ANALYTICAL SOLUTION FOR THE 1-D HEAT TRANSFER EQUATIONS WITH RADIATIVE LOSS

by

You-Chang LV[a,b*], Man WANG[a,b], and Ying-Wei WANG[a,b]

[a] State Key Laboratory of Coking Coal Exploitation and Comprehensive Utilization, China Pingmei Shenma Group, Pingdingshan, Henan, China
[b] State Institute of Energy and Chemical Industry, China Pingmei Shenma Group, Pingdingshan, Henan, China

Original scientific paper
https://doi.org/10.2298/TSCI17S1047L

In this paper, we consider the 1-D heat transfer equation with radiative loss. The variational iterative Sumudu type integral transform is used to obtain the analytical solution for the heat transfer problems. The presented method is efficient and accurate.

Key words: *heat transfer, radiative loss, variational iterative Sumudu type integral transform, analytical solution*

Introduction

In this paper, we consider the 1-D heat transfer equation with radiative loss compared with the surroundings of the form [1-2]:

$$\frac{\partial m(x,t)}{\partial t}=\frac{\kappa}{c_p\rho}\frac{\partial^2 m(x,t)}{\partial x^2}-\eta m(x,t) \tag{1}$$

where $m(x,t)$ is the temperature, κ – the thermal conductivity, c_p – the specific heat capacity, ρ – the mass density, and η – the parameter.

There are many methods for finding the solutions of the heat transfer problems, such as the finite volume [3], homotopy analysis [4], homotopy perturbation [5], variational iteration [5], fixed finite element [6] methods, and others [7-10].

The Sumudu integral transform was proposed by Weerakoon in 1998 [11]. Recently, the Sumudu-like integral transform was reported by Yang [10] for the first time. The variational iterative Sumudu type integral transform method was developed to solve the diffusion equation [12]. However, the method has not yet adopted to consider the 1-D heat transfer equation with radiative loss compared with the surroundings. In this manuscript, we plan to consider the variational iterative Sumudu type integral transform method to solve the radiative-loss heat transfer problem.

The Sumudu type integral transform method

The Sumudu type integral transform of the function $\Xi(t)$ is defined [10, 12, 13]:

* Corresponding author, e-mail: wsyjs2787617@163.com

$$\Xi(\sigma)=\Upsilon\left[\Xi(t)\right]=:\int_0^\infty \Xi(t)\mathrm{e}^{-\frac{t}{\sigma}}\mathrm{d}t \tag{2}$$

provided the integral exists for some t, where Υ is the Sumudu type integral transform operator.

The inverse Sumudu type integral transform operator is given [10, 12, 13]:

$$\Upsilon^{-1}\left\{\Upsilon\left[\Xi(t)\right]\right\}=\Upsilon^{-1}\left\{\Xi(\sigma)\right\}:=\Xi(t) \tag{3}$$

The properties of the Sumudu type integral transform are presented as follows [10, 13]:
(B1) Let $\Xi_1(\sigma)=\Upsilon\left[\Xi_1(t)\right]$ and $\Xi_2(\sigma)=\Upsilon\left[\Xi_2(t)\right]$. Then [10,13]:

$$\Upsilon\left[a\Xi_1(t)+b\Xi_2(t)\right]=\Xi_1(\sigma)+\Xi_2(\sigma) \tag{4}$$

where a and b are two constants.
(B2) Let $\Xi(\sigma)=\Upsilon\left[\Xi(t)\right]$. Then [10, 12, 13]:

$$\Upsilon\left[\frac{\mathrm{d}\Xi(t)}{\mathrm{d}t}\right]=\frac{1}{\sigma}\Xi(\sigma)-\Xi(0) \tag{5}$$

(B3) Let $\Xi(\sigma)=\Upsilon\left[\Xi(t)\right]$. Then [13]:

$$\Upsilon\left[\frac{\mathrm{d}^2\Xi(t)}{\mathrm{d}t^2}\right]=\frac{1}{\sigma^2}\Xi(\sigma)-\frac{1}{\sigma}\Xi(0)-\Xi^{(1)}(0) \tag{6}$$

