NUMERICAL SOLUTION OF THERMAL ELASTIC-PLASTIC FUNCTIONALLY GRADED THIN ROTATING DISK WITH EXPONENTIALLY VARIABLE THICKNESS AND EXPONENTIALLY VARIABLE DENSITY

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Thermal elastic-plastic stresses and strains have been obtained for rotating annular disk by using finite difference method with Von-Mises' yield criterion and non-linear strain hardening measure. The compressibility of the disk is assumed to be varying in the radial direction. From the numerical results, we can conclude that thermal rotating disk made of functionally graded material whose thickness decreases exponentially and density increases exponentially with non-linear strain hardening measure $(m = 0.2)$ *is on the safe side of the design as compared to disk made of homogenous material. This is because of the reason that circumferential stress is less for functionally graded disk as compared to homogenous disk. Also, plastic strains are high for functionally graded disk as compared to homogenous disk. It means that disk made of functionally graded material reduces the possibility of fracture at the bore as compared to the disk made of homogeneous material which leads to the idea of stress saving.*

Keywords: Disk, rotation, thin, thickness, density, thermal, non-homogenous.

1. Introduction

The study of stress distribution at high angular velocity and high temperature in rotating disk made of non-homogenous material is an active topic due to large number of industrial and mechanical applications. Some analytical solutions to rotating disk in elastic state and plastic state are available in the literature [1, 2] and research on them is always an important topic. Eraslan [3] investigated inelastic stresses and displacements in rotating solid disks of exponentially varying thickness using Tresca's and Von-Mises' yield criterion with linear strain hardening. Plastic limit angular velocities have been calculated for different disk profiles. Further, Eraslan et. al. [4] extended his work to find the numerical solution for elastic-plastic stresses in a rotating disk with Von-Mises' yield criterion using general non-linear strain hardening rule. The above work mainly concentrates on the material whose mechanical and physical properties are constant. In contrast, non-homogenous materials have different spatial distribution of material properties which can be designed according to different engineering applications. Gupta et al. [5] studied the effect of non-homogeneity on elastic-plastic stresses in a rotating disk using transition theory and concluded that non-homogeneous thin rotating disk require high angular speed for initial yielding as compared to homogenous disk. Further, Gupta et al. [6] studied rotating disk with variable thickness and variable density to analyze creep stresses and concluded that a rotating disk with variable thickness and density is on the safer side of design in comparison to flat disk with variable density. Yuriy et al. [7] discussed the control problem of thermal stresses in axissymmetrical infinite cylinder using technique of integral transform. Sharma et al. [8]

investigated elastic-plastic stresses for rotating disk made of transversely isotropic material and concluded that rotating disk made of transversely isotropic material is on the safer side of design as compared to rotating disc made of isotropic material. Reza et al. [9] discussed finite element simulation of residual stresses during the quenching process. A three dimensional nonlinear stress analysis model is used to estimate stress fields of UIC60. You et. al [10] calculated the elastic-plastic stresses with polynomial non-linear strain hardening model for rotating solid disk using perturbation technique. Further, with this polynomial non-linear strain hardening model You et. al [11] investigated elastic-plastic stresses for rotating disk having arbitrary variable thickness and density using Runge-Kutta method. Zhanling et al. [12] used finite element analysis of thermal-structure coupling to investigate stress and temperature when material properties are temperature dependent in a drag disk break application. Sharma et al. [13] investigated elastic-plastic stresses for rotating disk made up of isotropic material having exponentially variable thickness and exponentially variable density with non-linear strain hardening using finite difference method. They observed that disk whose thickness decreases radially and density increases radially is on the safer side of the design as compared to disk whose thickness and density varying exponentially as well as flat disk. Deepak et al. [14] studied creep stresses in a rotating disk made up of composite of silicon carbide particles in a matrix of pure aluminums and their study indicates that with the increase in particle gradient in the disk, the radial stress increases throughout the disk, whereas the tangential and effective stresses increase near the inner radius but decrease near the outer radius.

In this paper, we investigated thermal elastic-plastic stresses and strains for rotating disk made up of functionally graded material using finite difference method. The thickness and density are assumed to vary exponentially along the radius. Results have been discussed numerically and graphically.

2. Mathematical formulation

2.1. Distribution of material properties and thickness profile with basic equations

A thermal annular axi-symmetrical disk has been considered with inner radius a and external radius b rotating with angular velocity ω. The disk is made up of functionally graded material having exponentially varying thickness and exponentially varying density with plane stress condition i.e. $T_{zz} = 0$.

The coefficient of thermal expansion, compressibility, density, temperature distribution of the material and thickness profile of the rotating annular disks with radius r are expressed as:

$$\alpha(r) = \alpha_0 \left(\frac{r}{b}\right)^{-\alpha_1}; C(r) = C_0 \left(\frac{r}{b}\right)^{-k}; \ \rho(r) = \rho_0 e^{\left(\frac{r}{b}\right)^d}; \theta(r) = \frac{\theta_0}{\log(a/b)} \log(r/b); \ h(r) = h_0 exp^{-\left(\frac{r}{b}\right)^n}, \quad (1)$$

where α_0, C_0, ρ_0, θ_0, h_0 are material constants and k, d, α_1, and n are the geometric parameters. The eq. of equilibrium for the rotating disk is

$$\frac{d}{dr}\left(hrT_{rr}\right) - hT_{\theta\theta} + h\rho\omega^2 r^2 = 0. \quad (2)$$

The relation between strains and radial displacements are

$$e_r = du/dr; \ e_\theta = u/r; \ e_z = t, \quad (3)$$

where u is the radial displacement and t is a constant .
The eq. of compatibility can be derived from eq. (3) as

$$r\frac{de_\theta}{dr}+e_\theta-e_r=0\ , \tag{4}$$

The total radial and circumferential strains in rotating annular disks are

$$e_r=e_r^e+e_r^p+\alpha\theta\,;\ e_\theta=e_\theta^e+e_\theta^p+\alpha\theta\,. \tag{5}$$

The relationship between stresses and strains can be represented by Hooke's Law in elasticity as

$$e_r^e=\frac{1}{3}\left[(2-C)T_{rr}-(1-C)T_{\theta\theta}\right];\ e_\theta^e=\frac{1}{3}\left[(2-C)T_{\theta\theta}-(1-C)T_{rr}\right] \tag{6}$$

where C is the compressibility, T_{rr} and $T_{\theta\theta}$ are radial and circumferential stresses respectively.

