DETERMINATION OF AEROSOL PARTICLE SIZE DISTRIBUTION BY A NOVEL ARTIFICIAL BEE COLONY-DIFFERENTIAL EVOLUTION HYBRID ALGORITHM

by

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The aerosol size distribution, a vitally important environmental quality evaluation criterion, has a significant influence on radiative transfer and meteorological phenomena. To measure the aerosol size distribution effectively and accurately, the light scattering measurement method combined with a novel artificial bee colony-differential evolution hybrid algorithm which was based on the artificial bee colony algorithm and differential evolution algorithm, was proposed. First, the retrieval accuracy and convergence properties of the artificial bee colony-differential evolution algorithm were compared with those of the artificial bee colony algorithm. The results revealed that the artificial bee colony-differential evolution algorithm could avoid the phenomenon of local optima and low convergence accuracy which exited in artificial bee colony algorithm. Then, the parametric estimation of two commonly used monomodal aerosol size distribution, i. e. the Gamma distribution and the logarithmic normal distribution were studied under different random measurement errors. The investigation indicated that the retrieval results using the artificial bee colony-differential evolution showed better accuracy and robustness than those using the artificial bee colony. Moreover, the retrieval parameters with better monodromy characteristic would have better inverse accuracy. Finally, the actual measured aerosol size distribution over city of Harbin, China were also retrieved. All the results confirm that the artificial bee colony-differential evolution algorithm was an effective and reliable technique for estimating the aerosol size distribution.

Key words: *artificial bee colony algorithm, aerosol size distribution, inverse problem, light scattering measurement method*

Introduction

Aerosols play an important role in determining the properties of atmosphere environment, *e. g.* reducing visibility, affecting heat radiative balance and temperature, and changing ozone concentration and other related chemical constituents [1, 2]. Especially at the Earth's surface, where people live and the highest concentrations of aerosols are found, the aerosols present a serious health hazard [3]. The particle matter (PM), a criteria pollutant, has been a research topic for numerous studies in the context of air pollution [4]. Therefore, in-situ measuring the properties of aerosols presented in the atmosphere is of great relevance to understand the effect of aerosols on the atmosphere environment [5]. Generally, the aerosol size distribution (ASD) has significant influence on radiative transfer and meteorological phe-

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nomena and plays a crucial role in determining the climatic trends. Moreover, the ASD is also regarded as a vitally environmental quality evaluation criterion, and the influence of atmospheric aerosols on human health fundamentally depends on the aerosol size (PM) [6, 7]. Thus, without a good understanding of the ASD, the effects of the aerosols on the climate, meteorology, human health, and air quality, would remain highly uncertain. Nowadays, although there are several global ground-based aerosol observation networks that have been established to study the properties of the atmospheric aerosols, *e. g.* AERONET, MODIS [8], determining the ASD accurately is still regarded as an unsolved problem and needs further research.

During past several years, various methods have been introduced to determine particle size distribution, including aerodynamic, optical, ultrasonic, electrical mobility, and condensation methods [2]. Since properties of aerosols can be derived by measuring a variety of light scattering properties, such as extinction or other scattering information, the light scattering measurement methods combined with the optimization algorithms have been widely utilized to study the ASD owing to offering a useful and effective approach which only requires a simple optical layout and a commercial spectrophotometer [9]. The convergence accuracy and speed of optimization algorithms are usually treated as the key to obtain results accurately and efficiently. Nowadays, there are many intelligent optimization algorithms are used to study the particle size distributions [10-16]. However, to the best of the authors' knowledge, there have been no reports about the application of the artificial bee colony (ABC) algorithm in studying the inverse problem of ASD [17].

In present study, by introducing the differential evolution (DE) algorithm to improve the individual best position and avoid the local optima and low convergence accuracy, a novel ABC-DE hybrid algorithm was developed and combined with the light scattering measurement method to solve the problem of retrieving ASD. The remainder of this research was organized as follows: first, the convergence accuracy of the ABC-DE algorithm was compared with that of the ABC algorithm; then, the monomodal ASD, *i. e.* the Gamma distribution and logarithmic normal (L-N) distribution, were studied under different random measurement errors; and finally, the actual measurement ASD over city of Harbin, China were also retrieved by the ABC-DE algorithm. The main conclusions and prospects for further research were provided finally.

