NUMERICAL INVESTIGATION OF RELIABILITY OF PUMP SYSTEMS FROM THERMAL POWER PLANTS

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This article deals with numerical investigation of reliability of pump systems from thermal power plants. The reliability is examined by using intensity function for a system composed of n-linearly ordered pumps. The time between failures of pumps is taken as experimental. Firstly, appropriate model for the data is determined. Estimates of parameters are obtained with maximum likelihood method and intensity function models are written. Finally, reliability of consecutive 3-out-of-5:F systems are calculated for certain time periods by using the obtained intensity functions.

Key words: *reliability, time between failure, power law model, maximum likelihood method*

Introduction

Systems are generally classified as repairable and non-repairable systems. There is not recycling of non-repairable systems after once they are failure. Such systems can be modelled with for example, Weibull distributions. The system may not fail fully, but can degrade and there may exist several operation level of the system. The operation levels of the components can range from perfect functioning up to complete failure. The quality of the system is completely determined by components [1]. A repairable system is a system which brings back any intervention after failures. Repairable systems and modelling of these systems have been studied by [2-5].

The application of thermal sciences is not only limited to common engineering scenarios but also it is being potentially extended to the realms which have not been envisaged earlier [6]. Thermal power plants are industrial goods that produce electricity. Moreover, these plants are important to customer and are presumed to have a service life of greater than twenty years. The analysis is related to the pumps of thermal power plants. These pumps failures were revealed because of the pressure of pumps, leaks, vibrations or malfunction indications by sensors or alarms. In this study, let us consider of a system composed of n-linearly ordered pumps. The systems fails if and only if at least k consecutive pumps.

An n-component system that works if and only if at least k of the n components work is called a *k-out-of-n:G* system. An n-component system that fails if and only if at least k of the n components fail is called a *k-out-of-n:F* system. Thus, a *k-out-of-n:G* is equivalent to an *(n–k+1)-out-of-n:F* system [7]. A consecutive *k-out-of-n:F* system consists of n components such that the system failures if and only if at least k consecutive components fail. The first, consecutive *k-out-of-n:F* system was studied by Kontoleon [8].

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In many systems, reliability evaluation such as ones encountered in telecommunications, the design of integrated circuits, microwave relay stations, oil pipeline systems and vacuum systems in accelerators, computer ring networks, and spacecraft relay stations has been applied consecutive *k-out-of-n* system models [9].

The reliability of a system is probability of operation without the fault of the systems under a certain time period and conditions. The reliability of technical systems is one of the most important research subjects in the point reached by modern science [10]. The reliability of the subsystem is based on analysis of time between failures.

The $R(t)$ is the probability of non-failure of a system until the time, t [11]. Thus, $R(t)$ can be written as eq. (1):

$$R(t) = P(T > t) = 1 - F(t) \tag{1}$$

The most important step in reliability analysis is to establish an appropriate model for the data. For this, times between failures (TBF) are examined. If TBF are independent and in the same distribution, they are modelled by renewal process or homogeneous Poisson process. Suitable probabilistic models that can be used in this process are density function as Weibull, normal, and exponential. Otherwise, TBF are modelled with non-homogeneous Poisson process (NHPP) [12].

Non-homogeneous Poisson process and modelling

The NHPP is a process commonly used in reliability. The NHPP are used, for example, in modelling of failures of gas turbine engines [13] failures of hard drives in [14], failures of machine tools in [15], or for modelling software reliability [16].

An NHPP model is fully characterized by the intensity function $w(t)$, commonly denoted rate of occurrence of failures [17]. The NHPP becomes homogeneous Poisson process with $w(t) = w$.

Suppose $N(t)$ is number of failures experienced in the time interval from 0 to t. If $N(t)$ is Poisson distributed with parameter $\int_0^t w(t)\mathrm{d}t$, so that in particular is as eq. (2):

$$E[N(t)] = \int_0^t w(t)\mathrm{d}t = W(t) \quad \text{and} \quad \mathrm{Var}[N(t)] = \int_0^t w(t)\mathrm{d}t = W(t) \tag{2}$$

In NHPP, number of failures from any $(t_1, t_2]$ has Poisson distributed with $\int_{t_1}^{t_2} w(t)\mathrm{d}t$ mean. Accordingly, probability of k failures is as eq. (3):

$$P[N(t_2) - N(t_1) = k] = \left\{ \frac{\left[\int_{t_1}^{t_2} w(t)\,\mathrm{d}t \right]^k}{k!} \right\} \exp\left[-\int_{t_1}^{t_2} w(t)\,\mathrm{d}t \right] \tag{3}$$

