TRANSIENT PRESSURE AND PRODUCTIVITY ANALYSIS IN CARBONATE GEOTHERMAL RESERVOIRS WITH CHANGING EXTERNAL BOUNDARY FLUX

by

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In this paper, a triple-medium flow model for carbonate geothermal reservoirs with an exponential external boundary flux is established. The pressure solution under constant production conditions in Laplace space is solved. The geothermal wellbore pressure change considering wellbore storage and skin factor is obtained by Stehfest numerical inversion. The well test interpretation charts and Fetkovich production decline chart for carbonate geothermal reservoirs are proposed for the first time. The proposed Fetkovich production decline curves are applied to analyze the production decline behavior. The results indicate that in carbonate geothermal reservoirs with exponential external boundary flux, the pressure derivative curve contains a triple dip, which represents the interporosity flow between the vugs or matrix and fracture system and the invading flow of the external boundary flux. The interporosity flow of carbonate geothermal reservoirs and changing external boundary flux can both slow down the extent of production decline and the same variation tendency is observed in the Fetkovich production decline curve.

Key words: *carbonate geothermal reservoir, changing external boundary flux, triple-medium, well test interpretation chart, Fetkovich production decline*

Introduction

As a renewable and clean energy, geothermal energy has great potential for development and utilization [1, 2]. Carbonate heat storage systems for geothermal energy are the most important heat storage systems besides volcanic heat storage systems. Because of the unique karstification of carbonate rocks, there are many fractures in carbonate areas and many voids in the ground, which represent ideal reservoirs for storing hot water [3-5]. Well test analysis is an effective means for the dynamic identification in geothermal reservoirs. However, because of high heterogeneity and multiscale flow in carbonate geothermal reservoirs [6-8], conventional sandstone-based well test methods can not be used directly.

Wu *et al.* [9] proposed an analytical approach for pressure transient test analysis in naturally fractured vuggy reservoirs based on the triple-continuum concept. Jia *et al.* [10] study results show that the curves of well test type are dominated by interporosity flow factor, external-boundary conditions and fluid-storage capacitance coefficient. Xiong *et al.* [11] proposed a laboratory experiments method of well testing for fracture-cave carbonate gas reservoirs and the effects of fracture and large-scaled cave properties on the behavior of well testing curve

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were evaluated. Chen *et al.* [12] treated the large-scale cave as equipotential body and deemed matrix and fracture as double porous media. Gao *et al.* [13] established a radial composite reservoir model including the inner and outer regions with different characteristics. A well test model with changing external boundary flux was established by del Angel *et al.* [14], although the object of that study was homogeneous reservoirs.

Because of the high-storability vugs and high-permeability fractures in carbonate geothermal reservoirs, fluid inside the vugs or at the external boundary is more likely to invade the reservoir. Therefore, in this study, a mathematical model for carbonate geothermal reservoirs that considers the changing flux at the external boundary was established and studied. Fetkovich curves were also used to analyze production decline characteristics, which are significant for obtaining the dynamic parameters of geothermal reservoirs based on pressure and production data.

Well test model and solution

Model assumption

A physical model of a well penetrating a single-layer geothermal reservoir with constant production was developed according to the following basic assumptions: a triple-continuum geothermal reservoir for anisotropic formation is considered, with uniform thickness layers that are horizontal with closed upper and lower boundaries; both rock and fluid in the geothermal reservoir are slightly compressible and the compressibility is constant; the initial formation pressure is distributed homogeneously and equal to P_i; the invading fluid at the external boundary flows into the geothermal reservoir through the fracture system and only fluid in fractures flows into the wellbore; the interporosity flow is at pseudo-steady state [9]; and the fluid flow is considered to be 2-D.

