VALIDATION OF ACCURACY AND STABILITY OF NUMERICAL SIMULATION FOR 2-D HEAT TRANSFER SYSTEM BY AN ENTROPY PRODUCTION APPROACH

by

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The entropy production in 2-D heat transfer system has been analyzed systematically by using the finite volume method, to develop new criteria for the numerical simulation in case of multidimensional systems, with the aid of the CFD codes. The steady-state heat conduction problem has been investigated for entropy production, and the entropy production profile has been calculated based upon the current approach. From results for 2-D heat conduction, it can be found that the stability of entropy production profile exhibits a better agreement with the exact solution accordingly, and the current approach is effective for measuring the accuracy and stability of numerical simulations for heat transfer problems.

Key words: *heat transfer, entropy production, stability, accuracy, convergence*

Introduction

It seems to be revolutionary changes in the field of heat transfer and flow of fluids in this world of advanced technology. The concept of entropy production through heat transfer plays pivotal role in this change. Due to occurrence of this in many industrial applications such as heat transfer systems, high temperature working conditions and so on, the entropy production mechanism has essential importance in the field of engineering and science. The recent progress in the field of CFD, together with numerical heat transfer has becoming important in-order to analyze the systems involving heat transfer and fluid flow. Many researchers used numerical approaches based on computer softwares and suitable algorithms to understand the mechanism of heat transfer approaches because of their simplicity instead of experimental [1]. The travelling-wave solution for the non-linear transient heat conduction was proposed in [2, 3]. A finite difference scheme was developed based on parabolic equations for solving the micro-heat transport equations in a 3-D double-layer thin film and in a double-layer microsphere, respectively [4]. The numerical solution for the linear heat transfer problem was considered in [2]. The iterative differential quadrature method was proposed for the time-dependent 1-D non-linear heat conduction problem [5]. An informative entropy approach was discussed to

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evaluate the error caused by spatial discretization by numerical uncertainty for the radiative transfer [6, 7]

Entropy production associated with the heat transfer and fluid flow was briefly discussed based on methodology of the entropy generation minimization along with the fundamental equations for entropy production [8-10]. The CFD analysis was carried for the entropy production rates based on the turbulent convective heat transfer problems [11] with the use of the direct and indirect methods. In direct method the local part of the entropy production were calculated with the help of the basic thermodynamics equations by finite volume method (FVM), they involved some approximations for turbulence modeling whereas in indirect method they used time averaged entropy balance equation along with the approximations. Moreover, the accuracy and stability of the numerical simulation for the problems of the fluid flow and heat transfer was investigated via the entropy production method. Their approach effective for measuring the accuracy and stability of numerical simulation for the heat transfer and fluid flow related problems was provided in [12]. The numerical investigations were carried out for the heat transfer and fluid by being estimated the total entropy generation rate over turbulent dissipation, viscous direct dissipation, heat transfer and inner phase change during different stages [13, 14].

The main aim of this paper is to study the 2-D steady heat conduction problem instead of 1-D system to investigate the accuracy and stability of a certain numerical simulation technique with the use of FVM.

Numerical methodology

Governing equation for entropy production

The total entropy production for 3-D flow field is given by [9]:

$$\dot{S}_P''' = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] + \frac{\mu}{T}\left[2\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2\right\} + \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right)^2\right] \tag{1}$$

where $\dot{S}_P'''$ represents the total entropy production, T is the temperature, k – the thermal conductivity, v_x – the velocity vector along x-axis, and v_y – the velocity vector along y-axis.

The previous equation is composed of two parts: the first part shows the entropy production caused by the finite temperature gradient $S_{\text{temperature}}$, while the second part of the equation shows the entropy production caused by the finite velocity gradient S_{velocity}.

So, eq. (1) can be written:

$$\dot{S}_P''' = S_{\text{temperature}} + S_{\text{velocity}} \tag{2a}$$

where

$$S_{\text{temperature}} = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] \tag{2b}$$

and

$$S_{\text{temperature}} = \frac{\mu}{T}\left[2\left\{\left(\frac{\partial v_x}{\partial x}\right)^2 + \left(\frac{\partial v_y}{\partial y}\right)^2\right\} + \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y}\right)^2\right] \tag{2c}$$

We now consider the 2-D heat transfer and assume that velocity is independent of the location. Since the term S_{velocity} becomes zero, eq. (1) becomes:

$$\dot{S}_P''' = \frac{k}{T^2}\left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2\right] \tag{3}$$

Equation (3) is called the governing equation of the entropy production.