The temperature field satisfying Laplace heat eq. is with

$$\nabla^2\theta=0 \text{ with } \theta=\theta_0 \text{ at } r=a;\ \theta=0 \text{ at } r=b,\ \theta(r)=\frac{\theta_0}{\log\left(a/b\right)}\log\left(r/b\right),\ \theta_0 \text{ is a constant.} \tag{7}$$

Let us define radial and circumferential stress in terms of stress function as

$$T_{rr}=\frac{\phi}{hr};\ T_{\theta\theta}=\frac{1}{h}\frac{d\phi}{dr}+\rho\omega^2r^2\,. \tag{8}$$

Substituting eq. (8) into eq. (6) and expressing strain components of eq. (5) in terms of stress function as

$$\begin{bmatrix} e_r \\ e_\theta \end{bmatrix}=\frac{1}{3}\begin{bmatrix} 2-C & -(1-C) \\ -(1-C) & 2-C \end{bmatrix}\begin{bmatrix} \dfrac{\phi}{hr} \\ \dfrac{1}{h}\dfrac{d\phi}{dr}+\rho\omega^2r^2 \end{bmatrix}+\begin{bmatrix} e_r^p \\ e_\theta^p \end{bmatrix}+\begin{bmatrix} \alpha\theta \\ \alpha\theta \end{bmatrix}. \tag{9}$$

Substituting eq. (9) into eq. (4), we have a governing differential equation for thermo elastic-plastic strain hardening rotating disks

$$\begin{aligned} &r^2\phi''+r\phi'\left[1-r\frac{h'}{h}-r\left(\frac{C'}{2-C}\right)\right]-\phi\left[1-\left(\frac{1-C}{2-C}\right)r\frac{h'}{h}-r\left(\frac{C'}{2-C}\right)\right]+\frac{3}{2-C}hr^2\left[e_\theta^{p'}+\theta\alpha'+\alpha\theta'\right] \\ &=h\omega^2r^4\left[\rho\left(\frac{C'}{2-C}\right)-\rho'\right]-h\rho\omega^2r^3\left(\frac{7-4C}{2-C}\right)+\frac{3}{2-C}hr\left(e_r^p-e_\theta^p\right), \end{aligned} \tag{10}$$

where $\phi''=\dfrac{d^2\phi}{dr^2}$, $\phi'=\dfrac{d\phi}{dr}$, $C'=\dfrac{dC}{dr}$, $e_\theta^{p'}=\dfrac{de_\theta^p}{dr}$, $\theta'=\dfrac{d\theta}{dr}$, $\alpha'=\dfrac{d\alpha}{dr}$. $h'=\dfrac{dh}{dr}$, $\rho'=\dfrac{d\rho}{dr}$.

In the elastic region $\left(e_r^p=e_\theta^p=0\right)$, eq. (10) reduces to

$$\begin{aligned} &r^2\phi''+r\phi'\left[1-r\frac{h'}{h}-r\left(\frac{C'}{2-C}\right)\right]-\phi\left[1-\left(\frac{1-C}{2-C}\right)r\frac{h'}{h}-r\left(\frac{C'}{2-C}\right)\right]+\frac{3}{2-C}hr^2\left[\theta\alpha'+\alpha\theta'\right] \\ &=h\omega^2r^4\left[\rho\left(\frac{C'}{2-C}\right)-\rho'\right]-h\rho\omega^2r^3\left(\frac{7-4C}{2-C}\right). \end{aligned} \tag{11}$$

For the plastic deformation, the relations between stresses and plastic strains can be determined according to deformation theory in plasticity [2] as

$$e_r^p=\frac{e_e^p}{T_{ee}}\left(T_{rr}-\frac{1}{2}T_{\theta\theta}\right);\ e_\theta^p=\frac{e_e^p}{T_{ee}}\left(T_{\theta\theta}-\frac{1}{2}T_{rr}\right), \tag{12}$$

where e_r^p and e_θ^p are the plastic radial and circumferential strains, e_e^p is the equivalent plastic strain which depends on the material model used and T_{ee} is the equivalent stress.

The Von Mises' equivalent stress is given by the expression as,

$$T_{ee}=\frac{1}{\sqrt{2}}\sqrt{\left(T_{rr}-T_{\theta\theta}\right)^2+\left(T_{\theta\theta}-T_{zz}\right)^2+\left(T_{zz}-T_{rr}\right)^2}\ . \tag{13}$$

The stress-strain relationship for Swift's hardening law can be written as

$$e_e = \frac{T_{ee}}{E},\ e_e \le e_0; e_e^p = \frac{1}{\eta}\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right],\ e_e > e_0, \tag{14}$$

where T_0 is the yield limit, e_e is the equivalent total strain, e_0 is the yield strain and T_{ee} is the equivalent stress.

Substituting e_e^p from eq. (14) into eq. (12) results in

$$e_r^p = \left(\frac{1}{\eta}\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right] \Big/ T_{ee}\right)\left(T_{rr} - \frac{1}{2}T_{\theta\theta}\right);\ e_\theta^p = \left(\frac{1}{\eta}\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right] \Big/ T_{ee}\right)\left(T_{\theta\theta} - \frac{1}{2}T_{rr}\right). \tag{15}$$

