

NEW INTEGRAL TRANSFORMS FOR SOLVING A STEADY HEAT TRANSFER PROBLEM

by

Xiao-Jun YANG^{a,b}

^a State Key Laboratory for Geomechanics and Deep Underground Engineering,
China University of Mining and Technology, Xuzhou, China

^b School of Mechanics and Civil Engineering, China University of Mining and Technology,
Xuzhou, China

Original scientific paper
<https://doi.org/10.2298/TSCI17S1079Y>

The new Fourier-like integral transforms

$$\begin{aligned}\Pi(\kappa) &= \kappa \int_{-\infty}^{\infty} \Pi(t) e^{-i\kappa t} dt, & \Pi(\lambda) &= \frac{1}{\lambda} \int_{-\infty}^{\infty} \Pi(t) e^{-i\lambda t} dt \\ \Pi(\gamma) &= \frac{1}{\gamma} \int_{-\infty}^{\infty} \Pi(t) e^{-\frac{t}{\gamma}} dt, & \Pi(\varsigma) &= \varsigma \int_{-\infty}^{\infty} \Pi(t) e^{-\frac{t}{\varsigma}} dt\end{aligned}$$

are addressed for the first time. They are used to handle a steady heat transfer equation. The proposed methods are efficient and accurate.

Key words: heat transfer, Fourier-like integral transform, analytical solution

Introduction

The Fourier and Laplace integral transforms are successfully used to solve the real world problems in the field of mathematical physics and engineering, such as signals [1], heat conduction [2], wave equation and others [3]. We can recall the definitions of them.

The Laplace integral transform of the function $\Pi(t)$ is given [1-3]:

$$\Pi(s) = \mathbb{L}[\Pi(t)] =: \int_0^{\infty} \Pi(t) e^{-st} dt \quad (1)$$

provided the integral operator exists for some t , where \mathbb{L} is the Laplace transform operator.

The inverse Laplace integral transform operator of eq. (1) is given [1-3]:

$$\Pi(t) = \mathbb{L}^{-1}[\Pi(s)] =: \frac{1}{2\pi i} \int_{\omega-i\infty}^{\omega+i\infty} \Pi(s) e^{st} ds \quad (2)$$

where s is a real-valued constant.

The bilateral Laplace transform of the function $\Pi(t)$ is defined [4]:

$$\Pi(s) = \mathbb{L}[\Pi(t)] =: \int_{-\infty}^{\infty} \Pi(t) e^{-st} dt \quad (3)$$

provided the integral exists for some t .

The Fourier integral transform of the function $\Pi(t)$ is given [1-3]:

$$\Pi(\omega) = \mathbb{F}[\Pi(t)] =: \int_{-\infty}^{\infty} \Pi(t) e^{-i\omega t} dt \quad (4)$$

provided the integral exists for some t , where \mathbb{F} is the Fourier integral transform operator.

The inverse Fourier integral transform operator of eq. (4) is written [1-3]:

$$\Pi(t) = \mathbb{F}^{-1}[\Pi(\omega)] =: \frac{1}{2\pi i} \int_{-\infty}^{\infty} \Pi(\omega) e^{i\omega t} d\omega \quad (5)$$

where ω is a real-valued constant.

The relationship between the Laplace and Fourier transforms are discussed in [1]. The Laplace-like integral transforms were proposed by Carson [5, 6], Fryba [7], Aboodh [8], Yang [9, 10], Weerakoon [11] Belgacem and Karaballi [12], and Elzaki [13]. More recently, the Fourier-like integral transform was firstly discussed in [14]. In the inspiring idea, the main aim of the present paper is to derive the new integral transforms to solve the heat transfer problem.

New Fourier-like integral transforms

The Laplace-Carson integral transform of the function $\Pi(t)$ is defined [5-7]:

$$\Pi(\zeta) = \mathbb{k}[\Pi(t)] =: \zeta \int_0^{\infty} \Pi(t) e^{-\zeta t} dt \quad (6)$$

provided the integral exists for some t , where \mathbb{k} is the Laplace-Carson integral transform operator.