Governing equation for the heat transfer

The 3-D unsteady convective heat transfer in the Cartesian co-ordinate system with the internal heat source is given by [15]:

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) + S_\varnothing \tag{4}$$

where ρ is the fluid density, c_p – the specific heat at constant pressure, k – the thermal conductivity, and $S_\varnothing$ – the heat generation source.

Hence for the 2-D steady heat-conduction without heat generation, eq. (4) can be simplified:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{5}$$

Numerical scheme

In-order to derive the discretized equation for 2-D system, we consider the 2-D steady-state diffusion equation given by [1],

$$\frac{\partial}{\partial x}\left(\tau\frac{\partial \varphi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\tau\frac{\partial \varphi}{\partial y}\right) + S_\varnothing = 0 \tag{6}$$

where φ is the value of property per unit mass, τ – the diffusion coefficient, and $S_\varnothing$ – the source term.

For simplicity let us consider a portion of the 2-D grid used for the discretization is shown in fig. 1.

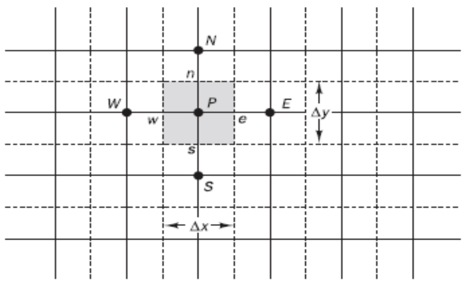

Figure 1. Part of 2-D grid

We need to apply the following steps for getting discretized equation for 2-D system by the FVM.

In the first step is to divide the domain into discrete control volumes and substitute a number of nodal points in the space. The boundaries (or faces) of control volumes are positioned mid-way between adjacent nodes. Thus, each node is surrounded by a control volume or cell. Let us take a general grid node, *P*, surrounded by four adjacent neighbor grids namely east (*E*), west (*W*), north (*N*), and south (*S*) neighbors, respectively. It is common practice to set up control volumes near the edge of the domain in such a way that the physical boundaries coincide with the control volume boundaries.

The east side face of the control volume is denoted by e, the west side face of the control volume is denoted by w, the north side face of the control volume is denoted by n, and the south side control volume face is referred by s. The distances between the nodes W and P, P and E, P and N, and between nodes P and S, are identified by δx_{PW}, δx_{PE}, and δx_{SP}, respectively. From fig. 1, it is clear that $\Delta x = \delta x_{we} = \delta x_{PW} = \delta x_{PE}$ and $\Delta y = \delta x_{ns} = \delta x_{PN} = \delta x_{SP}$.

When eq. (6) is formally integrated over the control volume, we obtain:

$$\int \frac{\partial}{\partial x}\left(\tau \frac{\partial \varphi}{\partial x}\right)\mathrm{d}x\,\mathrm{d}y + \int_{\Delta V}^{V} \frac{\partial}{\partial x}\left(\tau \frac{\partial \varphi}{\partial x}\right)\mathrm{d}x\,\mathrm{d}y + \int_{\Delta V}^{V} \mathrm{S}_{\varnothing}\mathrm{d}V = 0 \tag{7}$$

When $A_e = A_w = \Delta x$ and $A_n = A_s = \Delta y$, we get:

$$\left[\tau_e A_e \left(\frac{\partial \varphi}{\partial x}\right)_e - \tau_w A_w \left(\frac{\partial \varphi}{\partial x}\right)_w\right] + \left[\tau_n A_n \left(\frac{\partial \varphi}{\partial x}\right)_n - \tau_s A_s \left(\frac{\partial \varphi}{\partial x}\right)_s\right] + \bar{S}\,\Delta V = 0 \tag{8}$$

We can write expressions for the flux through control volume faces.