The boundary conditions for the rotating annular disks are

$$T_{rr} = 0 \text{ at } r = a; T_{rr} = 0 \text{ at } r = b. \tag{16}$$

Substituting eq. (15) into eq. (10), we have a non-linear differential equation in terms of ϕ as

$$\begin{aligned}
& r^2\phi''\left[1 + \frac{3}{2-C}\times\frac{1}{2\eta T_{ee}^2}\left\{2T_{ee}\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right] + \frac{1}{2T_{ee}}\left(T_{rr} - 2T_{\theta\theta}\right)^2 \times \left(1 + (m-1)\left(\frac{T_{ee}}{T_0}\right)^m\right)\right\}\right] \\
& - \frac{3}{2-C}\times\frac{1}{2\eta T_{ee}^2}\left[\begin{array}{l} T_{ee}\times\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right]\times\left\{\left(1 + 2r\frac{h'}{h}\right)r\phi' - \left(1 + r\frac{h'}{h}\right)\phi - 2h\rho'\omega^2 r^4 - 4h\rho\omega^2 r^3\right\} \\ + \frac{1}{2T_{ee}}\left(1 + (m-1)\left(\frac{T_{ee}}{T_0}\right)^m\right)\left(T_{rr} - 2T_{\theta\theta}\right)\times\left\{\begin{array}{l} \left(2T_{\theta\theta} - T_{rr}\right)\left[-r^2\frac{h'}{h}\phi' + h\rho'\omega^2 r^4 + 2h\rho\omega^2 r^3\right] + \\ \left(2T_{rr} - T_{\theta\theta}\right)\times\left[r\phi' - \left(1 + r\frac{h'}{h}\right)\phi\right] \end{array}\right\} \end{array}\right] \\
& - \frac{9}{2\eta T_{ee}}\frac{hr}{(2-C)}\times\left[\left(\frac{T_{ee}}{T_0}\right)^m - 1\right]\times\left(T_{rr} - T_{\theta\theta}\right) + r\phi'\left[1 - r\frac{h'}{h} - r\left(\frac{C'}{2-C}\right)\right] - \phi\left[1 - \left(\frac{1-C}{2-C}\right)r\frac{h'}{h} - r\left(\frac{C'}{2-C}\right)\right] \\
& + \frac{3}{2-C}hr^2\left[\theta\alpha' + \alpha\theta'\right] = h\omega^2 r^4\left[\rho\left(\frac{C'}{2-C}\right) - \rho'\right] - h\rho\omega^2 r^3\left(\frac{7-4C}{2-C}\right).
\end{aligned} \tag{17}$$

Eq. (17) is non-linear differential equation in the plastic region for rotating disk made up of functionally graded material with Swift's nonlinear strain hardening measure having non–uniform thickness and whose material properties subjected to thermal loading. When compressibility is assumed to be constants without thermal effects, eq. (17) becomes same as that obtained by Sharma et al. [13].

3. Finite Difference Algorithm

To determine thermal elastic-plastic non-homogeneous stresses and strains in thin rotating disks with non-linear strain hardening material, we have to solve the second order nonlinear differential eq. (17) under the boundary conditions (16). The general form of eq. (17) can be written as

$$\phi'' = f\left(r,\ \phi, \phi'\right). \tag{18}$$

(i) First partitioned the disk domain $r = [a\ b]$ into p subintervals of length Δr and express the differential operator ϕ' and ϕ'' in finite difference form as

$$\frac{d^2\phi}{dr^2} = \frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{(\Delta r)^2} \text{ and } \frac{d\phi}{dr} = \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta r}.$$

(ii) With $h = 1/p$, we have $p+1$ nodal points $\phi_1, \phi_2, \ldots \phi_{p+1}$. The values at the end points are given by eq. (16), i.e. $\phi_1 = 0,\ \phi_{p+1} = 0$. Using the finite difference approximation, we get the following system of equations:

$$\frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{(\Delta r)^2} = f\left(r, \phi_i, \frac{\phi_{i+1} - \phi_{i-1}}{2\Delta r}\right),\ i = 2, 3, \ldots p.$$

Stress function, stresses and strains can be obtained from the above system of $(p-1)$ nonlinear equations using Newton-Raphson method.

4. Numerical Discussion

The material properties of the annular disks made of functionally graded material with angular velocity $(\omega = 300, 700 \text{ rad/sec})$ and thermal effects $(\theta_0 = 0, 400, 700\ ^0\text{C})$ are defined as: material density $\rho_0 = 7850$ [kg/m^3], compressibility coefficient $C_0 = 0.5$, thermal expansion coefficient $\alpha_0 = 17.8 \times 10^{-6}$ [^{o} C^{-1}], Poisson's ratio $\nu = 0.3$. The inner and outer radii of the disk are taken as $a = 0.1$m and $b = 0.5$m respectively. The geometric parameters of the disk are taken as $n = 0.5, 1$ in thickness function, $d = 0.7, 1$ in density function, $\alpha_1 = 0$ in coefficient of thermal expansion and $k = 0, -0.5, -1, -1.5$ in compressibility function.

In order to explain the effect of rotation and compressibility on stresses in a disk made up of homogenous and non-homogenous materials, table 1 and curve have been drawn in figure 1 to figure 3 between stresses and radii r = 0.1:0.1:0.5.

Tab. 1 has made for circumferential (hoop) stresses with different parameters of compressibility and angular speed for non-linear strain hardening measure $m = 0.2$. From tab. 1, it is observed that without thermal effects circumferential stresses are maximum at the internal surface and increases significantly with the increase in angular speed. For non-homogenous disk, circumferential stresses increases with the increase in compressibility radially. It is also noticed that circumferential stress is high for homogenous disk $(k = 0)$ as compared to non-homogenous disk $(k = -0.5, -1, -1.5)$. Also, circumferential stresses are high for highly functionally graded disk $(k = -5)$ as compared to homogenous disk and these stresses increases with the increase in compressibility $(k = -5, -6, -7 \text{ etc.})$. With the introduction of thermal effects, circumferential stresses decrease and these stresses further decrease with the increase in temperature. With the increase in Swift's strain hardening measure, circumferential stress increases whereas these stresses decrease with the exponential decrease in thickness and increase in density.

From fig. 1, it is observed that circumferential stresses are maximum at internal surface for the disks made up of both homogenous and non-homogenous material. Also, circumferential stresses are maximum for homogenous disk as compared to non-homogenous disk. As compressibility measure changes from linear to non-linear, circumferential stresses decreases. With the increase in nonlinearity, stresses are again increases i.e. $\sigma_\theta = 257.843$MPa at $k = -0.5$, $\sigma_\theta = 258.195$MPa at $k = -1$ and $\sigma_\theta = 260.190$MPa at $k = -1.5$. From fig. 2, it has also been observed that with the increase in angular velocity from $\omega = 300$ rad/sec to $\omega = 700$ rad/sec, circumferential stresses increase significantly and maximum at internal surface of the disk. With the increase in thickness measure from $n = 0.5$ to $n = 1$ and density measure from $d = 0.7$ to $d = 1$, circumferential stress decreases. It can be observed from fig. 2 that circumferential stresses with thermal effects are maximum at internal surface for disk made of homogenous and functionally graded material. These circumferential stresses are maximum for homogenous disk as compared to functionally graded disk. With the increase in angular velocity circumferential stress increases significantly while with the increase in temperature, thickness and density, circumferential stress decreases as can be seen from fig. 3.