Mathematical model and solution

The following equations describe the flow of a single-phase fluid with slight compressibility in a carbonate geothermal reservoir:

$$\frac{3.6k_f}{\mu}\nabla^2 P_f - \phi_m C_m \frac{\partial P_m}{\partial t} - \phi_v C_v \frac{\partial P_v}{\partial t} = \phi_f C_f \frac{\partial P_f}{\partial t} \tag{1}$$

$$-\frac{\alpha_m k_m}{\mu}(P_m - P_f) = \phi_m C_m \frac{\partial P_m}{\partial t} \tag{2}$$

$$-\frac{\alpha_v k_v}{\mu}(P_v - P_f) = \phi_v C_v \frac{\partial P_v}{\partial t} \tag{3}$$

with a series of dimensionless parameters given:

$$r_D = \frac{r}{r_w}, \quad t_D = \frac{3.6k_f}{\mu r_w^2(\phi_m C_m + \phi_f C_f + \phi_v C_v)}t, \quad k_f = \sqrt{k_{fx}k_{fy}}$$

$$\omega_j = \frac{\phi_j C_j}{\phi_m C_m + \phi_f C_f + \phi_v C_v}, \; j = m, f, v, \quad \lambda_j = \alpha_j r_w{}^2 \frac{k_j}{k_f}, \; j = m, v$$

$$P_{Dj}(r_D, t_D) = \frac{k_f h}{1.842 \cdot 10^{-3} q\mu B}[P_i - P_j(r,t)] \quad (j = m, f, v)$$

where r is radius, C – the compressibility, t – the time, ω – the storability ratio, ϕ – the porosity, λ – the interporosity coefficient, k – the permeability, α – the shape factor, μ – the fluid viscosity, q – the production rate (constant), r_w – the wellbore radius, P_i – the initial pressure, and h – the geothermal reservoir thickness.

We assume that $k_{fx} = k_{fy}$ and the equations can be transformed into dimensionless form:

$$\frac{1}{r_D}\frac{\partial}{\partial r_D}\left(r_D\frac{\partial P_{Df}}{\partial r_D}\right)-\omega_m\frac{\partial P_{Dm}}{\partial t_D}-\omega_v\frac{\partial P_{Dv}}{\partial t_D}=\omega_f\frac{\partial P_{Df}}{\partial t_D} \tag{4}$$

$$-\lambda_m(P_{Dm}-P_{Df})=\omega_m\frac{\partial P_{Dm}}{\partial t_D} \tag{5}$$

$$-\lambda_v(P_{Dv}-P_{Df})=\omega_v\frac{\partial P_{Dv}}{\partial t_D} \tag{6}$$

The initial pressure in the geothermal reservoir is assumed to be P_i with a uniform distribution:

$$P_{Dm}(r_D,0)=P_{Df}(r_D,0)=P_{Dv}(r_D,0)=0 \tag{7}$$

For the inner boundary under constant production conditions:

$$\left.\frac{\partial P_{Df}}{\partial r_D}\right|_{r_D=1}=-1 \tag{8}$$

In eq. (9), the invasion flux gradually increases to the final fixed value. The fixed value increases with increasing, q_{R_D}. Furthermore, the time of arrival to the fixed value of invasion flux is determined by τ_D and a higher value of τ_D indicates a later arrival at the final fixed value.

$$\left(r_D\frac{\partial P_{Df}}{\partial r_D}\right)_{r_D=R_D}=-q_{R_D}\left(1-\mathrm{e}^{-\frac{t_D}{\tau_D}}\right) \tag{9}$$

where R_D dimensionless geothermal reservoir radius and t_D dimensionless time.

Laplace transformation was applied to the previous model. According to the definition:

$$\overline{P}_{Di}=\int_0^\infty \mathrm{e}^{-s\tau}P_{Di}\left(r_D,\tau\right)\mathrm{d}\tau,\quad i=m,f,v$$

the mathematical model in Laplace space is:

$$\frac{\mathrm{d}^2\overline{P}_{Df}}{\mathrm{d}r_D^2}+\frac{1}{r_D}\frac{\mathrm{d}\overline{P}_{Df}}{\mathrm{d}r_D}-sf(s)\overline{P}_{Df}=0 \tag{10}$$

For the inner boundary condition:

$$\left.\frac{\mathrm{d}\overline{P}_{Df}}{\mathrm{d}r_D}\right|_{r_D=1}=-\frac{1}{s} \tag{11}$$

For the external boundary condition with changing flux:

$$\left(r_D\frac{\mathrm{d}\overline{P}_{Df}}{\mathrm{d}r_D}\right)_{r_D=R_D}=\overline{q}_{Dext}=-\frac{q_{RD}}{s}+\frac{\tau_D q_{RD}}{s\tau_D+1} \tag{12}$$

where

$$f(s)=\frac{\omega_m\lambda_m}{\omega_m s+\lambda_m}+\frac{\omega_v\lambda_v}{\omega_v s+\lambda_v}+\omega_f$$

For the general solution of eq. (10):

$$\overline{P}_{Df}(r_D,s)=AI_0\left(\sqrt{sf(s)}r_D\right)+BK_0\left(\sqrt{sf(s)}r_D\right) \tag{13}$$

where $I_n(x)$ is the first category Bessel function of imaginary argument, n = 0, 1, 2 … and $K_n(x)$ is the second category Bessel function of imaginary argument, n = 0, 1, 2 ….