The inverse Laplace-Carson integral transform operator of eq. (6) is written:

$$\Pi(t) = \mathbb{k}^{-1}[\Pi(\zeta)] =: \frac{1}{2\pi i} \int_{\omega-i\infty}^{\omega+i\infty} \frac{\Pi(\zeta)}{\zeta} e^{\zeta t} d\zeta \quad (7)$$

where ζ is a real-valued constant.

The bilateral Laplace-Carson integral transform of the function $\Pi(t)$ is defined:

$$\Pi(\zeta) = \mathbb{k}[\Pi(t)] =: \zeta \int_{-\infty}^{\infty} \Pi(t) e^{-\zeta t} dt \quad (8)$$

provided the integral exists for some t .

Taking $\zeta = i\kappa$ in eq. (8), we have:

$$\Pi(i\kappa) = i\kappa \int_{-\infty}^{\infty} \Pi(t) e^{-i\kappa t} dt \quad (9)$$

which leads to the new Fourier-like integral transform of the function $\Pi(t)$:

$$\Pi(\kappa) = \mathfrak{M}[\Pi(t)] =: \kappa \int_{-\infty}^{\infty} \Pi(t) e^{-i\kappa t} dt \quad (10)$$

provided the integral exists for some t , where \mathfrak{M} is the new Fourier-like integral transform operator.

The inverse Fourier-like integral transform operator is defined:

$$\Pi(t) = \mathfrak{M}^{-1}[\Pi(\kappa)] =: \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\Pi(\kappa)}{\kappa} e^{i\kappa t} d\kappa \quad (11)$$

provided the integral exists for some κ .

As a direct result of eqs. (10) and (11), we have the following integral formula:

$$\Pi(t) = \mathfrak{M}^{-1}[\Pi(\kappa)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{1}{\kappa} \left[\kappa \int_{-\infty}^{\infty} \Pi(t) e^{-i\kappa t} dt \right] e^{i\kappa t} d\kappa \quad (12)$$

The properties of the Fourier-like integral transform operator are as follows.

(T1) (Duality of Fourier type) Let $\Pi(\omega) = \mathbb{F}[\Pi(t)]$ and $\Pi(\kappa) = \mathfrak{M}[\Pi(t)]$. Then:

$$\Pi(\omega) = \frac{1}{\kappa} \Pi(\kappa) \quad \text{and} \quad \Pi(\kappa) = \omega \Pi(\omega) \quad (13)$$

(T2) Let $\Pi(t) = e^{-at} u(t)$, where $u(t)$ is the Heaviside unit step function [1-3]. Then $\Pi(\kappa) = \kappa / (a + i\kappa)$.

(T3) Let $\Pi(t) = \delta(t)$, where $\delta(t)$ represents the Dirac function. Then $\Pi(\kappa) = \kappa$.

(T4) Let $\Pi(\kappa) = \mathfrak{M}[\Pi(t)]$. Then $\mathfrak{M}[\Pi(t - a)] = e^{-ia\kappa} \Pi(\kappa)$, where a is a constant.

(T5) Let $\Pi(\kappa) = \mathfrak{M}[\Pi(t)]$. Then:

$$\mathfrak{M} \left[\frac{d\Pi(t)}{dt} \right] = i\kappa^2 \Pi(\kappa) \quad (14)$$

(T6) Let $\Pi(\kappa) = \mathfrak{M}[\Pi(t)]$ and $\Theta(\kappa) = \mathfrak{M}[\Theta(t)]$. Then:

$$\mathfrak{M}[\Pi(t) + \Theta(t)] = \Pi(\kappa) + \Theta(\kappa) \quad (15)$$

(T7) Let $\Pi(\kappa) = \mathfrak{M}[\Pi(t)]$ and $\Theta(\kappa) = \mathfrak{M}[\Theta(t)]$. Then:

$$\mathfrak{M} \left[\int_{-\infty}^{\infty} \Pi(t - \tau) \Theta(\tau) d\tau \right] = \frac{1}{\kappa} \Pi(\kappa) \Theta(\kappa) \quad (16)$$