In NHPP, power law model is used commonly used. One reason for its popularity is that the rate of occurrence of failures as a function of t is of the same form as the hazard rate of a Weibull distribution [18]. The intensity function and cumulative intensity function of power law model is as following eqs. (4) and (5), respectively:

$$w(t) = \lambda\beta\, t^{\beta-1} \tag{4}$$

and

$$W(t) = \lambda t^{\beta} \tag{5}$$

Maximum likelihood estimation in the power law model

Suppose that the data which are available from m independent systems governed by NHPP with the same intensity function $w(t)$, where system j is observed in the time interval $[S_j, T_j]$, $j = 1, 2, ..., m$ with events observed at times $t_{1j}, t_{2j}, ..., t_{n_j j}$. The likelihood function of this data is given [19]:

$$L = \prod_{i=1}^{m} \left\{ \prod_{j=1}^{n_j} w(t_{ij}) \right\} \exp\{-[W(T_j)] - W(S_j)\} \tag{6}$$

In eq. (4), the log-likelihood function for the power-law model is:

$$l = \log L = n \log \lambda + n \log \beta + (\beta - 1) \sum_{j=1}^{m} \sum_{i=1}^{n_j} \log t_{ij} - \lambda \sum_{j=1}^{m} [T_j^{\beta} - S_j^{\beta}] \tag{7}$$

where $n = \sum_{j=1}^{m} n_j$. The $\hat{\lambda}$ and $\hat{\beta}$ parameter estimations are as eqs. (8) and (9), respectively:

$$\hat{\lambda} = \frac{n}{m t_n^{\beta}} \tag{8}$$

$$\hat{\beta} = \frac{n}{n \log t_n - \sum_{j=1}^{m} \sum_{i=1}^{n_j} \log t_{ij}} \tag{9}$$

where $T_j = t_n$ is total continuous period, and n is number of total failures.

Application

In this study, obtained as experimental, TBF for test the reliability of five pump machines are handled. Firstly, appropriate model for data is determined. Then, the distributions are deduced. Finally, reliabilities of consecutive *3-out-of-5:F* systems with $n = 5$ and $k = 3$ were calculated for certain time periods by using these distributions. Minitab 14 software package was used for the model analysis, see Appendix.

The first step is to establish an appropriate probabilistic model for the TBF data. Modelling should be done according to the change of TBF. Figure 1 is drawn for cumulative time between failures (CTBF) which is corresponding to the cumulative failure numbers (CFN). Thus, it is shown that the failures contain no linearity. So, the selection of the appropriate model and estimates should be made.

Power law model which is a form of NHPP will be used. From eqs. (8) and (9), estimations of parameters can be found easily for five machines.

From tab. 1, according to the obtained estimation of parameters, power law models are written as eq. (10).

$$\begin{aligned} w_1(t) &= (0.051)\,(0.8385)\,t^{0.1615} \\ w_2(t) &= (7.44 \cdot 10^{-5})\,(1.749)\,t^{0.749} \\ w_3(t) &= (1.512 \cdot 10^{-4})(1.667)\,t^{0.667} \\ w_4(t) &= (0.2033)\,(0.661)\,t^{0.339} \\ w_5(t) &= (1.284 \cdot 10^{-5})\,(1.9553)\,t^{0.9553} \end{aligned} \tag{10}$$

