A FULLY COUPLED THERMO-HYDRO-MECHANICAL MODEL ASSOCIATED WITH INERTIA AND SLIP EFFECTS

by

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The inertia and slip effects have a significant impact on the coal seam gas extraction. A fully coupled thermo-hydro-mechanical model is established in this study, which takes into account the influence of non-Darcy gas flow and Klinkenberg effect on the coal seam deformation and coalbed methane migration. The numerical result shows that the coalbed methane migration and transport evolution coal bed methane reservoir is not only dependent on the coal matrix deformation, gas pressure and gas adsorption, but also closely related to inertia effect and slip effect.

Key words: *coal seam gas, gas extraction, thermo-hydro-mechanical model, non-Darcy effect, Klinkenberg effect*

Introduction

Coal bed methane extraction is an effective method to reduce the gas disaster. Besides, gas drainage can take advantage of this clean energy source [1-4]. Coal seam is the dual porous media reservoir, containing matrix and fracture network. Generally, coal seam gas transportation process experiences three stages. First, gas flows to drainage gas well from the fracture network under the pressure gradient. Then, free gas in the coal matrix diffuses into the coal fracture due to reduced gas pressure of coal fracture. During this diffusion process, the gas desorption occurs in coal matrix [5-8]. Scholars have proposed many models to analyze the coal and methane interaction [7-9]. The gas flow and liquid flow are significantly different because of the gas compressibility and Klinkenberg effect. At the same time, non-Darcy effect also has a significant impact on the gas flow. However, most of the previous models do not take into account the effects of inertia effect and slip effect on gas drainage in coal seams. In this paper, a fully coupled model is established, and the inertia and slip effects are considered in this fully coupled model. The PDE in this finite element model are solved by numerical method. Through this finite element model, the effects of inertia (non-Darcy) and slip (Klinkenberg) on coal seams were quantitatively evaluated.

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Description of the mathematical model

Inertia effect of gas flow

Darcy flow is often used in the gas migration process. However, the permeability of the coal seams is usually low in most Chinese coal mines, and the hydraulic gradient and the infiltration velocity may not satisfy the linear relationship near the drilling holes, which makes that the Darcy's law is inapplicable [10, 11]. The non-Darcy flow caused by inertial effect may have a significant effect on gas reservoir performance. The estimation of the inertial effect is important. There is a higher flow rate through the addition of the velocity square term, *i. e.*, the Forchheimer's extension [12]:

$$-\nabla p = \frac{\upsilon}{k_g}\vec{\mu} + \beta\rho_g\vec{\mu}\left|\vec{\mu}\right| \tag{1}$$

where $\vec{\mu}$ is the gas velocity vector, ρ_g – the gas density, $\beta = 1.75/(150k_g\phi^3)^{1/2}$, ϕ – the porosity of coal, and k_g – the permeability of coal.

Equation (1) can be rewritten as the form of Darcy's law:

$$\vec{\mu} = -\frac{k_g}{\left(1+\dfrac{k_g}{\upsilon}\beta\rho_g\left|\vec{\mu}\right|\right)\upsilon}\nabla p = -\frac{k_g}{f_{qi}\upsilon}\nabla p \tag{2}$$

where $f_{qi}=1+(k_g/\upsilon)\beta\rho_g\left|\vec{\mu}\right|$ is a Forchheimer number.

The flow of gas in the coal satisfies the mass conservation law [13]:

$$\frac{\partial m}{\partial t} + \nabla(\rho_g\vec{\mu}) = Q_s \tag{3}$$

where ρ_g is the density of gas, $\vec{\mu}$ – the gas velocity vector, Q_s – the gas source by injection, t – the real time. In eq. (3), this mass content m is defined:

$$m = \rho_g\phi + \rho_{ga}\rho_c V_{sg} + \rho_{ga}\rho_c m_b \tag{4}$$

where ϕ is the porosity, ρ_{ga} – the gas density at standard conditions, ρ_c – the coal density, V_{sg} – the content of absorbed gas, and m_b – the average remaining gas content in the coal matrix.