The flux across the west face is:

$$\tau_w A_w \left(\frac{\partial \varphi}{\partial x}\right)\bigg|_w = \tau_w A_w \frac{\varphi_P - \varphi_W}{\delta x_{PW}} \tag{9}$$

The flux across the east face is:

$$\tau_e A_e \left(\frac{\partial \varphi}{\partial x}\right)\bigg|_e = \tau_e A_e \frac{\varphi_E - \varphi_P}{\delta x_{PE}} \tag{10}$$

The flux across the south face is:

$$\tau_s A_s \left(\frac{\partial \varphi}{\partial x}\right)\bigg|_s = \tau_s A_s \frac{\varphi_P - \varphi_S}{\delta y_{SP}} \tag{11}$$

The flux across the north face is:

$$\tau_n A_n \left(\frac{\partial \varphi}{\partial x}\right)\bigg|_n = \tau_n A_n \frac{\varphi_N - \varphi_P}{\delta y_{PN}} \tag{12}$$

By putting eqs. (9)-(12) in eq. (8), we get:

$$\tau_e A_e \frac{\varphi_E - \varphi_P}{\delta x_{PE}} - \tau_w A_w \frac{\varphi_P - \varphi_W}{\delta x_{PW}} + \tau_n A_n \frac{\varphi_N - \varphi_P}{\delta y_{PN}} - \tau_s A_s \frac{\varphi_P - \varphi_S}{\delta y_{SP}} + \bar{S}\,\Delta V = 0 \tag{13}$$

In the practical situations, as illustrated later, the source term $\overline{S}$ may be a function of the dependent variable. In such cases the FVM approximates the source term by means of a linear form:

$$\bar{S}\,\Delta V = s_u + s_p \varphi_P \tag{14}$$

Substituting eq. (14) in eq. (13) and rearranging the terms, we will get:

$$\left(\frac{\tau_w A_w}{\delta x_{PW}} + \frac{\tau_e A_e}{\delta x_{PE}} + \frac{\tau_s A_s}{\delta y_{SP}} + \frac{\tau_n A_n}{\delta y_{PN}} - s_p\right)\varphi_P = \left(\frac{\tau_w A_w}{\delta x_{PW}}\right)\varphi_W + \left(\frac{\tau_e A_e}{\delta x_{PE}}\right)\varphi_E +$$

$$+\left(\frac{\tau_s A_s}{\delta y_{SP}}\right)\varphi_S+\left(\frac{\tau_n A_n}{\delta y_{PN}}\right)\varphi_N+S_u \tag{15}$$

The generalized form of previous equation can be written:

$$a_P\varphi_P = a_W\varphi_W + a_E\varphi_E + a_S\varphi_S + a_N\varphi_N + S_u \tag{16}$$

The relationships among the discretized coefficients are listed in tab. 1.

Table 1. Discretized coefficients at corresponding nodes

a_W	a_E	a_S	a_N	a_P	S_u
$\frac{\tau_w A_w}{\delta x_{PW}}$	$\frac{\tau_e A_e}{\delta x_{PE}}$	$\frac{\tau_s A_s}{\delta y_{SP}}$	$\frac{\tau_n A_n}{\delta y_{PN}}$	$a_W + a_E + a_S + a_N$	$a_W + a_E + a_S + a_N - S_p$

Geometrical model

We consider a thin square metal plate with the dimensions 1 cm by 1 cm (length by width) for the 2-D steady-state heat conduction problem and the thermal conductivity of metal plate is $k = 237$. Two adjacent boundaries (edges) are maintained at the temperature 300 and the heat on the other boundaries increases linearly from the temperatures 300 to 500 where the sides meet as shown in fig. 2.

In order to find the temperature distribution and entropy production, we use the FVM to divide a metal plate into the many sub-volume nodal networks. After that, we apply the basic governing equations solved by the FVM.