Theories and methods

The principle of the light scattering measurement method

When a beam of collimated monochromatic laser impinges on a suspended polydisperse spherical particle system whose complex refraction index is different from that of the medium, the absorbing and scattering effects of particles will lead to an attenuation of the transmitted beam, see fig. 1. If the optical thickness is thin and the independent scattering dominates, the influence of the multiple scattering and interaction between particles can be neglected. The transmitted light intensity, I, with the incident wavelength, λ, can be calculated [15]:

$$\ln\frac{I(\lambda)}{I_0(\lambda)} = -\frac{3}{2}L\int_{D_{\min}}^{D_{\max}}\frac{N_0 f(D)}{D}Q_{\text{ext}}(\lambda,m,D)\text{d}D \tag{1}$$

where $I_0(\lambda)$ and $I(\lambda)$ denote the intensities of the incident and transmitted beams at wavelength λ, respectively, $I(\lambda)/I_0(\lambda)$, obtained by actual measurement, is the transmittance at wavelength λ, L – the geometrical thickness of aerosol particle system, $f(D)$ [m^{-3}μm^{-1}] is the unknown volume frequency distribution of the aerosol particle system with diameter D, N_0 – the total number concentration of aerosol particle dispersion system, D_{max} and D_{min} [μm] denote the upper and lower limits of the integral equation, $Q_{ext}(\lambda, m, D)$ – the extinction efficiency factor of a single particle, which can be calculated by Mie theory [18].

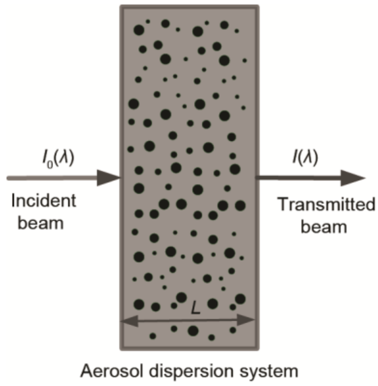

Figure 1. Schematic diagram of light scattering measurement method

The principle of the ABC algorithm

In ABC algorithm, each food source represents a possible solution to the concerned problem and the abundant degree of the corresponding nectar implies the quality of the solution represented by the fitness value. So, the bee colony usually divides to two equal parts. One half is the employed bees whose number is the same as the number of food source, which make sure exactly one employed bee for every food source [17]. The other half is the onlooker bees. At first, the food source positions $\mathbf{X}_i = [x_{i,1}, x_{i,2}, \ldots, x_{i,N_n}] (i = 1, 2, \ldots, SN)$ are initialized randomly in the search space, which contains the food source sites [17, 19]:

$$x_{i,j} = low_j + rand_1(high_j - low_j), \quad i \in \{1, 2, \ldots, SN\}, \quad j \in \{1, 2, \ldots, N_n\} \tag{2}$$

where SN is the number of food source, N_n – the dimension of unsolved problem, $high_j$ and low_j are the upper and lower limits of the search space $\mathbb{S}$ for the j^{th} dimension problem, $rand_1$ – the uniformly distributed random number in the range of [0, 1]. Evaluate $\mathbf{X}_i$ and record current personal best food source position $\mathbf{P}_i$ and $pbest_i$. The employed bee produces a candidate food source position $\mathbf{V}_i = [v_{i,1}, v_{i,2}, \ldots, v_{i,N_n}]$ according to:

$$\mathbf{V}_i = \mathbf{P}_i + \phi_i(\mathbf{P}_i - \mathbf{P}_k), \qquad i, k \in \{1, 2, \ldots, SN\}, \quad i \neq k \tag{3}$$

where ϕ_i is a uniformly distributed real random number in the range of [0, 1]. Evaluate the quality of $\mathbf{V}_i$ and calculate the corresponding fitness value Fit_i [20]:

$$Fit_i = \begin{cases} \dfrac{1}{1 + F_i} & \text{if } F_i \geq 0 \\ 1 + \text{abs}(F_i) & \text{if } F_i < 0 \end{cases} \tag{4}$$

where F_i is the cost value of the food source position $\mathbf{V}_i$. Comparing the Fit_i of $\mathbf{X}_i$ and that of $\mathbf{V}_i$, keep the one with more satisfactory fitness function value. If $\mathbf{X}_i$ cannot be improved, its counter holding the number of trials is incremented by 1, otherwise, the counter is reset to 0. When the employed bees bring back the nectar information of the food sources, the onlooker bees choose a food source depending on the probability P_i [20]:

$$P_i = \frac{Fit_i}{\sum_{i=1}^{SN} Fit_i} \tag{5}$$

If a position of food source cannot be improved further after a predetermined number $limit$ of cycles, the food source will be abandoned. Assume that the abandoned source is $\mathbf{X}_n, n \in \{1, 2, \ldots, SN\}$, , the scout bee will discover a new food source [19]:

$$x'_{n,j} = low_j + rand_2(high_j - low_j) \tag{6}$$

where $rand_2$ is a uniformly distributed random number in the range of [0, 1]. At the end of every generation, personal best position $\mathbf{P}_i$ and corresponding fitness value $pbest_i$ of each food source are recorded. Meanwhile, the global best position $\mathbf{P}_\mathrm{g}$ and corresponding fitness value $gbest$ of all the food sources are also recorded.

The principle of the ABC-DE hybrid algorithm

In standard ABC algorithm, the fast information flow between bees seems to be the reason for clustering of bees, which will also lead to the diversity decline rapidly and leave the ABC algorithm with great difficulties of escaping local optima. In this study, the DE algorithm is applied to improve the personal best position $\mathbf{P}_i$ of each food source and avoid local optima. The DE algorithm, developed by Storn and Price [21], is an evolutionary optimization algorithm based on population cooperation and competition of individuals and has been successfully applied to solve optimization problems particularly involving non-smooth fitness function [21-23].

The basic optimization process in the DE algorithm consists of mutation, cross-over, and selection [23]. Specifically, the mutation operation generates a new parameter vectors by adding the weighted difference between two population vectors to a third vector. Then, the mutated vector parameters are mixed with the parameters of another predetermined vector, named as the target vector, to yield the so-called trial vector. This operation is named as cross-over. The last operation, called selection, is to judge whether the fitness value yielded by the trial vector is superior to that of the target vector. If so, the trial vector replaces the target vector in the following generation. Each population vector has to serve once as the target vector so that *SN* competitions take place in one generation [21]. The mutant vector $\mathbf{P}_{i,\mathrm{mut}}$ can be generated according to [21]:

$$\mathbf{P}_{i,\mathrm{mut}} = \mathbf{P}_k + \omega[\mathbf{P}_j - \mathbf{P}_m] \tag{7}$$

where $k, l, m \in [1, SN], i \neq k \neq l \neq m;$; the mutant factor, ω, is a real and constant value, $\omega \in [0, 2]$. In order to increase the diversity of the perturbed parameter vectors, cross-over is introduced and the trial vector $\mathbf{P}_{i,\mathrm{tri}} = (p_{i1,\mathrm{tri}}, p_{i2,\mathrm{tri}}, p_{i3,\mathrm{tri}} \cdots, p_{iN_\mathrm{n},\mathrm{tri}})$ can be derived from [21]:

$$p_{ij,\mathrm{tri}}(t) = \begin{cases} p_{ij,\mathrm{mut}} & \text{if } rand_3(j) \le C_R \text{ or } j = rnbr(i) \\ p_{ij} & \text{if } rand_3(j) > C_R \text{ or } j \neq rnbr(i) \end{cases} \tag{8}$$

where $rand_3(j)$ is the j^th evaluation of a uniform random number generator, $rand_3(j) \in [0, 1]$, C_R is the crossover constant $C_R \in [0, 1]$, $rnbr(i)$ is a random integer uniformly selected from $[1, SN]$. The flowchart of the ABC-DE hybrid algorithm is shown in fig. 2.

Results and discussion

In this study, two kinds of widely used monomodal ASD, *e. g.* the *L-N* function distribution and the Gamma function distribution, were studied. The mathematical representations of their monomodal volume frequency distributions are [3]:

$$f_{L\text{-}N}(D)=\frac{1}{\sqrt{2\pi}D\ln\sigma}\exp\left[-\frac{1}{2}\left(\frac{\ln D-\ln\bar{D}}{\ln\sigma}\right)^2\right] \tag{9}$$

$$N_{\text{Gamma}}(D)=D^{\alpha}\exp(-\beta D^{\gamma}) \tag{10}$$

where $\bar{D}$ is the characteristic diameters of these distribution functions, σ, α, β, and γ are the narrowness indices of the distribution. Usually, in the modified form, $\gamma = 1$, so only parameters α and β in the Gamma distributions need to be investigated, respectively [3].