Combining eqs. (11) and (12), the solution in Laplace space is:

$$\overline{P}_{wD}=\frac{K_0\left(r_D\sqrt{u}\right)I_1\left(R_D\sqrt{u}\right)+I_0\left(r_D\sqrt{u}\right)K_1\left(R_D\sqrt{u}\right)}{s\sqrt{u}\left[K_1\left(\sqrt{u}\right)I_1\left(R_D\sqrt{u}\right)-I_1\left(\sqrt{u}\right)K_1\left(R_D\sqrt{u}\right)\right]}+$$
$$+\frac{q_{Dext}(s)}{R_D}\frac{K_0\left(r_D\sqrt{u}\right)I_1\left(\sqrt{u}\right)+I_0\left(r_D\sqrt{u}\right)K_1\left(\sqrt{u}\right)}{\sqrt{u}\left[K_1\left(\sqrt{u}\right)I_1\left(R_D\sqrt{u}\right)-I_1\left(\sqrt{u}\right)K_1\left(R_D\sqrt{u}\right)\right]} \tag{14}$$

where $u = sf(s)$.

Solution of Wellbore pressure considering Wellbore storage and skin factor

During the process of drilling and completion, the permeability of the geothermal reservoir around the well could be affected. Furthermore, fluid in the well hole could expand because of pressure drop in geothermal development. In order to consider skin factor and wellbore storage, the dimensionless skin factor and wellbore storage are introduced into Laplace space through Duhamel's law, eq. (15) shows the bottom-hole pressure solution in Laplace space:

$$\overline{P}_D(z)=\frac{z\overline{P}_{wD}(z)+S}{z\left\{1+C_Dz\left[z\overline{P}_{wD}(z)+S\right]\right\}} \tag{15}$$

Subsequently, the Stehfest numerical inversion method is applied to obtain the pressure solution of geothermal well P_D considering the wellbore storage and skin factor in real space.

Pressure response and sensitivity analysis

Pressure characteristic curve

A single well at the center of a cylindrical carbonate geothermal reservoir produces at a constant rate and the flux at the external boundary obeys the exponential function. Concrete parameter values can be found: R_D = 20000, ω_f = 0.003, ω_v = 0.03, λ_{vf} = $1\cdot10^{-4}$, λ_{mf} = $5\cdot10^{-7}$, q_{R_D} = 0.8, τ_D = $1\cdot10^5$, C_D = 0.01, and S =1.

As shown in fig. 1, in the stage of pure wellbore storage, the pressure (P_D) and pressure derivative (dP_D) curves coincide to a straight line with a slope of 1. Then, a *hump* can be seen in the pressure derivative curve, the pattern of which is determined by the skin factor. After that, the pressure derivative curve exhibits three dips. With parameters shown in fig. 1, the first dip represents the interporosity flow between vugs and fracture systems, the second dip reflects the interporosity flow between matrix and fracture systems, and the third dip describes the characteristics of the external boundary fluid flowing into the geothermal reservoir, which can be seen as the interporosity flow between external flux and geothermal reservoir. For carbonate geothermal reservoirs, this part of the fluid is likely to be the fluid in the large-scale vugs or the discontinuous water of the external boundary. In the late stage, when the pressure wave reaches the boundary, the pressure and the pressure derivative curve again coincide to a straight line with a slope of 1.