Proof. (T1) With the use of the definition eq. (10), we easily have:

$$\Pi(\omega) = \frac{1}{\kappa} \Pi(\kappa) \quad \text{and} \quad \Pi(\kappa) = \omega \Pi(\omega)$$

(T2)

$$\mathfrak{M}[\Pi(t)] = \kappa \int_{-\infty}^{\infty} e^{-at} u(t) e^{-i\kappa t} dt = \kappa \int_0^{\infty} e^{-at} e^{-i\kappa t} dt = \frac{\kappa}{a + i\kappa}$$

(T3)

$$\mathfrak{M}[\Pi(t)] = \kappa \int_{-\infty}^{\infty} \delta(t) e^{-i\kappa t} dt = \kappa$$

(T4)

$$\mathfrak{M}[\Pi(t - a)] = \kappa \int_{-\infty}^{\infty} \Pi(t - a) e^{-i\kappa t} dt = \kappa \int_{-\infty}^{\infty} \Pi(t) e^{-i(\kappa+a)t} dt = e^{-ia\kappa} \Pi(\kappa)$$

(T5)

$$\mathfrak{M}\left[\frac{d\Pi(t)}{dt}\right] = \kappa \int_{-\infty}^{\infty} \frac{d\Pi(t)}{dt} e^{-i\kappa t} dt = i\kappa \Pi(\kappa)$$

(T6)

$$\mathfrak{M}[\Pi(t) + \Theta(t)] = \kappa \int_{-\infty}^{\infty} [\Pi(t) + \Theta(t)] e^{-i\kappa t} dt = \Pi(\kappa) + \Theta(\kappa)$$

(T7)

$$\mathfrak{M}\left[\int_{-\infty}^{\infty} \Pi(t-\tau)\Theta(\tau)d\tau\right] = \frac{1}{\kappa} \left(\kappa \int_{-\infty}^{\infty} \Theta(\vartheta) e^{-i\kappa\vartheta} d\vartheta\right) \left(\kappa \int_{-\infty}^{\infty} e^{-i\kappa(t-\vartheta)} \Pi(t-\vartheta) dt\right) = \frac{1}{\kappa} \Theta(\kappa) \Pi(\kappa)$$

The integral transform of the function $\Pi(t)$ is defined [8, 9]:

$$\mathfrak{M}\left[\int_{-\infty}^{\infty} \Pi(t-\tau)\Theta(\tau)d\tau\right] = \frac{1}{\kappa} \Pi(\kappa) \Theta(\kappa) \quad (17)$$

provided the integral exists for some t , where \mathbb{S} is the integral transform operator.

The inverse integral transform operator of eq. (17) is defined [9]:

$$\Pi(t) = \mathbb{S}^{-1}[\Pi(\varpi)] =: \frac{1}{2\pi i} \int_{\varpi-i\infty}^{\varpi+i\infty} \Pi(\varpi) \varpi e^{\varpi t} d\varpi \quad (18)$$

where ϖ is a real-valued constant.

The bilateral integral transform of the function $\Pi(t)$ is defined as:

$$\Pi(\varpi) = \mathbb{S}[\Pi(t)] =: \frac{1}{\varpi} \int_{-\infty}^{\infty} \Pi(t) e^{-\varpi t} dt \quad (19)$$

provided the integral exists for some t , where \mathbb{S} is the integral transform operator.