Slip effect of gas flow

The slip (Klinkenberg) effect between the methane and coal rock can seriously affect the flow rate during the gas extraction process. Klinkenberg effect is obvious in porous media when the sizes of the mean free path of gas molecules and pore dimension are in the same order of magnitude. The effective gas permeability, k_g, can be expressed as a as function of the gas pressure:

$$k_g = k_\infty\left(1+\frac{b}{p}\right) \tag{5}$$

where k_∞ is the intrinsic permeability to liquids, and b is the Klinkenberg coefficient, which is usually a function of the mean free path of the gas molecules, gas pressure, temperature, and gas molecular weight. The Klinkenberg factor $b = \alpha_k k_\infty^{-0.36}$ increases with the decrease of permeability [14, 15]. The α_k is the Klinkenberg effect coefficient, which is fitted to be 0.251.

Gas diffusion in coal matrix

The source term $\bar{Q}_m$ from the sorption/adsorption of coal matrix can be expressed:

$$\bar{Q}_m = -\rho_c \rho_{ga} \frac{\mathrm{d}m_b}{\mathrm{d}t} \tag{6}$$

The methane exchange rate is related to the current gas content and the equilibrium gas content, therefore, the eq. (7) is used to calculate the exchange rate:

$$\frac{\mathrm{d}m_b}{\mathrm{d}t} = -\frac{1}{\tau}\left[m_b - m_e(p)\right] \tag{7}$$

where m_e is the equilibrium gas content under pressure, p. The diffusion time can be expressed:

$$\tau = \frac{1}{aD} \tag{8}$$

where a is a shape factor, and D – the diffusion coefficient.

Coal permeability

If the initial pressure is p_0 and the initial porosity is ϕ_0, then the porosity is expressed [7]:

$$\phi = \frac{1}{1+S}\left[(1+S_0)\phi_0 + \alpha(S - S_0)\right] \tag{9}$$

where $S = \varepsilon_V + p/K_s - \varepsilon_s - \alpha_T T$, and $S_0 = \varepsilon_{V0} + p_0/K_s - \varepsilon_{s0} - \alpha_T T_0$.

The evolution model of apparent permeability:

$$\frac{k_g}{k_{\infty 0}} = \frac{k_\infty}{k_{\infty 0}}\left(1 + \frac{b}{p}\right) = \left(\frac{\phi}{\phi_0}\right)^3\left(1 + \frac{b}{p}\right) \tag{10}$$

Mechanical equilibrium equation

The stress-strain relationships for an isothermal linear elastic porous medium may be written:

$$\Delta\boldsymbol{\varepsilon}_{ij} = \frac{1}{2G}\Delta\boldsymbol{\sigma}_{ij} - \left(\frac{1}{6G} - \frac{1}{9K}\right)\Delta\sigma_{kk}\delta_{ij} + \frac{\alpha}{3K}\Delta p\delta_{ij} + \frac{\Delta\varepsilon_s}{3}\delta_{ij} + \frac{\alpha_T}{3}\Delta T\delta_{ij} \tag{11}$$

where $\boldsymbol{\varepsilon}_{ij}$ is the strain tensor, $\boldsymbol{\sigma}_{ij}$ – the stress tensor, ε_s – the sorption-induced volumetric strain, K – the bulk modulus, G – the shear modulus, p – the gas pressure, T – temperature, δ_{ij} – the Kronecker delta, and α – the Biot coefficient.

The volume strain of coal is:

$$\varepsilon_V = \frac{1}{K}\left(\bar{\sigma} + \alpha p\right) + \alpha_T T + \varepsilon_s \tag{12}$$

where $\varepsilon_V = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$ is the volumetric strain, $\bar{\sigma} = \sigma_{kk}/3$ is the mean stress, and the effective stress is defined as $\boldsymbol{\sigma}_{eij} = \boldsymbol{\sigma}_{ij} + \alpha p\delta_{ij}$.