In order to explain the effect of rotation and compressibility on strains in a disk made up of homogenous and non-homogenous materials, table 2 and graphs have been drawn in figure 4 to figure 6 between strains and radii r = 0.1:0.1:0.5.

It has been noticed from tab. 2 that without thermal effects, circumferential strains are maximum at external surface of the disk. Also, these circumferential strains are compressive in nature. With the increase in angular speed, circumferential strains decrease significantly. Also, these circumferential strains increase when the thickness of disk decreases while density of the disk increases exponentially. These strains decreases with the increase in temperature while increases with the increase in angular speed. It has also been noted that circumferential strains are less for homogenous disk $(k=0)$ as compared to disk made of functionally graded material and these strains decreases with the increase in compressibility measure $(k=-0.5,-1,-1.5,-2\ etc.)$.

From fig. 4, it is observed that plastic strains are maximum at external surface for homogenous as well as functionally graded disk. Plastic strains decreases significantly for non-homogenous disk when compressibility measure changes from $k=-1$ to $k=-1.5$ as can be seen from figure 4. Plastic strains decreases significantly when angular velocity changes from $\omega=300$ rad/sec to $\omega=700$ rad/sec, but as thickness measure changes from $h=0.5$ to $h=1$ and density measure from $d=0.7$ to $d=1$, these strains increases significantly. With the increase in temperature, plastic strains increases remarkably. From fig. 5, it has been observed that circumferential strains increases with the introduction of temperature. These strains further increases with the increase in temperature as can be seen from fig. 6. Also circumferential strains decrease significantly with the increase in angular velocity while increases with the increase in thickness and density.

Conclusion

From the above analysis, It can be conclude that disk made of functionally graded material having nonlinear strain hardening $(m=0.2)$ with thermal effects whose thickness decreases exponentially and density increases exponentially is on the safer side of the design as compared to disk made of homogenous material. This is because of the reason that circumferential stress is less for functionally graded disk as compared to homogenous disk. Also, plastic strains are high for functionally graded disk as compared to homogenous disk. It means that disk made of functionally graded material reduces the possibility of fracture at the bore as compared to the disk made of homogeneous material which leads to the idea of stress saving.

Nomenclature of Symbols

T_{rr}, $T_{\theta\theta}$: Radial and circumferential stresses

e_r, e_θ, e_z : Total radial, circumferential and axial strains

r,ω : Radius and angular velocity of the disk

α,θ : Thermal expansion, temperature

e_θ^e, e_θ^p : Circumferential elastic and plastic strains

α_0, C_0, ρ_0, θ_0, h_0 : Material constants

E,ν : Young's modulus, Possion's ratio

u, e_0 : Radial displacement, yield strain

h,ρ,C : Thickness, density and compressibility of the disk

T_{ee}, e_e : Equivalent stress and strain

a,b : Inner and outer radii of the disk

e_r^e, e_r^p : Radial elastic and plastic strains

k, d, α_1, n : Geometric parameters

ϕ : Stress function

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Table 1. Hoop Stress in rotating annular disk having different geometric parameters of compressibility $(k = 0, -0.5, -1, -1.5)$ with non-linear strain hardening measure $m = 0.2$, angular velocity ω in rad/sec. and temperature θ_0 in ^{o}C.

θ_0 in ^{o}C	$T_{\theta\theta}$ in MPa	$\omega = 300$ rad/sec., $d = 0.7$, $l = 0.5$			$\omega = 300$ rad/sec., $d = 1$, $l = 1$			$\omega = 700$ rad/sec., $d = 0.7$, $l = 0.5$			$\omega = 700$ rad/sec., $d = 1$, $l = 1$		
	k \ r	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
0	0	265.0684	136.5472	71.7840	225.9867	118.5655	63.1811	1442.7691	743.4235	390.8258	1230.0670	645.5231	343.9862
	-0.5	257.8431	137.4753	73.4469	219.7997	119.5076	64.6199	1403.4647	748.4767	399.8788	1196.4099	650.6527	351.8196
	-1	258.1946	137.4663	74.3184	220.0074	119.5696	65.3629	1405.3682	748.4275	404.6234	1197.5330	650.9898	355.8642
	-1.5	260.1903	137.1896	74.7894	221.6020	119.3645	65.7561	1416.2223	746.9213	407.1871	1206.2053	649.8734	358.0047
400	0	265.0684	136.5472	71.7840	225.9867	118.5655	63.1811	1442.7691	743.4235	390.8258	1230.0670	645.5231	343.9863
	-0.5	257.8431	137.4753	73.4469	219.7997	119.5076	64.6199	1403.4647	748.4767	399.8788	1196.4099	650.6527	351.8196
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	-1.5	260.1903	137.1896	74.7894	221.6020	119.3645	65.7561	1416.2223	746.9213	407.1871	1206.2053	649.8734	358.0048
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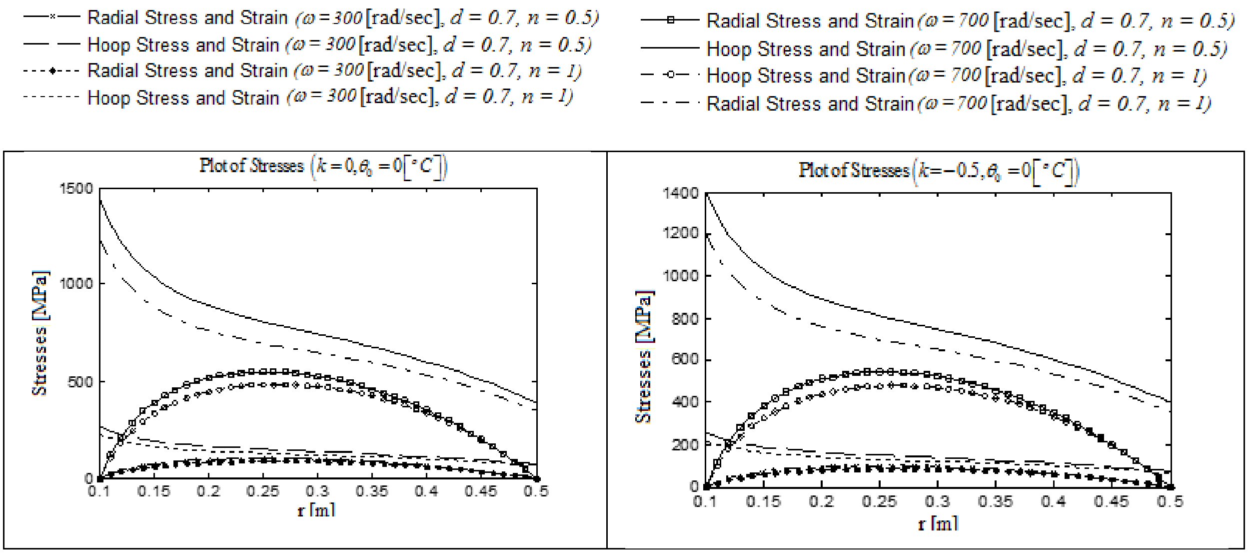
	-0.5	257.8431	137.4753	73.4469	219.7997	119.5076	64.6199	1403.4647	748.4767	399.8788	1196.4099	650.6527	351.8196
	-1	258.1946	137.4663	74.3184	220.0074	119.5696	65.3629	1405.3682	748.4275	404.6234	1197.5330	650.9898	355.8642
	-1.5	260.1903	137.1896	74.7894	221.6020	119.3645	65.7561	1416.2224	746.9213	407.1872	1206.2053	649.8734	358.0047