The size measurement range of the particles is selected from 0.1 μm to 10 μm, which is the optimal measurement range in the light scattering measurement method [13]. The spectral complex refractive indices of soot aerosols with different wavelengths are available in [24]. According to [25], for there are two parameters, *e. g.* α and β in the gamma function, need to be retrieved, three wavelengths are choosed to be as the measurement wavelengths to improve the retrieval accuracy of ASD, see tab. 1.

Table 1. The complex refractive indices of aerosols [24]

Wavelength, λ [μm]	n	k
0.200	1.500	0.350
0.488	1.750	0.450
1.060	1.750	0.440

The retrieval of the ASD is solved by minimizing the fitness function value, which is the sum of the square residuals between the estimated and measured transmittance:

$$Fit=\sum_{i=1}^{3}\left\{\frac{\left[\dfrac{I(\lambda)}{I_0(\lambda)}\right]_{\text{est}}-\left[\dfrac{I(\lambda)}{I_0(\lambda)}\right]_{\text{mea}}}{\left[\dfrac{I(\lambda)}{I_0(\lambda)}\right]_{\text{mea}}}\right\}^2 \tag{11}$$

For the algorithm is a stochastic optimization method and all optimizations have certain randomness, all the calculations are repeated 50 times. Moreover, for the purpose of investigating the reliability and feasibility of the optimization algorithm, the relative deviation, δ, which means the sum of the deviation error between the probability distribution estimated from the inverse calculation

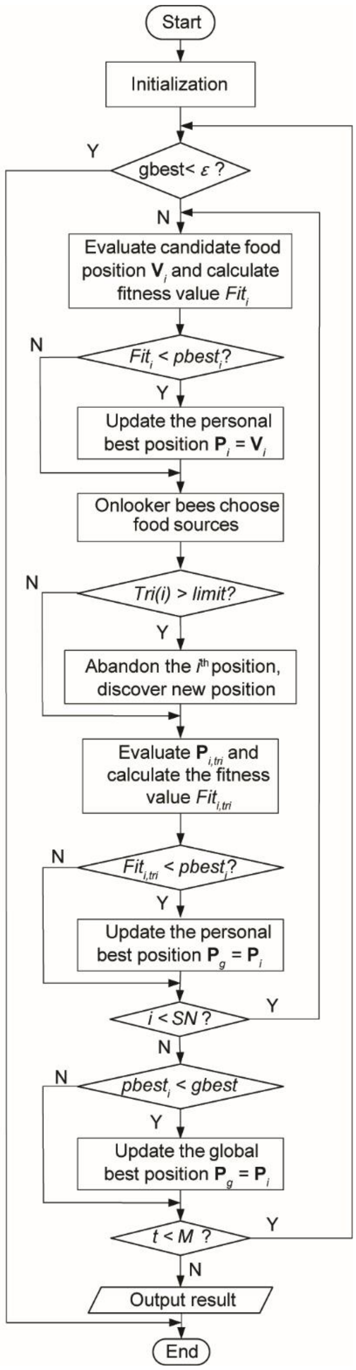

Figure 2. Flowchart of the ABC-DE hybrid algorithm

and the true ASD in every subinterval, are studied to evaluate the quality of inverse results, and its mathematic expression is descripted:

$$\delta = \frac{\left\{\sum_{i=1}^{N'}\left[f_{\text{est}}(\tilde{D}_i) - f_{\text{true}}(\tilde{D}_i)\right]^2\right\}^{1/2}}{\left\{\sum_{i=1}^{N'}\left[f_{\text{true}}(\tilde{D}_i)\right]^2\right\}^{1/2}} \tag{12}$$

where N' represents the number of subintervals that the size ranges of D are divided, $\tilde{D}_i$ – the midpoint of the i^{th} subinterval $[D_i, D_{i+1}]$. Moreover, the standard deviation of the retrieval results, η, is also investigated to evaluate the reliability and feasibility of the results, and the corresponding mathematical expressions are described:

$$\eta_x = \sqrt{\frac{1}{50}\sum_{i=1}^{50}(\bar{x}_{\text{est}} - x_{\text{est},\,i})^2} \tag{13}$$

$$\bar{x}_{\text{est}} = \frac{1}{50}\sum_{i=1}^{50} x_{\text{est},\,i} \qquad x = \bar{D}, \sigma, \alpha, \beta \tag{14}$$

where x denotes the retrieval parameters in the distribution functions, *e. g.* $\bar{D}$, σ, $\bar{x}_{\text{est}}$ is the average retrieval results of 50 calculation.

