Casson fluid flow over a convectively heated upper surface of a paraboloid revolution, *Journal of Molecular Liquids*, 229 (2017), pp. 380-388.

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