Table 2. Hoop Strain in rotating annular disk having different geometric parameters of compressibility $(k = 0, -0.5, -1, -1.5)$ with non-linear strain hardening measure $m = 0.2$, angular velocity ω in rad/sec. and temperature θ_0 in ^{o}C.

θ_0 in ^{o}C	e_{θ}^{p}	$\omega = 300$ rad/sec., $d = 0.7$, $l = 0.5$			$\omega = 300$ rad/sec., $d = 1$, $l = 1$			$\omega = 700$ rad/sec., $d = 0.7$, $l = 0.5$			$\omega = 700$ rad/sec., $d = 1$, $l = 1$		
	k \ r	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5	0.1	0.3	0.5
0	0	-0.00648	-0.00475	-0.00658	-0.00649	-0.00464	-0.00659	-0.0063	-0.00465	-0.00644	-0.00632	-0.00453	-0.00645
	-0.5	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1.5	-0.00648	-0.00478	-0.00658	-0.00649	-0.00468	-0.00659	-0.0063	-0.00467	-0.00644	-0.00632	-0.00457	-0.00645
400	0	-0.00648	-0.00475	-0.00658	-0.00649	-0.00464	-0.00659	-0.0063	-0.00464	-0.00644	-0.00632	-0.00453	-0.00645
	-0.5	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1.5	-0.00648	-0.00478	-0.00658	-0.00649	-0.00468	-0.00659	-0.0063	-0.00467	-0.00644	-0.00632	-0.00457	-0.00645
700	0	-0.00648	-0.00475	-0.00658	-0.00649	-0.00464	-0.00659	-0.0063	-0.00464	-0.00644	-0.00632	-0.00453	-0.00645
	-0.5	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1	-0.00648	-0.0048	-0.00658	-0.0065	-0.00469	-0.00659	-0.0063	-0.00468	-0.00644	-0.00632	-0.00458	-0.00645
	-1.5	-0.00648	-0.00478	-0.00658	-0.00649	-0.00468	-0.00659	-0.0063	-0.00467	-0.00644	-0.00632	-0.00457	-0.00645



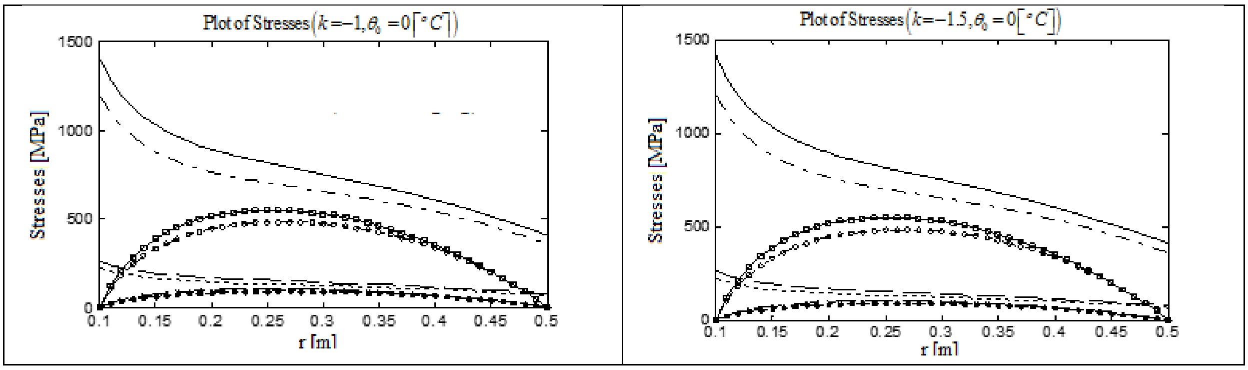


Figure 1. Elastic-Plastic stresses in a rotating disk with parameters $(k=0,-0.5,-1,-1.5)$**.**

