THERMOECONOMIC ANALYSIS OF SPIRAL HEAT EXCHANGER WITH CONSTANT WALL TEMPERATURE

by

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Thermoeconomic analysis of spiral heat exchanger is conducted. Different geometrical parameters, such as outer diameter, plate height, passage gap, etc. are used and varied in a wide range. Detailed thermal and total costs analyses were performed for two spiral heat exchanger with different process fluids (water and thermal oil) with temperature changes, while the wall temperature was kept constant (condensation). The results were shown graphically. It is determined that optimum values of number of entropy generation units correspond to minimum total annual cost. The optimal solution could be found in the recommended range of geometric sizes for defined inlet and outlet temperatures and process fluid-flow rate.

Key words: spiral heat exchangers, thermal analysis, economic analysis, optimal solution, thermoeconomics

Introduction

Thermoeconomists claim that human economic systems can be modeled on the basis of thermodynamic systems, i.e. on the basis of the First and Second law of thermodynamics, Burley et al. [1]. Valero and Torres [2] say that thermoeconomics is the science of natural resources saving that connects physics and economics by means of the Second Law of thermodynamics.

Maheshwari and Patel [3] said that Prigogine’s minimum entropy generation principle is the most debated one. The entropy generation minimization approach, widely applied to modeling and optimization of thermal systems that owe their thermodynamic imperfection to heat transfer, mass transfer, and fluid-flow irreversibility’s, demonstrates some inconsistencies and paradoxes in application to heat exchanger design. Xu [4] stated that even McClintock [5] had found that it is not possible to neglect irreversibility in heat exchanger design. Bejan [6-9] in his studies and researches proposed the criterion of minimization of entropy generation and widely used it for optimization of thermal equipment. Hesselgreaves [10] and Bertola and Cafaro [11], found that entropy generation minimization approach led to some contradictions. Hesselgreaves [10] proposed non-dimensionalisation method for the entropy generation in heat exchanger in order to avoid the entropy generation paradoxes, but this method has induced some other contradictions.

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Ahmadi et al. [12] conducted analysis of the optimal design of the plate fin heat exchanger (PFHE) using the thermoeconomic approach where the number of entropy generation units and the total annual cost were objective functions and different design parameters were considered (such as fin pitch, fin height, fin offset length, cold stream length, etc.). It was shown that the results from exergy analysis revealed that the exergy efficiency of the heat exchanger increases, the total cost of the heat exchanger increases, respectively. They concluded that higher exergy efficiency leads to have efficient heat exchangers in both thermodynamic and thermo economic points of view.

Nag and Mukherjee [13] analyzed entropy generation rate due to fluid friction and heat transfer, considering the phase change of one side fluid-flow and a constant wall temperature in a duct. Entropy generation rate was optimized with non-dimensional temperature difference where non-dimensional duty and duct ratio (duct length over duct radius) were used as parameters and with ratio of heat transfer rate to pumping power (non-dimensional duty was used as parameter).

Bermejo et al. [14] conducted an optimization of the micro-channel evaporator design based on entropy generation number. Zhou et al. [15] optimized PFHE according to optimal allocation of total heat transfer area between fluid sides using entropy generation minimization method of PFHE. Specific entropy generation rate was an optimization objective while total PFHE area was constraint and the criterion was a ratio of hot-side heat transfer area to total heat area (allocation ratio) while fin height, fin spacing, etc., were used as geometrical parameters. Nguyen and San [16] showed the relation of heat transfer effectiveness of spiral heat exchanger (SHE) with number of transfer unit (NTU), ratio of flow capacity rates, number of spiral turns. They concluded that it is possible to find heat transfer maximum for optimal parameters previously mentioned.

Kaushik and Manjunath [17] conducted entropy generation analysis of air cooled wire-and-tube condenser to investigate influence of geometrical and operating parameters. The analysis also merges thermoeconomics. It is combining the Second law of thermodynamics with economics where geometrical parameters of the condenser like tube outer diameter, wire diameter, number of wire pairs, number of tube rows, tube pitch, wire pitch, and refrigerant properties like mass velocity, saturation temperature and dryness fraction were varied.

Melhem et al. [18] studied the entropy generation due to the laminar flow and heat transfer with four different fluids, applied to circular cylinder confined between parallel planes. Pourmahmoud et al. [19] investigated the effects of non-dimensional rib height on the total entropy generation, so as entropy generation attributed to the friction, and the heat transfer. It is concluded that the optimum rib height with the minimum total entropy generation rate depend on Reynolds number and wall heat flux.