Thermal transport equation

Ignoring the thermal-filtration effect, the total heat flux, q_T, is expressed:

$$q_T = -\lambda_M \nabla T + \rho_g C_g q_g (T_{ar} + T) \tag{13}$$

where q_T is thermal flux, ρ_s – the gas density, C_g – the gas specific heat constants, q_g – the Darcy velocity, and λ_M – the thermal conductivities of coal, which is defined by $\lambda_M = (1-\phi)\lambda_s + \phi\lambda_g$, with solid components, λ_g, and gas components, λ_s.

Ignoring the inter-convertibility of thermal and mechanical energy, the thermal balance can be expressed [7]:

$$\frac{\partial[(\rho C)_M (T_{ar} + T)]}{\partial t} + (T_{ar} + T) K_g \alpha_g \nabla\left(\frac{k_g}{f_{qi}\upsilon}\nabla p\right) + (T_{ar} + T) K \alpha_T \frac{\partial \varepsilon_V}{\partial t} = -\nabla q_T \tag{14}$$

where $(\rho C)_M$ is the specific heat capacity of solid medium, which is defined by $(\rho C)_M = \phi(\rho_g C_g) + (1-\phi)(\rho_s C_s)$, with the mass density, ρ_s, gas heat constants, C_g, solid heat constants, C_s, and thermal expansion coefficient of the gas, α_g.

The fully coupled thermo-hydro-mechanical model is defined by previously field equations. A finite element program is implemented within the framework of Comsol Multiphysics (the non-linear PDE solver) to solve the fully coupled processes.

Numerical simulation of coal bed methane migration

Model description

The coal seam gas extraction is related to the inertia and slip effects of gas flow. In order to study the coupling effect of these two factors, a gas drainage model is established in this section, as shown in fig. 1. The length of the model is 20 m, and the width is 13 m. The thickness of the coal seam is 3 m, distributed horizontally between the two layers. Because of the gravity of upper strata, the stress of 12.5 MPa is loaded on the upper boundary. The gas extraction borehole is distributed along the right boundary. The other boundaries are normal restricted. It is assumed that the gas only flows in the coal seam. The diameter of the gas extraction borehole is 0.1 m, the initial gas pressure is 2.5 MPa, the gas extraction pressure is 0.1 MPa, the Youngs modulus of coal is 1800 MPa, the methane Langmuir pressure constant is 1.57 MPa, the density of coal is 1380 kg/m^3, the density of methane at standard condition is 0.717 kg/m^3, the initial