Compare the fitness function values of the ABC and ABC-DE algorithms

The performance of the ABC-DE algorithm is studied through comparison with the ABC algorithm. Table 2 lists the system control parameters for both the ABC and ABC-DE algorithms. Figure 3 displays the comparison of the fitness function values of the ABC and ABC-DE in retrieving the L-N distribution. The termination criteria are set as: when the iteration accuracy is below 10^{-16} and when the maximum generation number 1000 is reached. It can be found that the fitness function value of the ABC-DE converges much faster than those of others. Moreover, the ABC-DE can obtain lower fitness function value within a smaller number of generations than the ABC, which means the ABC-DE can avoid local optima that exits in the ABC.

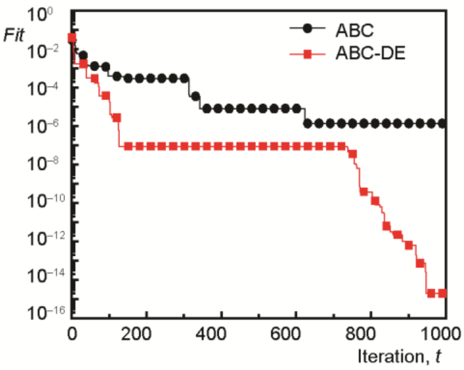

Figure 3. Comparison of fitness function values of ABC and ABC-DE algorithms

Table 2. The system control parameters of the ABC algorithms

Parameter value	*SN*	N_n	*M*	ε	*limit*	C_R	ω	[*low*, *high*]
ABC	50	2	1000	10^{-16}	100	–	–	[0, 10]
ABC-DE	50	2	1000	10^{-16}	100	0.4	0.4	[0, 10]

Retrieval of the ASD using the ABC and ABC-DE algorithms

Table 3 lists the results of ASD retrieved by the ABC and ABC-DE algorithms respectively, and the corresponding inverse curves are depicted in fig. 4. The system control parameters are also listed in tab. 2. From tab. 3 and fig. 4, it can be found that when there is no random measurement error added to the measurement signals, the results retrieved by the ABC-DE algorithm show higher accuracy than those retrieved by the ABC algorithm. When the random error increases, the value of δ becomes lager, and the retrieval results deteriorate. However, the results retrieved by the ABC-DE algorithm still show satisfactory convergence accuracy even with 10% random errors, which also means results retrieved by ABC-DE algorithm show high robustness than those retrieved by the ABC algorithm. Moreover, it can be found that the retrieval results of *L-N* distribution shows higher accuracy than those of Gamma distribution under the same random measurement errors.

Table 3. The retrieval results of the ASD by the ABC algorithms

Algorithms	Err	*L-N* $(\bar{D},\sigma)=(2.2,1.8)$					Gamma (α, β) = (5, 6)				
		$\bar{D}_{est}$	$\eta_{\bar{D}}$	$\bar{\sigma}_{est}$	η_σ	δ	$\bar{\alpha}_{est}$	η_α	$\bar{\beta}_{est}$	η_β	δ
ABC	0%	2.18	0.151	1.77	0.159	0.01447	5.00	0.018	5.99	0.028	0.00184
	5%	2.14	0.234	1.73	0.248	0.04080	5.59	1.230	6.04	0.099	0.25611
	10%	2.29	0.406	1.95	0.567	0.07816	5.86	1.706	6.14	0.212	0.26642
	15%	2.04	0.616	1.46	0.775	0.22025	6.25	1.713	6.12	0.232	0.33787
ABC-DE	0%	2.20	0.003	1.80	0.003	0.00020	5.00	0.001	5.99	0.001	0.00003
	5%	2.17	0.227	1.77	0.239	0.01714	5.35	0.897	6.02	0.063	0.11418
	10%	2.15	0.307	1.73	0.368	0.04158	5.78	1.296	6.11	0.160	0.24207
	15%	2.42	0.360	2.02	0.495	0.12232	6.03	1.537	6.04	0.208	0.28752