The aim of this paper is to analyze both, thermal and economic aspects of SHE. The results of optimization according to minimum total annual cost achieved by varying plate height, effective plate height, passage gap, outer diameter of SHE, and the size of heat transfer area are given, and results are presented graphically. Using the applied methodology of thermoeconomics it is shown that it is possible to determine the optimum SHE size according to minimum entropy generation.

Modeling

Thermal analysis

In this study spiral plate heat exchanger with rectangular cross-section is modeled as a single stream heat exchanger. The steam, as a utility fluid, flows through one passage, achieving the condition of constant wall temperature. Through the second spiral passage a process flu-
id-flows, and the variation of bulk fluid temperature along the spiral passage can be obtained from heat exchange analysis.

The rectangular cross-sectional area of the passages is used as a thermodynamic control volume, as shown in Fig. 1. The wall temperature, $T_w$, of the passage is kept constant during the process (i.e. saturated or superheated water vapor is used). An incompressible fluid, with mass-flow rate, $m$, and inlet temperature, $T_{in}$, enters the passage. The energy balances of an incompressible steady fluid-flow within the control volume of the passage (length $dx$):

$$\delta \dot{Q} = \dot{m} \delta h = \dot{m} c_p dT = \alpha P_Q (T_w - T) dx$$

From eq. (1) we get fluid temperature on SHE surface, $T(x)$ as:

$$T(x) = T_w - (T_w - T_{in}) \cdot \exp(-Bx)$$

where parameter $B$ is defined:

$$B = \frac{\alpha P_Q}{\dot{m} c_p} = \frac{NTU_2}{T_{eff}} = \frac{2St}{b_c} = \frac{4St}{D_h}$$

The hydraulic diameter, $D_h$ [m], of the spiral passage is defined:

$$D_h = \frac{4(B_{eff} b_c)}{2(B_{eff} + b_c)} = \frac{2b_c}{1 + \frac{b_c}{B_{eff}}} \approx 2b_c$$

where $B_{eff}$ is effective plate height [m], and it is calculated:

$$B_{eff} = B_{ph} - 0.02$$

and $B_{ph}$ is actual plate height [m].

The $NTU$ for cold (process) fluid ($NTU_2$) is defined [20]:

$$NTU_2 = \frac{\alpha S_{HE}}{\dot{m} c_p} = \ln \frac{T_w - T_{in}}{T_w - T_{out}}$$

**Entropy generation**

Entropy generation for thermodynamic control volume, for constant wall temperature, can be written:

$$d \dot{S}_{gen} = \dot{m} ds - \frac{\delta \dot{Q}}{T_w}$$

If we consider well known thermodynamic relation:

$$dh = Tds + \frac{dp}{\rho}$$
from eqs. (7) and (8):

$$dS_{gen} = \dot{m} c_p \left( \frac{dT}{T} - \frac{dT}{T_w} \right) - \dot{m} \frac{dp}{pT} \tag{9}$$

Heat transfer coefficient, for radial fluid-flow without phase change in SHE, is usually obtained according to Holger [21] and Minton [22]. According to Saravanan and Rajavel [23] Holger’s correlation:

$$Nu = \frac{St \ Re \ Pr}{\lambda} = 0.04 \ Re^{0.74} \ Pr^{0.4} \tag{10}$$
gives results that have a good matching with experimental data in range of Reynolds number between 400 and 30000. In eq. (10), \( \lambda \) is thermal conductivity \( [\text{Wm}^{-1}\text{K}^{-1}] \) and dimensionless numbers are:

- Nusselt number

$$Nu = St \ Re \ Pr = \frac{aD_h}{\lambda} \tag{11}$$

- Reynolds number

$$Re = \frac{wD_h \rho}{\mu} \tag{12}$$

- Prandtl number

$$Pr = \frac{c_p \mu}{\lambda} \tag{13}$$

The fluid velocity for radial flow:

$$w = \frac{m}{\rho A_c} \tag{14}$$

and the cross-section of the rectangular passage is given:

$$A_c = B_{eff} b_c \tag{15}$$

For radial fluid stream without phase change, Holger [21] recommends the following equation for pressure drop:

$$\Delta p = f_D \frac{L_{eff} \rho w^2}{b_c} \tag{16}$$

where \( \rho \) [kgm\(^{-3}\)] is density, \( w \) [ms\(^{-1}\)] – the average velocity, and \( f_D \) – the Darcy friction factor.