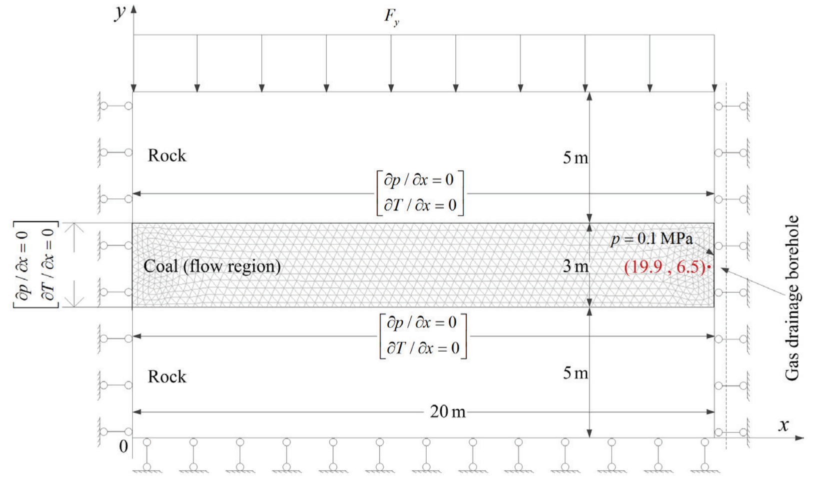

Figure 1. Calculation model

porosity is 0.01, the initial permeability is $1.09\cdot10^{-18}$ m^2, the initial gas pressure is 2.16 MPa, the drainage pressure is 20 kPa, the pressure coefficient is 0.07 MPa^{-1}, the temperature coefficient is 0.02 MPa^{-1}, the pressure coefficient is 0.07 MPa^{-1}, the temperature coefficient is 0.02 MPa^{-1}, the specific heat capacity of gas is $1.005\cdot10^3$ J/kgK, the volumetric thermal expansion of the solid matrix coefficient is $2.4\cdot10^{-5}$K^{-1}. The co-ordinates of the monitoring point is (19.9, 6.5).

Gas pressure evolution of coal seam

The gas pressure of the coal seam at different times is presented in fig. 2. The gas pressure decreases with time in the coal seam. For example, $p_{\max} = 2.10$ MPa and $p_{\max} = 0.77$ MPa correspond to the gas drainage time of $t = 1\cdot10^6$ second and $t = 1\cdot10^9$ second, respectively. The gas pressure distribution along the horizontal line $y = 6.5$ m is shown in fig. 3. It should be noted that the annotation *Darcy without Klinkenberg* denotes that both of non-Darcy effect and Klinkenberg effect are not considered in the model.

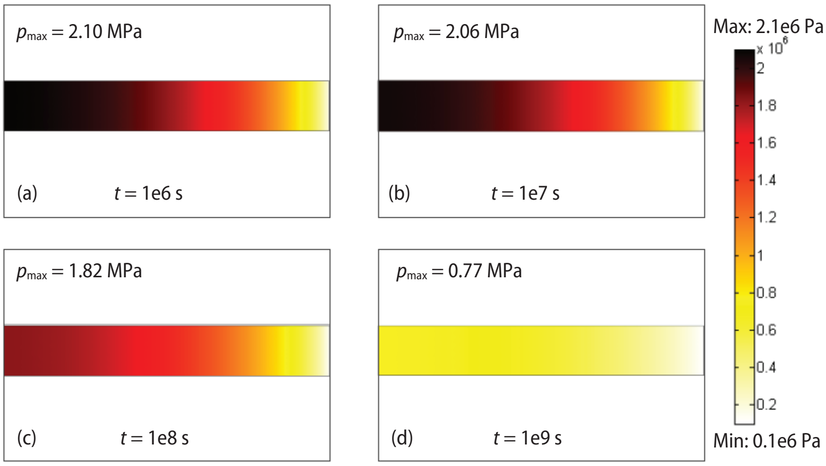

Figure 2. Gas pressure distribution in the coalbed at different times
(*for color image see journal web site*)

It can be seen from fig. 3 that the gas pressure of coal becomes smaller with the decreasing distance to the borehole. At the same time, the gas pressure of coal seam decreases with the increase of time. The slip effect, *i. e.*, the Klinkenberg effect has a significant influence on gas flow while the influence of inertia effect (non-Darcy flow) on the gas drainage is not obvious and can be ignored. The research results of Jones and Owens [14] also showed that inertia effect had little effect on gas flow. This may be caused by the fact that inertia effect is usually more pronounced in high gas pressure and high permeability coal seams, while this gas drainage condition are rare in most Chinese coal mines.