*The mean inverse results and relative deviation of the 50 times calculations are shown in the tables

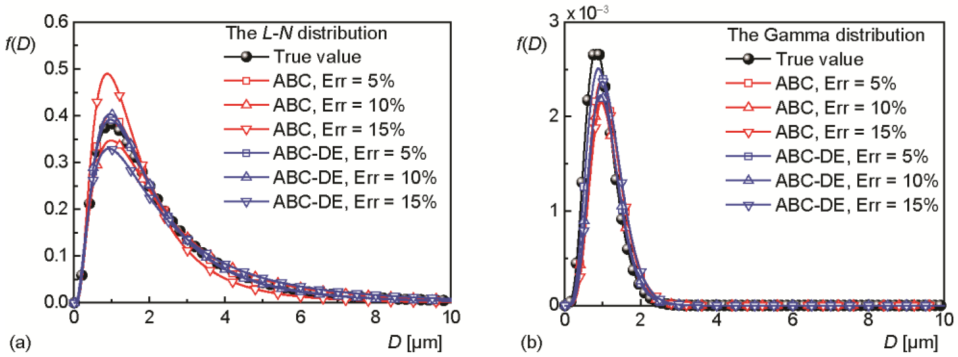

Figure 4. Reproducibility of the ASD by the ABC and ABC-DE algorithms; (a) *L-N* distribution, (b) Gamma distribution

Comparing the retrieval results of different parameters in the ASD functions, we could find that they have different inverse accuracies and standard deviations in the same distribution function and at the same random measurement errors. For example, when studying the Gamma distribution, the inverse accuracy of retrieval results for parameter α is lower than those for parameter β, and standard deviation, η_α, for parameter α is larger than those η_β for parameter β. The likely explanation of the phenomenon could be that the monodromy characteristics of the estimated results for parameter β are better than those for parameter α. The retrieval results for parameter α show more serious multi-value characteristics than those for parameter β. The difficulties of inverse estimation of the ASD are the multiple-value characteristics of the results, *i. e.*, several results may exist according to the same experimental data. In retrieving the Gamma distribution, there are many couples of (α, β) values corresponding to the same transmittance measured by photoelectric detector. This finding implies that the inverse problem may not have a unique solution. This phenomenon can be observed from fig. 5, which depicts the distribution of the fitness function values for *L-N* and Gamma distributions. It is easy to find that the minimum value regions of the fitness function are not a point but are similar to a valley. This phenomenon indicates that the optimal solution is not unique. For example, based on the distribution trend of the minimum fitness function value region, appropriate values for parameter α are available to satisfy the minimum fitness function value under the same parameter β. The same solution can also be concluded in the fitness function values distribution of *L-N* function.

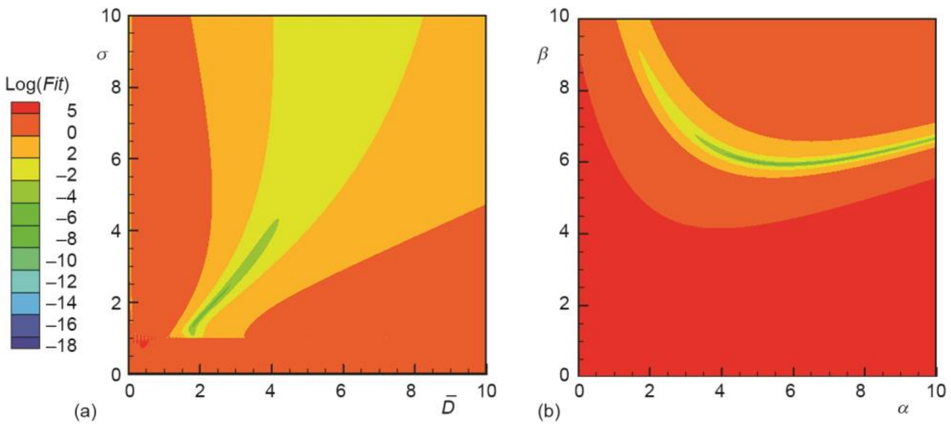

Figure 5. The distribution of the fitness function values for different ASD; (a) *L-N* distribution, (b) Gamma distribution (*for color image see journal web site*)