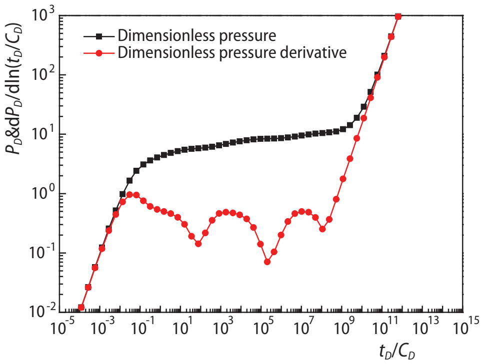

Figure 1. Transient pressure in a carbonate geothermal reservoir with changing external boundary flux
(*for color image see journal web site*)

The influence of q_{R_D} on pressure response

As shown in fig. 2(a), when $q_{R_D} < 1$, the influx is less than well production, which represents a feature of the closed boundary. When $q_{R_D} = 1$, the influx at the external boundary is equal to the well production, which is equivalent to the constant pressure external boundary condition. When $q_{R_D} > 1$, the velocity of the boundary fluid invasion is greater than that of production, and the pressure of the whole reservoir system rises, while the dimensionless bottom hole pressure gradually decreases. This paper focuses on the case of $q_{R_D} < 1$. As is shown in fig. 2(b), the external boundary fluid invasion has no effect on the early and middle stages of well testing, while on the late stage with increasing q_{R_D}, the time to appearance of the boundary response will be later and the dip in the pressure derivative curve will be deeper. Finally, $q_{R_D} = 0$ represents a closed external boundary.

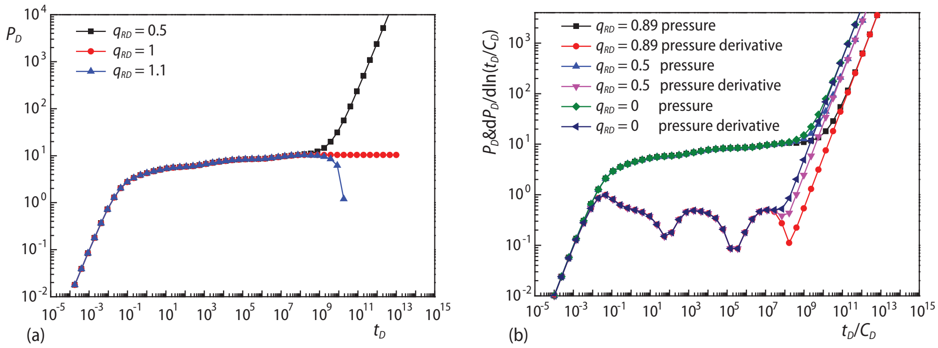

Figure 2. The influence of q_{R_D} on the pressure and pressure derivative curves
(*for color image see journal web site*)

The influence of τ_D on pressure response

The duration of the late transition period is affected by the dimensionless parameter τ_D. With increasing τ_D, the external boundary fluid invasion velocity becomes slower, the dip representing the late transition period in the well test curve appears later, and the cavity amplitude becomes smaller. As shown in fig. 3, when $\tau_D = 1\cdot10^9$, the pressure wave first reaches the boundary of the reservoir, and then reflects the nature of the fluid invasion, such that the pres-

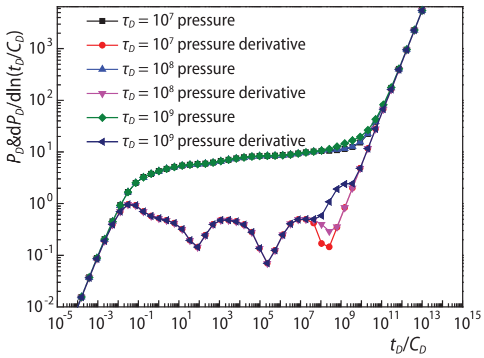

Figure 3. The influence of τ_D on the pressure and pressure derivative curves
(*for color image see journal web site*)

sure derivative curve no longer appears as a dip, but instead as a *sidestep* that represents the influx at the external boundary.

Fetkovich production decline curve analysis

The geothermal development process involves transporting energy to the ground by geothermal reservoir fluid, and when the reservoir fluid production drops beyond a certain level, it will be unreliable to ensure economic development. Therefore, it is very important to analyze the productivity of carbonate geothermal reservoirs during their development.