Permeability evolution of coal seam

Figure 4 shows the relative permeability distribution at different times. With the increase of time, the coal seam desorbs the gas gradually and releases the pressure. This induces the shrinkage of coal matrix and the increase of porosity, further resulting in the change of the effective stress. The permeability ratio increases from 1.048 to 1.418 with the time from $t = 1\cdot10^6$ second to $t = 10^9$ second. Therefore, the permeability of the coal seam increases grad-

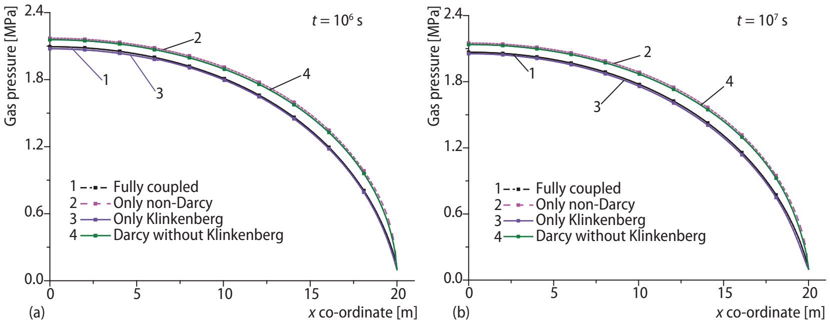

Figure 3. Gas pressure distribution along the horizontal line $y = 6.5$ m, $x \in [0$ m, 20 m$]$
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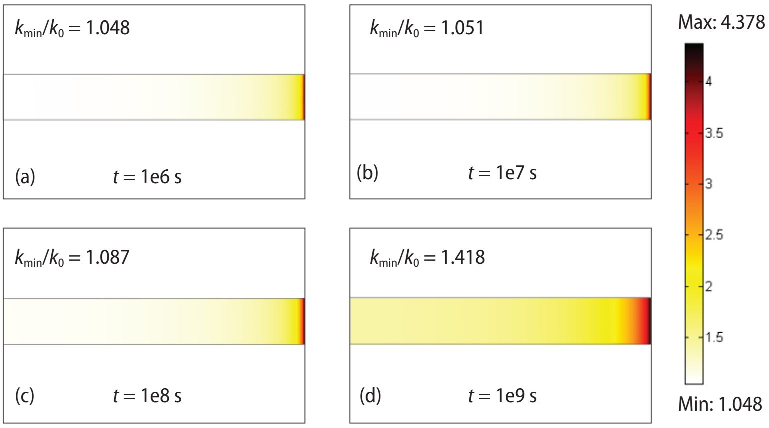

Figure 4. Relative permeability distribution in the coal seam at different times
(*for color image see journal web site*)

ually during the gas extraction process. The permeability evolution of coal seam and the gas migration process are not only related to the deformation of coal seams and pore pressure, but also affected by the inertia and slip effects of gas reservoir.

Figure 5 shows the relative permeability profile at point (19.9, 6.5) and the gas extraction rate of the whole borehole under different times. It can be seen from fig. 5(a) that the permeability of coal seams with considering the Klinkenberg effect is significantly higher than that without considering the Klinkenberg effect. The permeability ratio without considering the Klinkenberg effect is 2.77, only the 68.2% of the permeability ratio with considering the Klinkenberg effect under gas pressure $p_0 = 2.5$ MPa. Therefore, the Klinkenberg effect is one of the factors that must be considered in the gas extraction process. Because of the low gas pressure in this simulation case, the effect of inertial effect on the gas seepage process is difficult to detect.

To analyze the coupling effect of inertia and slip effects on gas drainage, the gas flow rate of extraction borehole is shown in fig. 5(b). Considering the slip effect, the gas extraction rate of the borehole is $4.11 \cdot 10^{-2}$ m^3/s at $p_0 = 2.5$ MPa for $t = 10^2$ second, while the gas extraction rate of the borehole is $2.37 \cdot 10^{-2}$ m^3/s without considering the slip effect at the same state. This