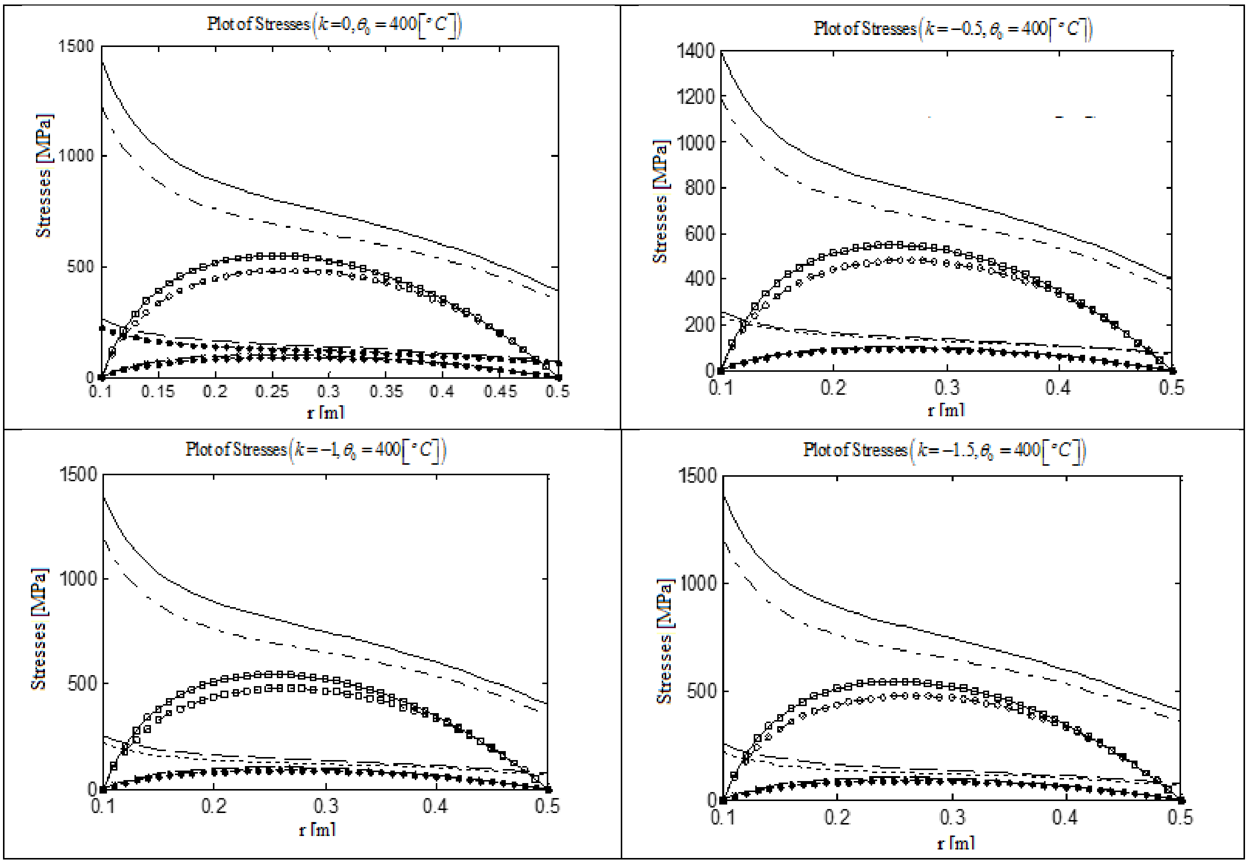


Figure 2. Thermal elastic-plastic stresses in a rotating disk with thermal effects ($\theta_0 = 400\,^{o}C$ **) with parameters** $(k=0,-0.5,-1,-1.5)$**.**

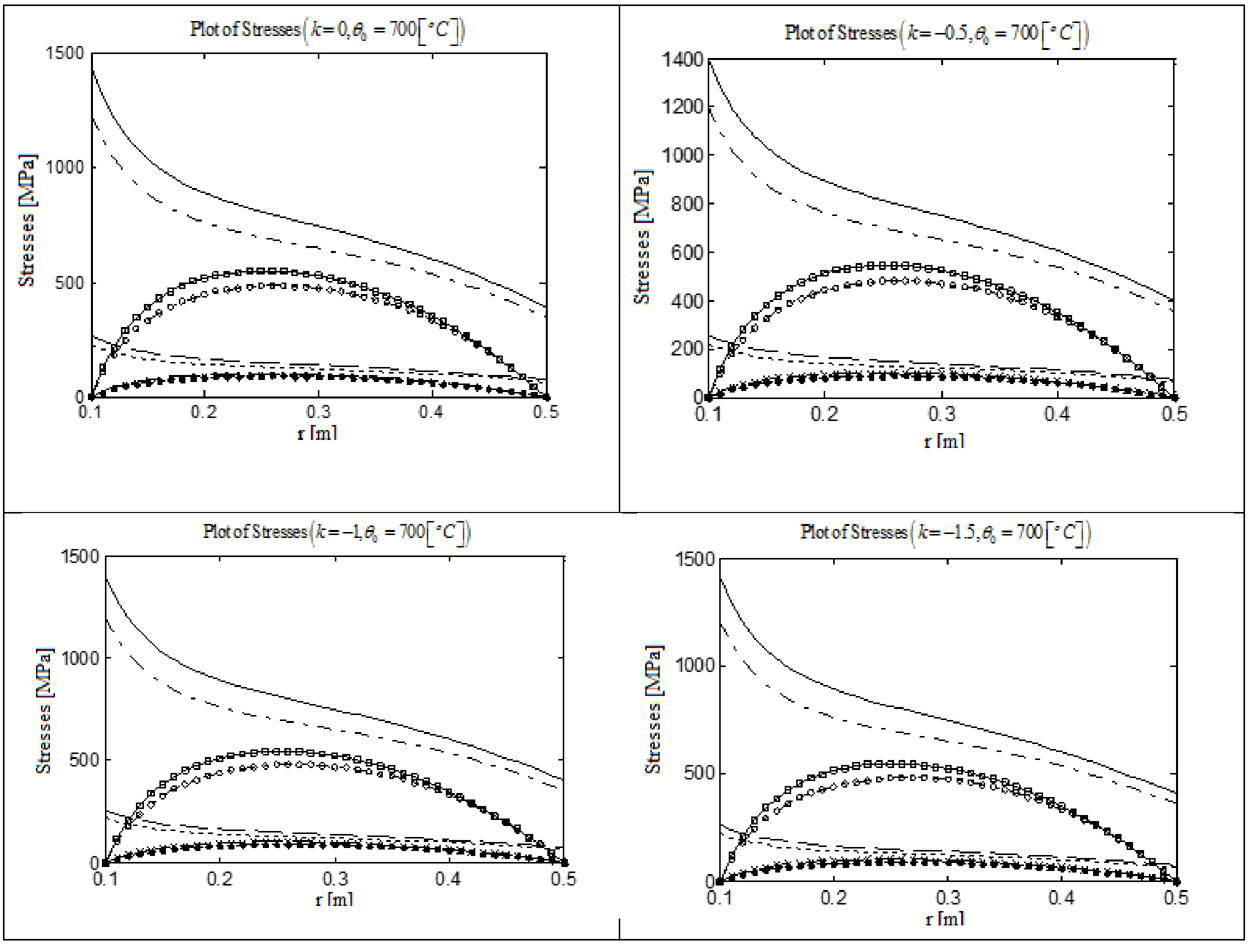
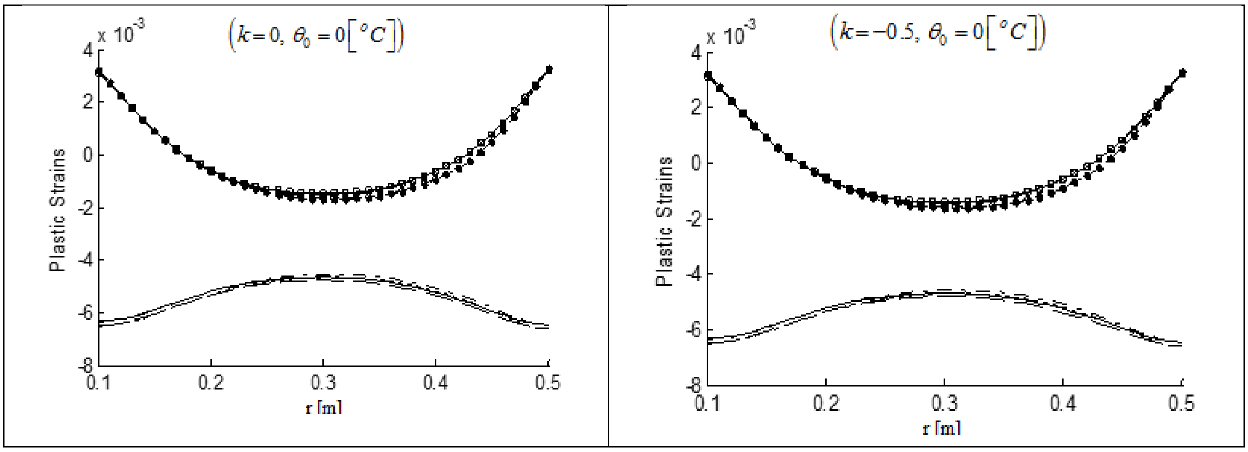