The widely used Arps decline curve can only be applied to analyze the boundary dominated flow stage, whereas the Fetkovich method also accounts for the unsteady flow stage. In this paper, the Fetkovich dimensionless production decline curve is used to study the production decline of carbonate geothermal reservoirs with variable external boundary flux. The dimensionless variable is redefined:

$$q_{Dd} = \alpha q_D \tag{16}$$

$$t_{Dd} = \beta t_D \tag{17}$$

where $\alpha = \ln[R_D - (1/2)]$ and

$$\beta = \frac{2}{\left(R_D^2 - 1\right)\left(\ln R_D - \frac{1}{2}\right)}$$

Due to the different definitions of the dimensionless time t_{Dd}, τ_{Dd} needs to be redefined:

$$\tau_{Dd} = \beta \tau_D \tag{18}$$

In order to further investigate the characteristics of the Fetkovich decline curve in carbonate geothermal reservoirs with changing external boundary flux, the Fetkovich decline curves of four typical geothermal reservoir types were analyzed: (a) homogeneous geothermal reservoirs with a closed boundary, (b) homogeneous geothermal reservoirs with changing external boundary flux, (c) carbonate geothermal reservoirs with a closed boundary, and (d) carbonate geothermal reservoirs with changing external boundary flux.

As shown in fig. 4, for geothermal reservoir type (a), the production declines very slowly in the early transient flow stage and then exhibits exponential decline in the subsequent boundary dominated flow. For geothermal reservoir type (b), the Fetkovich decline curve behaves the same as that for geothermal reservoir type (a) in the early transient flow stage. In the subsequent boundary dominated flow, the geothermal wellbore production first declines exponentially and then slows down because of the invasion of external boundary flux. In this paper, we focus on the condition of $q_{R_D} < 1$, which indicates that the boundary still behaves as closed after large flux invasion. This explains why the production also declines exponentially in the final stage. For geothermal reservoir type (c), only the fracture system contributes to the production in the early transient flow stage, which results in relatively lower production compared with the corresponding production for homogeneous geothermal reservoirs. After the interporosity flow period between the vugs and fractures, there occurs the first *sidestep* in the Fetkovich chart. The geothermal wellbore production decline slows down and the second *sidestep* appears after the vugs and fractures have both supplied fluid for a while, with the interporosity flow between the matrix and fractures. For geothermal reservoir type (d), there then appears a third *sidestep*, which represents the invasion of external boundary fluid, namely, the interporosity flow between the external boundary fluid and the geothermal reservoir. From the previous analysis, it is apparent that the carbonate geothermal reservoir and variable external boundary flux can both slow down the tendency of production decline.

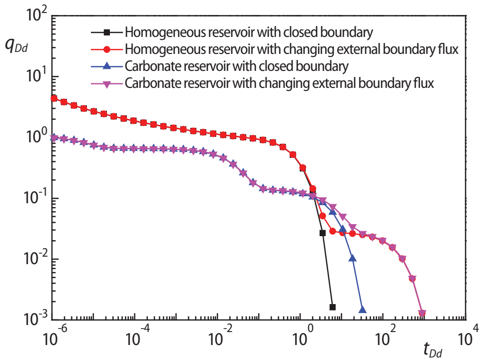

Figure 4. Fetkovich production decline chart under different geothermal reservoir conditions (*for color image see journal web site*)

Conclusions

A mathematical model for carbonate geothermal reservoirs with exponential external boundary flux has been developed in this paper, and a pressure solution with constant production of the geothermal well has been obtained. Furthermore, well test interpretation charts and Fetkovich production decline chart has been constructed. The following conclusions can be drawn.

- Changing external boundary flux results in the appearance of triple dips in the pressure derivative curves for carbonate geothermal reservoirs.
- Different values of q_{R_D} reflect different boundary properties when the external boundary flux obeys an exponential law. While τ_{Dd} determines the time when the external boundary fluid invades into the geothermal reservoir.
- The interporosity flow in carbonate geothermal reservoirs and the changing external boundary flux show consistent patterns in the Fetkovich chart. The production declines rapidly in the initial period and subsequently enters a stable production period reflecting the characteristics of interporosity flow. After reaching the closed external boundary, the production shows exponential decline.

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Nomenclature

C – compressibility, [MPa^{-1}]
h – geothermal reservoir thickness, [m]
k – permeability, [μm^2]
P_i – initial pressure, [MPa]
q – production rate (constant), [m^{3}d^{-1}]
R_D – dimensionless geothermal reservoir radius, [m]
r – radius, [m]
r_w – Wellbore radius, [m]
t – time, [h]
t_D – dimensionless time, [s]

Greek symbols

α – shape factor, [–]
λ – interporosity coefficient, [–]
μ – fluid viscosity, [mPa·s]
ϕ – porosity, [–]
ω – storability ratio, [–]

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