$$f_D = 1.5 \frac{64}{Re} + 0.2 \text{Re}^{-0.1} \tag{17}$$

Entropy generation number for process stream is defined as non-dimensional number composed of heat transfer and viscous friction contributions, Garcia and Moreles [24]:

$$N_s = N_s(\Delta T) + N_s(\Delta p) \tag{18}$$

Entropy generation number of stream contributed by heat transfer:
\[ N_s(\Delta T) = \ln \left[ \frac{1 - \tau \exp(-NTU_2)}{1 - \tau} \right] - \tau \left[ 1 - \exp(-NTU_2) \right] = \ln \left( \frac{T_{\text{out}}}{T_{\text{in}}} \right) - \frac{T_{\text{out}} - T_{\text{in}}}{T_w} \]  \hspace{1cm} (19)

and the entropy generation number of stream contributed by fluid friction:

\[ N_s(\Delta p) = \frac{1}{NTU_2} \frac{\Delta p}{c_p \rho} \left[ \ln \left( \frac{T_{\text{out}}}{T_{\text{in}}} \right) + NTU_2 \right] = \frac{1}{c_p \rho T_s} \left[ \ln \left( \frac{T_{\text{out}}}{T_{\text{in}}} \right) + \ln \frac{T_{\text{w}} - T_{\text{in}}}{T_w - T_{\text{out}}} \right] = \frac{\Delta p}{c_p \rho T_s} \]  \hspace{1cm} (20)

The non-dimensional temperature difference:

\[ \tau = \frac{T_w - T_{\text{in}}}{T_w} \]  \hspace{1cm} (21)

and characteristic temperature, \( T_s \), is defined:

\[ T_s = T_w \frac{\ln T_{\text{w}} - \ln T_{\text{in}}}{\ln \left( \frac{T_{\text{out}}}{T_{\text{in}}} \right) + \ln \frac{T_{\text{w}} - T_{\text{in}}}{T_w - T_{\text{out}}} } \]  \hspace{1cm} (22)

It can be shown that the characteristic temperature, \( T_s \), depends on flow pattern trough SHE (cocurrent flow, counter-current flow, plug flow with constant wall temperature, etc.). It has to be noted that Guo et al. [25] \( T_s \) defined:

\[ T_s = \frac{T_{\text{out}} - T_{\text{in}}}{\ln \left( \frac{T_{\text{out}}}{T_{\text{in}}} \right) } \]  \hspace{1cm} (23)

which is not correct because eq. (23) can be used as an approximation of eq. (22) only in case when \( T_{\text{out}} \) is very close to \( T_{\text{in}} \) and \( T_w \). In that specific case it is equally good to use expression:

\[ T_s = \frac{T_{\text{in}} + T_{\text{out}}}{2} \]  \hspace{1cm} (24)

There are two criteria for describing the entropy generation. The first one is Bejan number which shows the rate of entropy generation contributed by heat transfer on the overall generation of entropy taken from Yilmaz et al. [26]:

\[ \text{Be} = \frac{N_s(\Delta T)}{N_s(\Delta T) + N_s(\Delta p)} \]  \hspace{1cm} (25)

Apparently, the Bejan number values are in range from 0 to 1; limiting cases \( \text{Be} = 1 \) and \( \text{Be} = 0 \) represent the irreversibility occurred only by heat transfer and only by fluid friction, respectively. Other criterion is the relative importance of the two irreversibility mechanisms, and it is described by the irreversibility distribution ratio, \( \Phi \), which is defined in [7]:

\[ \Phi = \frac{N_s(\Delta p)}{N_s(\Delta T)} \]  \hspace{1cm} (26)
It is obvious that entropy generation number can be expressed:

$$N_s = (1 + \Phi)N'_s(\Delta T)$$  \hspace{1cm} (27)

### Economic optimization

Economic optimization is conducted according to criterion of minimum annual costs:

$$C_{tot} = a \cdot C_{inv} + C_{oper}$$  \hspace{1cm} (28)

where $a = 0.1 \text{ 1/year}$ is the amortization rate, $C_{inv} \quad [\text{€}]$ are the overall investment costs, and $C_{oper} \quad [\text{€/year}^{-1}]$ presents the operating costs. The investment costs are calculated with the assumption that pump/blower is already installed on SHE pipeline, so:

$$C_{inv} = C_{HE}$$  \hspace{1cm} (29)

Installed cost of spiral heat exchanger made of SS304, according to [27], can be expressed:

$$C_{HE} = 3720 + 8720S_{HE}^{0.57}$$  \hspace{1cm} (30)

where $S_{HE} \quad [\text{m}^2]$ is heat exchanger surface calculated:

$$S_{HE} = 2B_{eh}L_{eff}$$  \hspace{1cm} (31)

The operating costs mainly consist of annual cost for electric power consumption $P_W \quad [\text{W}]$ is pump power consumption:

$$C_{oper} = P_W \tau_{year} c_{kWh}$$  \hspace{1cm} (32)

where the number of annual working hours is $\tau_{year} = 8000 \text{ h/year}$, and the average price of electricity in Europe is given as $c_{kWh} = 0.15 \text{ €/kWh}$.