Figures 6 and 7 show the distributions of 50 times estimated results for retrieving the *L-N* and Gamma distributions. It can be found that the distributions of the estimated results disperse with increasing random measurement errors. The larger the random error is, the more dispersive the distribution will be. Moreover, the distributions of the results retrieved by the ABC algorithm show more dispersive than those retrieved by the ABC-DE algorithm. The dispersity of retrieval results for parameter $\bar{D}$ is similar to that for parameter σ in retrieving *L-N* distribution function, while the dispersity of retrieval results for parameter α is larger than that for parameter β in retrieving Gamma distribution function. All these findings can also be used to confirm the conclusions obtained in fig. 4 and tab. 3.

Retrieval of the actual measured ASD over city of Habin, China

The reliability of the ABC-DE algorithm is also verified by experiments. Figure 8 show the actual measured ASD, named as the *true value*, at different locations over Harbin,

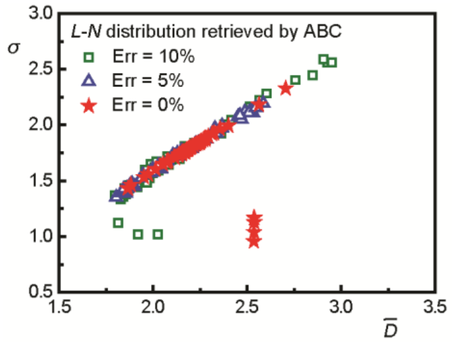

Figure 6. The distribution of results of *L-N* distribution for 50 calculation retrieved by the ABC and ABC-DE algorithms, respectively

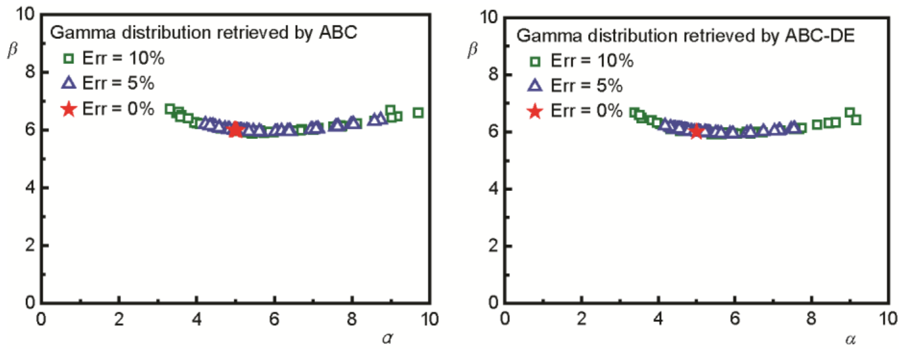

Figure 7. The distribution of results of Gamma distribution for 50 calculation retrieved by the ABC and ABC-DE algorithms, respectively

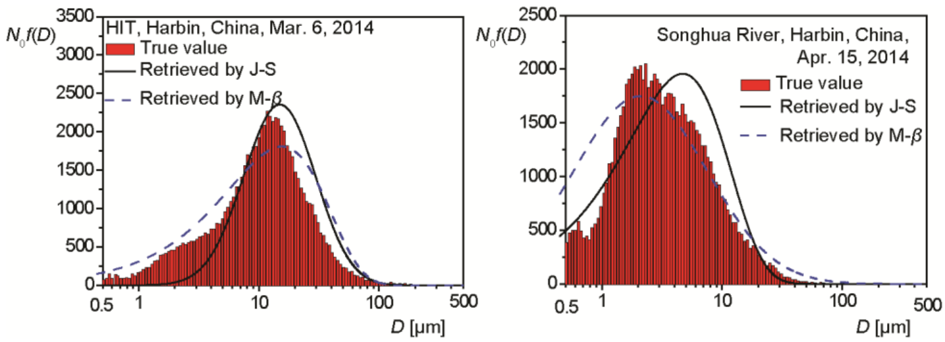

Figure 8. Reproducibility of actual measured ASD over Harbin, China by the ABC-DE algorithm