Taking $\varpi = i\lambda$ in eq. (19), we obtain:

$$\Pi(i\lambda) = \frac{1}{i\lambda} \int_{-\infty}^{\infty} \Pi(t) e^{-i\lambda t} dt \quad (20)$$

which yields the new Fourier-like integral transform of the function $\Pi(t)$:

$$\Pi(\lambda) = \mathfrak{A}[\Pi(t)] =: \frac{1}{\lambda} \int_{-\infty}^{\infty} \Pi(t) e^{-i\lambda t} dt \quad (21)$$

provided the integral exists for some t , where \mathfrak{A} is the new Fourier-like integral transform operator.

The inverse Fourier-like integral transform operator is defined:

$$\Pi(t) = \mathfrak{A}^{-1}[\Pi(\lambda)] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Pi(\lambda) \lambda e^{i\lambda t} d\lambda \quad (22)$$

provided the integral exists for some λ .

As a direct result of eqs. (21) and (22), we have the following integral formula:

$$\Pi(t) = \mathfrak{A}^{-1}[\Pi(\lambda)] =: \frac{1}{2\pi} \int_{-\infty}^{\infty} \lambda \left(\frac{1}{\lambda} \int_{-\infty}^{\infty} \Pi(t) e^{-i\lambda t} dt \right) e^{i\lambda t} d\lambda \quad (23)$$

The properties of the Fourier-like integral transform operator are as follows.

(U1) (Duality of Fourier type) Let $\Pi(\omega) = \mathbb{F}[\Pi(t)]$ and $\Pi(\kappa) = \mathfrak{A}[\Pi(t)]$. Then:

$$\Pi(\omega) = \lambda \Pi(\lambda) \text{ and } \Pi(\lambda) = \frac{1}{\omega} \Pi(\omega) \quad (24)$$

(U2) Let $\Pi(t) = e^{-at}u(t)$. Then $\Pi(\lambda) = 1/[\lambda(a + i\lambda)]$.

(U3) Let $\Pi(t) = \delta(t)$. Then $\Pi(\omega) = 1/\lambda$.

(U4) Let $\Pi(\lambda) = \mathfrak{A}[\Pi(t)]$. Then $\mathfrak{A}[\Pi(t - a)] = e^{-i\kappa a} \Pi(\kappa)$.

(U5) Let $\Pi(\lambda) = \mathfrak{A}[\Pi(t)]$. Then:

$$\mathfrak{A} \left[\frac{d\Pi(t)}{dt} \right] = i\lambda \Pi(\lambda) \quad (25)$$

(U6) Let $\Pi(\lambda) = \mathfrak{A}[\Pi(t)]$ and $\Theta(\lambda) = \mathfrak{A}[\Theta(t)]$. Then:

$$\mathfrak{A}[\Pi(t) + \Theta(t)] = \Pi(\lambda) + \Theta(\lambda) \quad (26)$$

(U7) Let $\Pi(\lambda) = \mathfrak{A}[\Pi(t)]$ and $\Theta(\lambda) = \mathfrak{A}[\Theta(t)]$. Then:

$$\mathfrak{A} \left[\int_{-\infty}^{\infty} \Pi(t - \tau) \Theta(\tau) d\tau \right] = \lambda \Pi(\lambda) \Theta(\lambda) \quad (27)$$

Proof. (U1) According to the definition eq. (20), we obtain:

$$\Pi(\omega) = \lambda \Pi(\lambda) \text{ and } \Pi(\lambda) = \frac{1}{\omega} \Pi(\omega)$$

(U2)

$$\mathfrak{A}[\Pi(t)] = \frac{1}{\lambda} \int_{-\infty}^{\infty} e^{-at} u(t) e^{-i\lambda t} dt = \frac{1}{\lambda} \int_0^{\infty} e^{-at} e^{-i\lambda t} dt = \frac{1}{\lambda(a + i\lambda)}$$

(U3)

$$\mathfrak{A}[\Pi(t)] = \frac{1}{\lambda} \int_{-\infty}^{\infty} \delta(t) e^{-i\lambda t} dt = \frac{1}{\lambda}$$