#### Pump power consumption

Power of pump is calculated:

$$P_W = \frac{1}{E} \left(\frac{m}{\rho}\right) \Delta p$$  \hspace{1cm} (33)

where $\Delta p \quad [\text{Pa}]$ is pressure drop.

Overall efficiency $E$ consist of pump, $E_p$, and motor, $E_M$, efficiencies:

$$E = E_p E_M$$  \hspace{1cm} (34)

and they can be calculated using the following equations.

Estimation of the motor in eq. (35) and pump in eq. (36) efficiencies according to [28] defined for $m/\rho = 11-1100 \text{ m}^3/\text{h}$ and $P_W = 0.75-1100 \text{ kW}$ are:

$$E_M = \left(\frac{m}{\rho} \Delta p\right)^{0.25} \frac{1.14 + 1.03 \left(\frac{m}{\rho} \Delta p\right)^{0.25}}{0.25}$$  \hspace{1cm} (35)
Analysis and discussion

Thermodynamic properties of fluids were calculated according to [29]. The presented numerical model is programmed and resolved with Visual Basic language. Optimal design of SHE should be calculated on the basis of inlet and outlet temperatures and process fluid-flow rate. Optimum values of number of entropy generation units correspond to minimum total annual cost. The solutions have to be found in the recommended range of geometric sizes for this type of heat exchangers.

In the first example is determined the size for SHE working with SAE15W40 as process fluid. Results are shown on fig. 2. Ranges for geometrical parameters and Reynolds number used are $B_{eff} = 1000-2500$ mm, $b_c = 3-69$ mm, $D_{out} = 323-5107$ mm, $S_{HE} = 11.8-453$ m$^2$, and $Re = 387-989$. Working conditions used for this SHE are $Q = 1000$ kW, $t_w = 150$ °C, $t_{2p} = 60$ °C, and $t_{2k} = 110$ °C, where $Q$ is SHE heat duty, $t_w$ is constant wall temperature and $t_{2p}$, $t_{2k}$ are, respectively, inlet and outlet temperatures of SAE15W40.

The second example is performed in case when water is used as process fluid. Results are shown on fig. 3. Ranges for geometrical parameters and Reynolds number used in this case are $B_{eff} = 1000-2500$ mm, $b_c = 5-61$ mm, $D_{out} = 284-2082$ mm, $S_{HE} =$ = 4.9-117 m$^2$, and Reynolds number in range 25283-65222. In this case, $Q = 4000$ kW, $t_w = 100$ °C, $t_{2p} = 20$ °C, and $t_{2k} = 70$ °C.

For defined heat duty and temperature-operating mode, there is an optimal entropy generation number $N_{s, opt}$ that determines the minimal total annual costs for adopted plate height and the corresponding passage gap (the spacing between consecutive rounds of the spiral plate). The entropy generation of stream contributed by heat transfer is constant value and it depends on the inlet and outlet temperatures. The entropy generation of stream contributed by fluid friction depends on the pressure drop and the characteristic temperature, $T_e$ (eq. (22), characteristic temperature, $T_e$, depends on the pattern of fluid-flow in a heat exchanger). According to the criterion of the technoeconomic optimization that gives about a constant pressure drop regarding the size of SHE, it follows that $N_{s, opt}$ is the same for all the SHE of this class (irreversibility being totally dominated by heat transfer).
The increase in \( N_s \) is followed by the increase of the total annual costs, primarily due to the increase in operating costs (increased pressure drops of fluid streaming through SHE).

With decrease of \( N_s \) (due to reduced pressure drop) total annual costs increase primarily due to the increase in investment costs (increased surface area for heat exchange). This means that the heat exchanger with a zero pressure drop, when \( N_s = N_{s, \min} = N_s(\Delta t) \), is not an optimal solution according to the criteria defined in this article. The adopted model of heat exchange gives \( N_s(\Delta t) \to 0 \) when \( NTU \to \infty \), and it is so-called vanishing heat exchanger limit, Hesselgreaves [10].

The \( N_{s, \min} \) is value for zero pressure drop entropy generation. It is found from the examples that for the oil is \( N_{s, \text{opt}} = 1.0073 \times N_{s, \min} \), and for water is \( N_{s, \text{opt}} = 1.001 \times N_{s, \min} \).