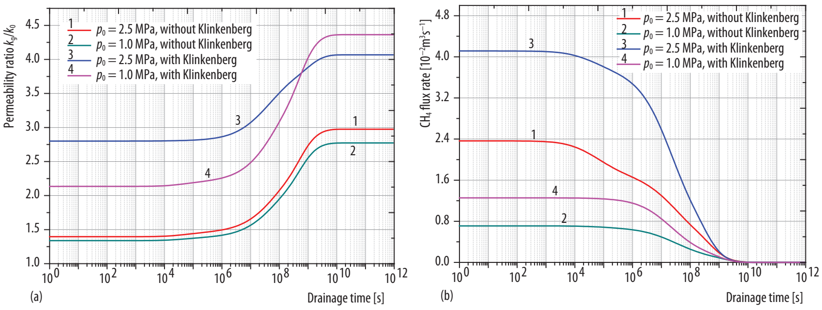

Figure 5. Gas pressure, permeability, and gas production changes over time near the well under different models
(*for color image see journal web site*)

indicates that the gas drainage rate with considering the Klinkenberg effect is significantly larger than that without considering the Klinkenberg effect, and the gas extraction rate decreases with the reduce of the initial pressure. In conclusion, Klinkenberg effect plays an important role in coalbed gas migration process, and it cannot be neglected in the analyses of gas extraction process.

Conclusions

In this paper, a fully coupled model is established, and the inertia and slip effects are considered in this fully coupled model. The PDE in this finite element model are solved by numerical method. The numerical result shows that the coalbed methane reservoir evolution and gas migration are not only related to the change of coal deformation and pore pressure, but also affected by inertia and slip effects. Slip effect plays an important role in coalbed methane migration. For example, when the initial gas pressure is 2.5 MPa, considering the slip effect, the gas extraction rate of the borehole increased 73.4%, compared to the case without considering slip effect. Therefore, slip effect should be considered in the analyses of gas extraction process. In the coal seams with high permeability and high gas pressure, the non-Darcy effect is more obvious, while the Klinkenberg effect is more remarkable in the coal seam when the permeability and pore pressure are both lower. The permeability and gas extraction rate will be underestimated without considering the slip effect.

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Nomenclature

b – Klinkenberg coefficient, [MPa]
C_g – specific heat capacity of gas, $[\mathrm{Jkg^{-1}K^{-1}}]$
C_s – specific heat capacity of coal, $[\mathrm{Jkg^{-1}K^{-1}}]$
D – diffusion coefficient, $[\mathrm{s^{-1}}]$
Fy – strata gravity, [MPa]
f_{qi} – Forchheimer number, [–]
K – bulk modulus of coal, [MPa]
K_s – bulk modulus of coal grians, [MPa]
k_g – permeability of coal, $[\mathrm{m^2}]$
k_0 – initial permeability, $[\mathrm{m^2}]$
k_∞ – intrinsic permeability, $[\mathrm{m^2}]$
m – mass content, $[\mathrm{kgm^{-3}}]$
m_e – equilibrium gas content, $[\mathrm{kgm^{-3}}]$
p – gas pressure, [MPa]
p_0 – initial gas pressure, [MPa]
Q_s – gas source by injection, $[\mathrm{kgs^{-1}m^{-3}}]$
T – initial coal temperature, [K]
T_{ar} – the absolute reference temperature, [K]
t – time, [s]
V_{sg} – content of absorbed gas, [–]

Greek symbols

α_g – thermal expansion coefficient of the gas, [K^{-1}]
α_{sg} – coefficient for sorption-induced volumetric strain, [kgm^{-3}]
α_T – volumetric thermal expansion of the solid matrix coefficient, [K^{-1}]
ε_s – sorption-induced volumetric strain, [–]
λ_s – thermal conductivity of coal, [$\mathrm{Jm}^{-1}\mathrm{s}^{-1}\mathrm{K}^{-1}$]
$\bar{\mu}$ – gas dynamic viscosity, [Nsm^{-2}]
ρ_c – density of coal, [kgm^{-3}]
ρ_g – density of CH_4 at standard condition, [kgm^{-3}]
ρ_{ga} – gas density at standard conditions, [kgm^{-3}]
τ – adsorption time, [s]
ϕ – coal porosity, [–]
ϕ_0 – initial porosity, [–]

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