China. The spectral complex refractive indices used in the manuscript are listed in tab. 4. For the prior distribution information of ASD cannot be known beforehand, the J-S_B and M-β functions are proposed as the general functions to reconstruct the ASD. The mathematical expression of two general functions can be derived [11]:

$$f_{\text{J-S}_\text{B}}(D)=\frac{\sigma'}{\sqrt{2\pi}}\frac{D_{\max}-D_{\min}}{(D-D_{\min})(D_{\max}-D)}\exp\left\{-\frac{(\sigma')^2}{2}\left[\ln\left(\frac{D-D_{\min}}{D_{\max}-D}\right)-\ln\left(\frac{M'-D_{\min}}{D_{\max}-M'}\right)\right]^2\right\} \tag{15}$$

$$f_{\text{M}-\beta}(D)=\frac{(D-D_{\min})^{\alpha'm'}(D_{\max}-D)^{m'}}{\int\limits_{D_{\min}}^{D_{\max}}(D-D_{\min})^{\alpha'm'}(D_{\max}-D)^{m'}\mathrm{d}D} \tag{16}$$

where σ', M', α', and m' are the characteristic parameters. The retrieval results are also illustrated in fig. 8. It can be found that the results retrieved by the ABC-DE algorithm are still acceptable, which demonstrates that the ABC-DE algorithm can be applied to study actual measured ASD.

Table 4. The spectral complex refractive indices of aerosols over Harbin, China

Wavelength λ [μm]	Urban		Songhua River	
	n	k	n	k
2.58497	1.65729	0.00914	1.40245	0.00413
3.38973	1.70714	0.02032	1.40785	0.00409
3.75553	1.69412	0.02387	1.40887	0.01585

Conclusions

Combined with the light scattering measurement method, a novel hybrid algorithm, based on the ABC and DE algorithms, is proposed to determine the ASD. Compared with the ABC algorithm, the novel ABC-DE algorithm shows many satisfactory convergence properties in retrieving the ASD. The following conclusions can be drawn.

- The ABC-DE algorithm can converge much faster and obtain lower fitness function value within a smaller number of generations than the ABC algorithm in retrieving the ASD. The accuracy and robustness of the results retrieved by the ABC-DE algorithm are better than those retrieved by the ABC algorithm even with random measurement errors.
- Different retrieval parameters in the same distribution function show different inverse accuracies, because monodromy characteristics of the estimated results for each retrieved parameter are different. Generally, the retrieval parameters with better monodromy characteristics will show better inverse accuracy.
- The actual measured ASD in Harbin, China can also be retrieved by the ABC-DE algorithm, when there is no prior information about the ASD.

As a whole, the ABC-DE hybrid algorithm is demonstrated to be a potential and effective optimal algorithm to retrieve the ASD. Moreover, for the aerosol particles treated as the homogenic spherical particles, the methodology presented here can also be used to study the particle size distribution of other spherical particle dispersion medium, *e. g.* algae dispersion medium. Further study will focus on the applications of the ABC-DE algorithm in the retrieval of ASD of the non-spherical particles.

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Nomenclature

C_R – cross-over constant, [–]
D – diameter of particles, [μm]
F – cost value of food source position, [–]
Fit – fitness function value, [–]
$f(D)$ – volume frequency distribution, [–]
$high$ – high limit of the search space
I – radiative intensity, [$\mathrm{Wm^{-2}sr^{-1}}$]
k – imaginary part of refractive index, [–]
L – geometrical thickness, [m]
$limit$ – limit number for employed bee, [–]
low – low limit of the search space
m – complex refractive index, [–]
M – total iteration number, [–]
N_0 – total number of particles, [–]
n – real part of refractive index, [–]
P – probability to choose a food source, [–]
$\mathbf{P}_\mathrm{i}$, $\mathbf{P}_\mathrm{g}$ – individual and global best positions, [–]
Q_ext – extinction efficiency, [–]
SN – number of food source, [–]
$\mathbf{V}_i$ – candidate position of food source, [–]
$\mathbf{X}_i$ – the position of food source, [–]

Greek symbols

δ – relative deviation of particle size distribution, [–]
ε – the tolerance, [–]
η – standard deviation of retrieval results, [–]
λ – the wavelength of laser, [μm]
ω – constant value, [–]

Subscripts

est – estimated value
ext – extinction efficiency
Gamma – Gamma distribution
L-N – logarithmic normal distribution
max – maximum value
mea – measurement value
min – minimum value
true – true value

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