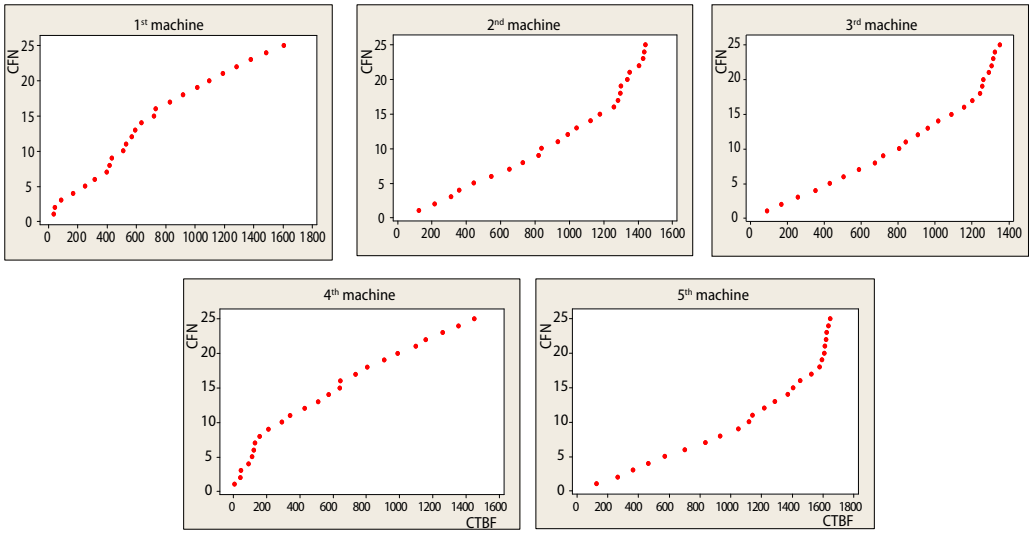

Figure 1. Trend graphs

Gokdere *et.al.* [9] have given R-Project codes based on our proposed method to compute the reliability of the systems which have a great number of components. Thus, the reliability of consecutive *3-out-of-5:F* systems can be easily calculated by using this code. The reliability of consecutive *3-out-of-5:F* systems are shown as tab. 2.

Table 1. Estimation of parameters

	1st machine	2nd	3rd	4th	5th
$\hat{\lambda}$	0.0513	$7.4\cdot10^{-5}$	$1.5\cdot10^{-4}$	0.2033	$1.2\cdot10^{-5}$
$\hat{\beta}$	0.8385	1.7490	1.6670	0.6610	1.9553

Table 2. Reliability of consecutive *3-out-of-5:F* systems

t	p_1	p_2	p_3	p_4	p_5	$R(t)$
10	0.9861	0.9998	0.9997	0.9633	0.9999	1
50	0.9464	0.9972	0.9959	0.8929	0.9989	0.9999
100	0.9043	0.9906	0.9870	0.8307	0.9958	0.9999
250	0.7936	0.9535	0.9403	0.6896	0.9750	0.9983
500	0.6310	0.8439	0.8106	0.5035	0.9032	0.9721
1000	0.3402	0.4754	0.3985	0.2244	0.6247	0.6253

Conclusion

A modelling of the systems is studied by many authors. When the data has been independent and not identically distributed, NHPP is used. The most widely used process model is power law model. In this study, NHPP power law model is used in order to test the reliability of the machine. Estimates of parameters using standard statistical techniques such as maximum likelihood method are made. The reliabilities of consecutive *3-out-of-5:F* systems are calculated for the certain time periods by using probabilities which are obtained by power law models. Finally, it is shown that while the time is increasing reliability is decreasing.

Nomenclature

E – expected value
L – likelihood function
$N(t)$ – number of failures in the time interval from 0 to t
$P(x)$ – probability of x
Var – variance
$w(t)$ – intensity function
$W(t)$ – distribution function

Greek symbols

β – parameter of function
$\hat{\beta}$ – estimation of β
λ – parameter of function
$\hat{\lambda}$ – estimation of λ

Appendix

Data of analysis

	1st machine		2nd machine		3rd machine		4th machine		5th machine	
CFN	TBF	CTBF	TBF	CTBF	TBF	CTBF	TBF	CTBF	TBF	CTBF
1	36	36	124	124	87	87	5	5	125	125
2	6	42	90	214	78	165	36	41	140	365
3	43	85	96	310	90	255	4	45	100	365
4	84	169	48	358	95	350	45	90	98	463
5	80	249	85	443	79	429	21	111	109	572
6	65	314	102	545	75	504	10	121	130	702
7	82	396	105	650	82	586	9	130	132	834
8	20	416	78	728	87	673	26	156	97	931
9	15	431	94	822	45	718	56	212	120	1051
10	78	509	15	837	88	806	78	290	66	1117
11	19	528	95	932	36	842	52	342	26	1143
12	41	569	58	990	63	905	86	428	74	1217
13	22	591	50	1040	55	960	80	508	71	1288
14	43	634	83	1123	56	1016	65	573	84	1372
15	85	719	56	1179	71	1087	66	639	32	1404
16	12	731	79	1258	70	1157	6	645	47	1451
17	98	829	25	1283	45	1202	89	734	73	1524
18	90	919	12	1295	42	1244	73	807	52	1576
19	95	1014	6	1301	12	1256	99	906	18	1594
20	82	1096	38	1339	5	1261	84	990	14	1608
21	92	1188	13	1352	31	1292	109	1099	4	1612
22	96	1284	55	1407	14	1306	60	1156	7	1619
23	96	1380	21	1428	10	1316	102	1258	3	1622
24	105	1485	9	1437	9	1325	98	1356	15	1637
25	120	1605	7	1444	25	1350	94	1450	19	1646

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