(U4)

$$\mathfrak{A}[\Pi(t - a)] = \frac{1}{\lambda} \int_{-\infty}^{\infty} \Pi(t - a) e^{-i\lambda t} dt = \frac{1}{\lambda} \int_{-\infty}^{\infty} \Pi(t) e^{-i(\lambda+a)t} dt = e^{-i\lambda a} \Pi(\lambda)$$

(U5)

$$\mathfrak{A} \left[\frac{d\Pi(t)}{dt} \right] = \frac{1}{\lambda} \int_{-\infty}^{\infty} \frac{d\Pi(t)}{dt} e^{-i\lambda t} dt = i\lambda \Pi(\lambda)$$

(U6)

$$\mathfrak{A}[\Pi(t) + \Theta(t)] = \frac{1}{\lambda} \int_{-\infty}^{\infty} [\Pi(t) + \Theta(t)] e^{-i\lambda t} dt = \Pi(\lambda) + \Theta(\lambda)$$

(U7)

$$\mathfrak{A} \left[\int_{-\infty}^{\infty} \Pi(t - \tau) \Theta(\tau) d\tau \right] = \lambda \left[\frac{1}{\lambda} \int_{-\infty}^{\infty} \Theta(\varrho) e^{-i\lambda \varrho} d\varrho \right] \left[\frac{1}{\lambda} \int_{-\infty}^{\infty} e^{-i\lambda(t-\varrho)} \Pi(t - \varrho) dt \right] = \lambda \Theta(\lambda) \Pi(\lambda)$$

The Sumudu integral transform of the function $\Pi(t)$ is given [12, 13]:

$$\Pi(\sigma) = \mathbb{Z}[\Pi(t)] = \frac{1}{\sigma} \int_0^{\infty} \Pi(t) e^{-\frac{t}{\sigma}} dt \quad (28)$$

provided the integral exists for some t , where \mathbb{Z} is the Sumudu integral transform operator.

The bilateral Sumudu integral transform of the function $\Pi(t)$ is given [12, 13]:

$$\Pi(\sigma) = \mathbb{Z}[\Pi(t)] = \frac{1}{\sigma} \int_{-\infty}^{\infty} \Pi(t) e^{-\frac{t}{\sigma}} dt \quad (29)$$

provided the integral exists for some σ , where \mathbb{Z} is the Sumudu integral transform operator.

Taking $\sigma = \gamma/i$ in eq. (27), we get:

$$\Pi\left(\frac{\gamma}{i}\right) = \frac{i}{\gamma} \int_{-\infty}^{\infty} \Pi(t) e^{-\frac{t}{\gamma}} dt \quad (30)$$

which results in the new Fourier-like integral transform of the function:

$$\Pi(\gamma) = \mathfrak{N}[\Pi(t)] = \frac{1}{\gamma} \int_{-\infty}^{\infty} \Pi(t) e^{-\frac{t}{\gamma}} dt \quad (31)$$

provided the integral exists for some t , where \mathfrak{N} is the new Fourier-like integral transform operator.

The properties of the Fourier-like integral transform operator are as follows.

(V1) Let $\Pi(t) = e^{-at} u(t)$. Then $\Pi(\gamma) = 1/(i + \gamma a)$.

(V2) Let $\Pi(t) = \delta(t)$. Then $\Pi(\gamma) = 1/\gamma$.

(V3) Let $\Pi(\gamma) = \mathfrak{N}[\Pi(t)]$. Then $\mathfrak{N}[\Pi(t - a)] = e^{-ia/\gamma} \Pi(\gamma)$.