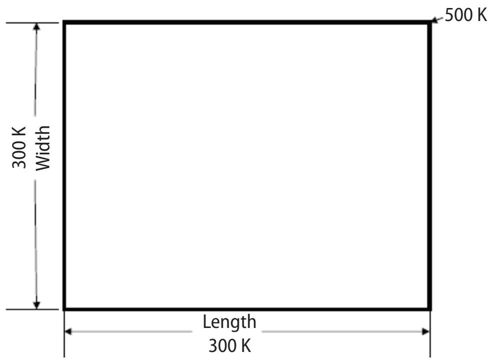

Figure 2. The profile of thin metal plate

Results and discussion

The temperature distribution and entropy production of the 2-D heat-conduction problem for the grid ratio $r = 0.5$ have been calculated and displayed in graphs as shown in fig. 3,

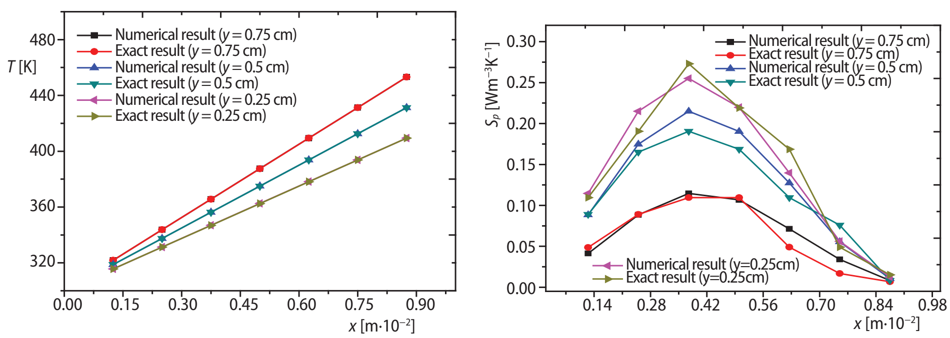

Figure 3. The temperature and entropy production profile of the 2-D heat-conduction problem for the non-uniform grid at $r = 0.5$
(*for color image see journal web site*)

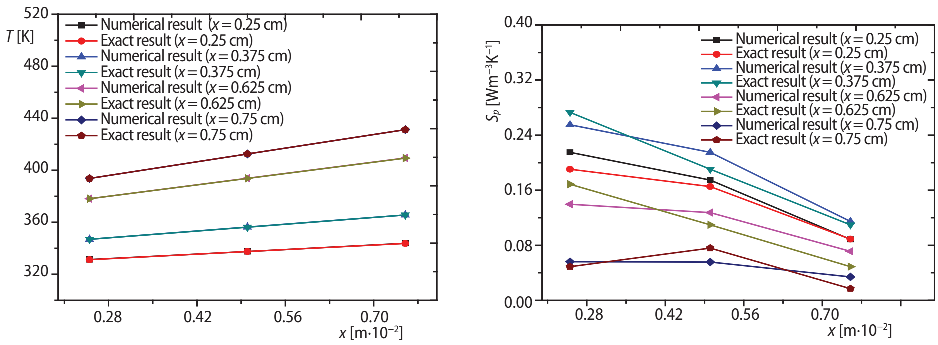

Figure 4. The temperature and entropy production profile of the 2-D heat-conduction problem for non-uniform grid at *r* = 0.5
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and fig. 4. It is shown that entropy production is always presented in the heat conduction problems, and that the numerical results of the temperature and entropy profile converges to its exact result in a satisfactory manner along x-axis because of the increased number of the nodal points.

Figure 5 illustrate an unsatisfactory convergence towards exact result, while fig. 6 give a satisfactory convergence towards the exact result. It is clear that we get a better convergence of numerical result towards the exact result along y axis of the metal plate since the nodal points are more in number as compared to x-axis.

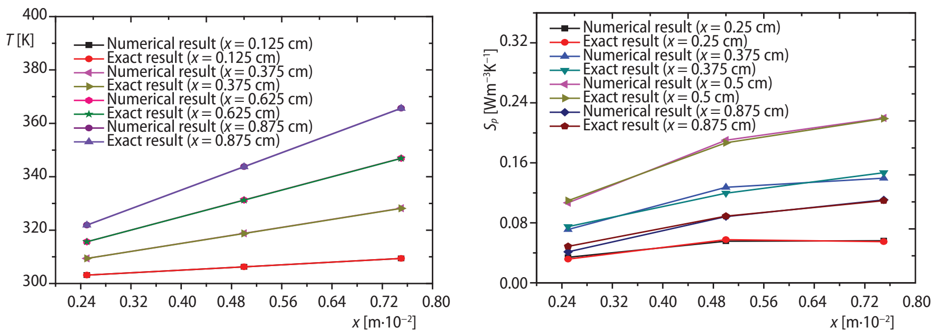

Figure 5. The temperature and entropy production profile of the 2-D heat-conduction problem for the non-uniform grid at *r* = 2
(*for color image see journal web site*)