**Conclusion**

In this paper it is analyzed spiral heat exchangers using thermoeconomic aspects. Entropy generation number was determined for various working conditions of incompressible fluid-flow through a spiral passage subjected to isothermal boundary condition, and it has been correlated to the total annual costs. Through two examples it was shown that the optimal value of entropy generation number \( N_{s, \text{opt}} \) defined for minimum annual cost, is very close (just slightly greater) to minimum of entropy generation \( N_{s, \min} \).

Presented model can be used for design of an optimal SHE (for defined flow rates and inlet and outlet temperatures), without performing economic analysis and calculation of the minimal annual costs.

**Nomenclature**

- \( A_c \) – cross-section area, \([m^2]\)
- \( a \) – amortization rate, \([\text{year}^{-1}]\)
- \( B_{ph} \) – plate height, \([m]\)
- \( B_{eff} \) – effective plate height, \([m]\)
- \( b_c \) – passage gap (i.e. the spacing between consecutive rounds of the spiral plate, \( b - \delta \), \([m]\))
- \( Be \) – Bejan number, [-]
- \( C_{HE} \) – heat exchanger cost, \([\text{€}]\)
- \( C_{tot} \) – total costs, \([\text{€year}^{-1}]\)
- \( c_{kw} \) – unity power price, \([\text{€kWh}^{-1}]\)
- \( C_{inv} \) – overall investment costs, \([\text{€}]\)
- \( C_{oper} \) – operating costs, \([\text{€}]\)
- \( c_p \) – specific heat at constant pressure, \([\text{Jkg}^{-1}\text{K}^{-1}]\)
- \( D_h \) – hydraulic diameter, \([m]\)
- \( D_{out} \) – outer diameter of SHE, \([m]\)
- \( E \) – overall efficiency, [-]
- \( E_M \) – motor efficiency, [-]
- \( E_P \) – pump efficiency, [-]
- \( f_D \) – Darcy friction factor, [-]
- \( h \) – specific enthalpy, \([\text{Jkg}^{-1}]\)
- \( m \) – mass-flow rate, \([\text{kg} \cdot \text{s}^{-1}]\)
- \( NTU \) – number of transfer unit, [-]
- \( Nu \) – Nusselt number, [-]
- \( N_{s, \text{opt}} \) – optimal entropy generation number, [-]
- \( N_{s, \min} \) – entropy generation number contributed by fluid friction, [-]
- \( N_s(\Delta T) \) – entropy generation number contributed by heat transfer, [-]
- \( L_{eff} \) – effective spiral length (fig. 1. \( m \) to \( M \), so as \( n \) to \( N_s \), \([m]\))
- \( P_Q \) – heat transfer perimeter, \([m]\)
- \( P_W \) – pump power, \([\text{W}]\)
- \( p \) – pressure, \([\text{Pa}]\)
- \( \Delta p \) – pressure drop, \([\text{Pa}]\)
- \( Pr \) – Prandtl number, [-]
- \( \dot{Q} \) – heat transfer rate, \([\text{W}]\)
- \( r_{f1}, r_{f2} \) – radius of consecutive spirals, \([m]\)
- \( Re \) – Reynolds number, [-]
- \( s \) – specific entropy, \([\text{Jkg}^{-1}\text{K}^{-1}]\)
- \( S_{gen} \) – entropy generation rate, \([\text{W} \cdot \text{K}^{-1}]\)
- \( S_{HE} \) – heat exchanger surface area, \([m^2]\)
- \( St \) – Stanton number, [-]
- \( T \) – temperature, \([\text{K}]\)
- \( w \) – average velocity, \([\text{m} \cdot \text{s}^{-1}]\)
- \( x \) – length, \([m]\)

**Greek symbols**

- \( \alpha \) – convective heat transfer coefficient, \([\text{Wm}^{-2}\text{K}^{-1}]\)
- \( \delta \) – thickness of wall, \([m]\)
- \( \Phi \) – irreversibility distribution ratio, [-]
- \( \lambda \) – thermal conductivity, \([\text{Wm}^{-1}\text{K}^{-1}]\)
- \( \mu \) – dynamic viscosity, \([\text{Pa} \cdot \text{s}]\)
- \( \rho \) – density, \([\text{kgm}^{-3}]\)
\[ \tau_{\text{year}} \quad \text{number of annual working hours, [h_{\text{year}}^{-1}]} \]
\[ \tau \quad \text{non-dimensional temperature difference, [-]} \]
Subscripts
\[
\begin{align*}
in & \quad \text{inlet} \\
\text{min} & \quad \text{minimum} \\
\text{opt} & \quad \text{optimum} \\
\text{oper} & \quad \text{operating} \\
w & \quad \text{wall} \\
2 & \quad \text{cold fluid stream}
\end{align*}
\]

References