(V4) Let $\Pi(\gamma) = \mathfrak{N}[\Pi(t)]$. Then:

$$\mathfrak{N} \left[\frac{d\Pi(t)}{dt} \right] = \frac{i}{\gamma} \Pi(\gamma) \quad (32)$$

(V5) Let $\Pi(\gamma) = \mathfrak{N}[\Pi(t)]$ and $\Theta(\gamma) = \mathfrak{N}[\Theta(t)]$. Then:

$$\mathfrak{N} \left[\int_{-\infty}^{\infty} \Pi(t - \tau) \Theta(\tau) d\tau \right] = \gamma \Pi(\gamma) \Theta(\gamma) \quad (33)$$

Proof.

$$(V1) \quad \Pi(\gamma) = \frac{1}{\gamma} \int_{-\infty}^{\infty} e^{-at} u(t) e^{-\frac{t}{\gamma}} dt = \frac{1}{\gamma} \int_0^{\infty} e^{-\left(\frac{i}{\gamma} + a\right)t} dt = \frac{1}{i + \gamma a}$$

(V2)

$$\Pi(\gamma) = \frac{1}{\gamma} \int_{-\infty}^{\infty} \delta(t) e^{-i\frac{t}{\gamma}} dt = \frac{1}{\gamma}$$

(V3)

$$\aleph[\Pi(t-a)] = \frac{1}{\gamma} \int_{-\infty}^{\infty} \Pi(t-a) e^{-i\frac{t}{\gamma}} dt = e^{-i\frac{a}{\gamma}} \Pi(\gamma)$$

(V4)

$$\aleph\left[\frac{d\Pi(t)}{dt}\right] = \frac{1}{\gamma} \int_{-\infty}^{\infty} \frac{d\Pi(t)}{dt} e^{-i\frac{t}{\gamma}} dt = \frac{i}{\gamma} \Pi(\gamma)$$

(V5)

$$\aleph\left[\int_{-\infty}^{\infty} \Pi(t-\tau)\Theta(\tau)d\tau\right] = \gamma \left[\frac{1}{\gamma} \int_{-\infty}^{\infty} e^{-i\frac{\tau}{\gamma}} \Theta(\tau)d\tau\right] \left[\frac{1}{\gamma} \int_{-\infty}^{\infty} \Pi(t-\tau) e^{-i\frac{t-\tau}{\gamma}} dt\right] = \\ = \gamma \Pi(\gamma)\Theta(\gamma)$$

The Elzaki integral transform of the function $\phi(\tau)$ is given [13]:

$$\Pi(\nu) = \aleph[\Pi(t)] = \nu \int_0^{\infty} \Pi(t) e^{-\frac{t}{\nu}} dt \quad (34)$$

provided the integral exists for some t , where \aleph is the Elzaki integral transform operator.

The bilateral Elzaki integral transform of the function $\phi(\tau)$ is given:

$$\Pi(\nu) = \aleph[\Pi(t)] =: \nu \int_{-\infty}^{\infty} \Pi(t) e^{-\frac{t}{\nu}} dt \quad (35)$$

provided the integral exists for some t .

Taking $\nu = \zeta/i$ in eq. (30), we have:

$$\Pi\left(\frac{\zeta}{i}\right) = \frac{\zeta}{i} \int_{-\infty}^{\infty} \Pi(t) e^{-i\frac{t}{\zeta}} dt \quad (36)$$

which results in the new Fourier-like integral transform of the function $\Pi(t)$:

$$\Pi(\zeta) = \aleph[\Pi(t)] =: \zeta \int_{-\infty}^{\infty} \Pi(t) e^{-i\frac{t}{\zeta}} dt \quad (37)$$

provided the integral exists for some ζ , where \aleph is the new Fourier-like integral transform operator.

The properties of the Fourier-like integral transform operator are presented as follows.

(S1) Let $\Pi(t) = e^{-at}u(t)$. Then $\Pi(\zeta) = \zeta^2/(i + \zeta a)$.

(S2) Let $\Pi(t) = \delta(t)$. Then $\Pi(\zeta) = \zeta$.

(S3) Let $\Pi(\gamma) = \aleph[\Pi(t)]$. Then $\aleph[\Pi(t-a)] = e^{-ia/\zeta} \Pi(\zeta)$.