Figure 3. Thermal elastic-plastic stresses in a rotating disk with thermal effects ($\theta_0 = 700\,^oC$) with parameters $(k = 0, -0.5, -1, -1.5)$.



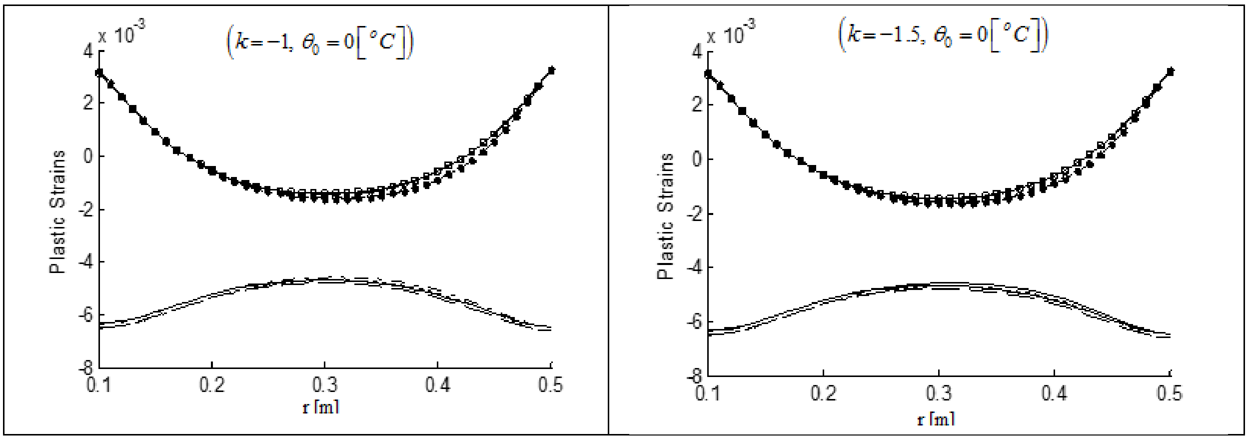


Figure 4. Plastic strains in a rotating disk with parameters $\left(k=0,-0.5,-1,-1.5\right)$.