As shown in fig. 7, it is demonstrated that the entropy production tends to zero as the temperature difference comes closer. The numerical result of entropy production profile converges towards exact result in a better way if the temperature difference goes in decreasing manner near the wall.

Conclusion

In this work, the accuracy and stability of the numerical simulation for the 2-D heat transfer system for the entropy production were investigated. The FVM is adopted to analyze the 2-D heat conduction problems. The entropy production profile for the 2-D heat conduction system was calculated and displayed. The results revealed that the convergence of numerical

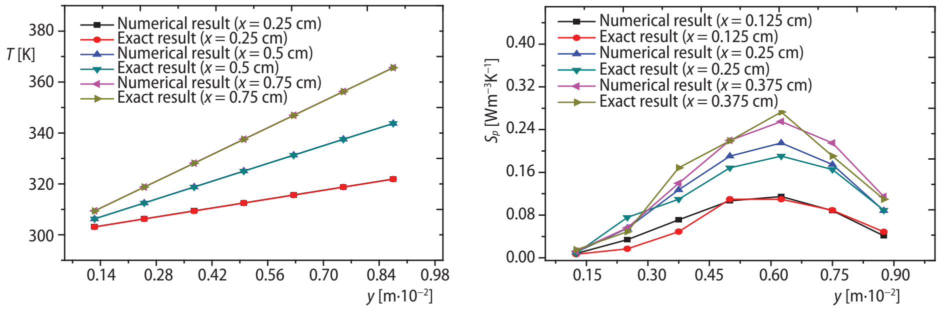

Figure 6. The temperature and entropy production profile of the 2-D heat-conduction problem for the non-uniform grid at *r* = 2
(*for color image see journal web site*)

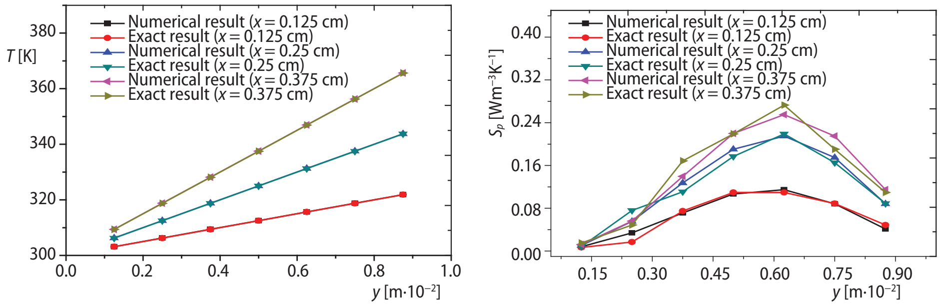

Figure 7. The temperature and entropy production profile of the 2-D heat-conduction problem at *r* = 1
(*for color image see journal web site*)

solution has a better agreement towards the exact solution at $r = 1$ instead of $r = 0.5$ and $r = 2$. Thus, it was found that the accuracy of the numerical result for the entropy production profile depends upon the grid size; it is to say, as the number of grid size increases, the better convergence of numerical solution will be experienced towards the exact solution. Moreover, the entropy production profile has not always a positive correlation with the temperature gradient. As the temperature difference between the nodes comes closer, the entropy production tends to zero.

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Nomenclature

c_p – specific heat constant, [Jkg^{-1}K^{-1}]
k – thermal conductivity, [Wm^{-1}K^{-1}]
$S_{\varnothing}$ – heat generation, [J]
$v_x v_y$ – velocity components [ms^{-1}]
$\Delta x, \Delta y$ – grid size in x- and y-directions, [–]

Greek symbols

μ – viscosity of the fluid, [Pa·s]
ρ – fluid density [kgm^{-3}]
φ – value of property per unit mass, [–]

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