(S4) Let $\Pi(\gamma) = \aleph[\Pi(t)]$. Then:

$$\aleph\left[\frac{d\Pi(t)}{dt}\right] = \frac{i}{\zeta} \Pi(\zeta) \quad (38)$$

(S5) Let $\Pi(\gamma) = \aleph[\Pi(t)]$ and $\Theta(\gamma) = \aleph[\Theta(t)]$. Then:

$$\aleph \left[\int_{-\infty}^{\infty} \Pi(t-\tau)\Theta(\tau) d\tau \right] = \frac{1}{\zeta} \Pi(\zeta)\Theta(\zeta) \quad (39)$$

Proof.

$$(S1) \quad \Pi(\zeta) = \zeta \int_{-\infty}^{\infty} e^{-at} e^{-\frac{t}{\zeta}} dt = \zeta \int_{-\infty}^{\infty} e^{-at} e^{-\left(\frac{i}{\zeta}+a\right)t} dt = \frac{\zeta^2}{i + \zeta a}$$

(S2)

$$\Pi(\zeta) = \zeta \int_{-\infty}^{\infty} \delta(t) e^{-\frac{t}{\zeta}} dt = \zeta$$

(S3)

$$\aleph[\Pi(t-a)] = \zeta \int_{-\infty}^{\infty} \Pi(t-a) e^{-\frac{t}{\zeta}} dt = e^{-\frac{a}{\zeta}} \Pi(\zeta)$$

(S4)

$$\aleph \left[\frac{d\Pi(t)}{dt} \right] = \zeta \int_{-\infty}^{\infty} \frac{d\Pi(t)}{dt} e^{-\frac{t}{\zeta}} dt = \frac{i}{\zeta} \Pi(\zeta)$$

(S5)

$$\aleph \left[\int_{-\infty}^{\infty} \Pi(t-\tau)\Theta(\tau) d\tau \right] = \frac{1}{\zeta} \left[\zeta \int_{-\infty}^{\infty} e^{-\frac{\tau}{\zeta}} \Theta(\tau) d\tau \right] \left[\zeta \int_{-\infty}^{\infty} \Pi(t-\tau) e^{-\frac{t-\tau}{\zeta}} dt \right] = \frac{1}{\zeta} \Pi(\zeta)\Theta(\zeta)$$

Solving a steady heat transfer problem

Let us consider the differential equation in the steady heat transfer problem [9]:

$$\Pi^{(1)}(t) + \phi\Pi(t) = \delta(t) \quad (40)$$

where ϕ represents the thermal diffusivity, and $\Pi(t)$ is the temperature.

Taking the integral transforms of eq. (10) yields the forms of the equations:

$$(i\kappa + \phi)\Pi(\kappa) = \kappa \quad (41)$$

$$(i\lambda + \phi)\Pi(\lambda) = \frac{1}{\lambda} \quad (42)$$

$$\left(\frac{i}{\gamma} + \phi \right) \Pi(\gamma) = \frac{1}{\gamma} \quad (43)$$

$$\left(\frac{i}{\zeta} + \phi \right) \Pi(\zeta) = \zeta \quad (44)$$

Thus, we have the solution of eq. (40) in the form:

$$\Pi(t) = e^{-\phi t} u(t) \quad (45)$$

Conclusion

In this paper, we presented the Fourier-like integral transforms to derive from the Laplace-Carson, Aboodh and Yang, Sumudu and Elzaki. The solution for the steady heat transfer equation was discussed. The proposed methods are efficient and accurate for solving the heat transfer problems.

Nomenclature

t – space co-ordinate, [m]

Greek symbols

$\Pi(t)$ – temperature, [K]

ϕ – the thermal diffusivity, [m^2s^{-1}]

Acknowledgment

This work is supported by the State Key Research Development Program of the People Republic of China (Grant No. 2016YFC0600705), the Natural Science Foundation of China (Grant No. 51323004), and the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD2014).

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