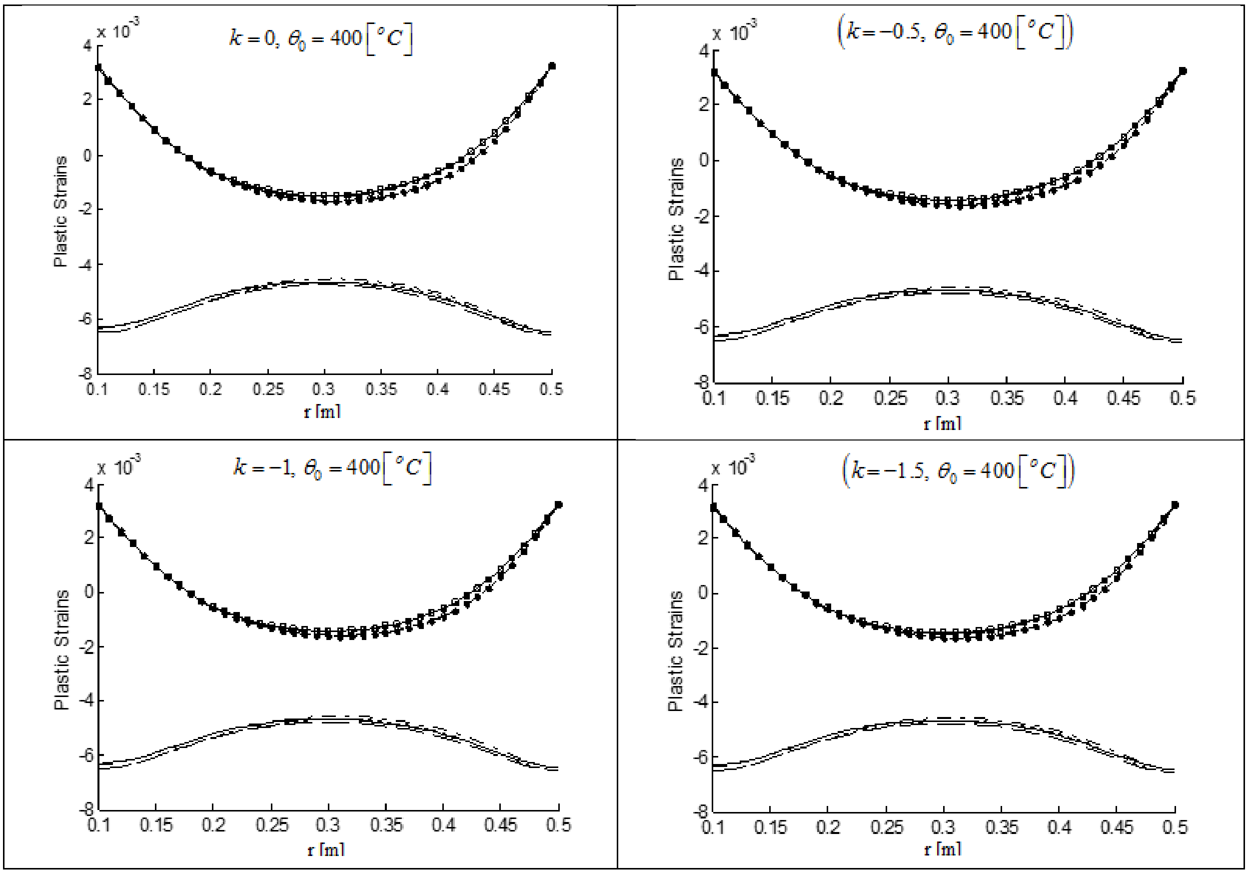


Figure 5. Thermal plastic strains in a rotating disk with thermal effects ($\theta_0=400\,{}^{\circ}C$ **) with parameters (** $k=0,-0.5,-1,-1.5$ **).**

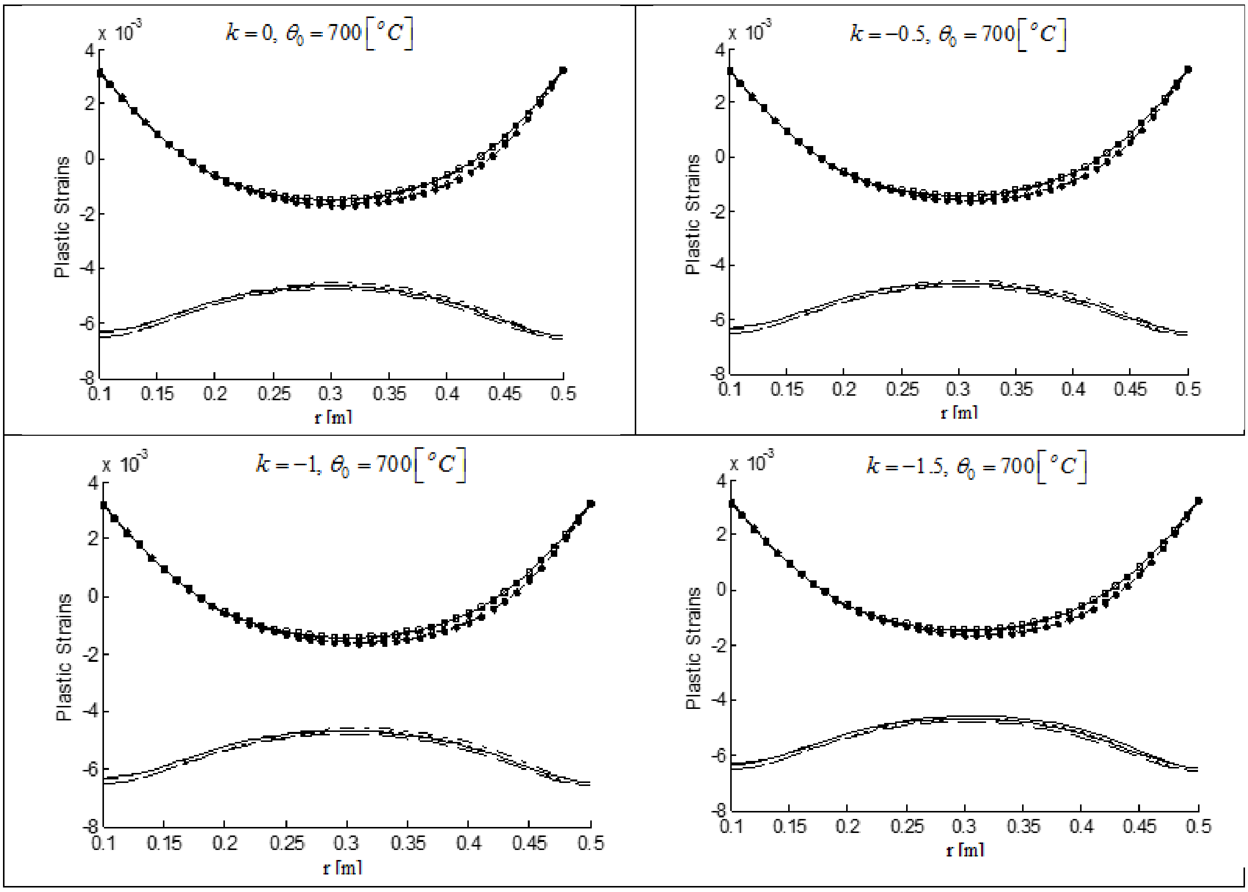


Figure 6. Thermo plastic strains in a rotating disk with thermal effects ($\theta_0 = 700\,{}^{o}C$) with parameters$\left(k = 0, -0.5, -1, -1.5\right)$.