(B4) Let $\Xi_1(\sigma)=\Upsilon\left[\Xi_1(t)\right]$ and $\Xi_2(\sigma)=\Upsilon\left[\Xi_2(t)\right]$. Then [13]:

$$\Upsilon\left[\int_0^t \Xi_1(\tau)\Xi_2(t-\tau)\mathrm{d}\tau\right]=\Xi_1(\sigma)\Xi_2(\sigma) \tag{7}$$

(B5) Let $\Xi(\sigma)=\Upsilon\left[\Xi(t)\right]$. Then:

$$\Upsilon\left[\Xi(at)\right]=\int_0^\infty \Xi(at)\mathrm{e}^{-\frac{t}{\sigma}}\mathrm{d}t \tag{8}$$

Proof. With the use of eq. (2), we obtain:

$$\Upsilon\left[\Xi(at)\right]=\frac{1}{a}\int_0^\infty \Xi(t)\mathrm{e}^{-\frac{t}{a\sigma}}\mathrm{d}t=\frac{1}{a}\Upsilon\left[\Xi\left(\frac{t}{a}\right)\right] \tag{9}$$

The properties of the Sumudu type integral transform operator are listed in tab. 1.

The variational iterative Sumudu type integral transform method

In this section we recall the variational iterative Sumudu type integral transform method proposed in [12].

We present a differential equation as:

$$\mu\Xi+\lambda\Xi=\eta m \tag{10}$$

where $\mu=\partial/\partial t$, $\lambda=-\beta\partial^2/\partial x^2$ with the parameter β, and m is the source term.

Table 1. Table for the Sumudu type integral transform operator

	Sumudu type integral transforms	Functions		Sumudu type integral transforms	Functions
1	$\frac{\sigma}{1+a^2\sigma^2}$	$\cos(at)$	7	$\frac{a\sigma^2}{(b\sigma-1)^2+a^2\sigma^2}$	$e^{bt}\sin(at)$
2	$\frac{a\sigma^2}{1+a^2\sigma^2}$	$\sin(at)$	8	σ^{k+1}	$\frac{t^k}{k!}$
3	$\frac{\sigma}{1-a^2\sigma^2}$	$\cosh(at)$	9	$\frac{\sigma}{(1-a\sigma)(1-b\sigma)}$	$\frac{1}{a-b}(ae^{at}-be^{bt})$
4	$\frac{a\sigma^2}{1-a^2\sigma^2}$	$\sinh(at)$	10	$\frac{a^3\sigma^4}{(1-a^2\sigma^2)^2}$	$\frac{1}{2}[at\cosh(at)-\sinh(at)]$
5	$\left(\frac{\sigma}{1-a\sigma}\right)^{k+1}$	$\frac{t^k}{k!}e^{at}$	11	$\frac{\sigma}{(1-a^2\sigma^2)^2}$	$\cosh(at)+\frac{a}{2}t\sinh(at)$
6	$\frac{\sigma(1-b\sigma)}{(1-b\sigma)^2+a^2\sigma^2}$	$e^{bt}\cos(at)$	12	$\frac{a\sigma^2}{(1+a^2\sigma^2)^2}$	$\frac{1}{2}[\sin(at)+at\cos(at)]$

Making use of the variational iteration method [12], the functional can be written:

$$\Xi_{n+1}(x,t)=\Xi_n(x,t)+\int_0^t\Phi(t-\tau)\left[\mu\Xi_n(x,\tau)+\lambda\Xi_n(x,\tau)-\eta m(x,\tau)\right]\mathrm{d}\tau \tag{11}$$

Adopting the integral transform to eq. (11), it follows:

$$\begin{aligned}\Xi_{n+1}(x,\sigma)&=\Xi_n(x,\sigma)+\mathrm{Y}\left\{\int_0^t\Phi(t-\tau)\left[\mu\,\Xi_n(x,\tau)+\lambda\Xi_n(x,\tau)-\eta m(x,\tau)\right]\mathrm{d}\tau\right\}=\\&=\Xi_n(x,\sigma)+Y\{\Phi(t)\}Y\{[\mu\,\Xi_n(x,\tau)+\lambda\Xi_n(x,\tau)-\eta m(x,\tau)]\}=\\&=\Xi_n(x,\sigma)+\Phi(\sigma)\mathrm{Y}\{[\mu\Xi_n(x,\tau)+\lambda\Xi_n(x,\tau)-\eta m(x,\tau)]\}\end{aligned} \tag{12}$$

Considering the variation of eq. (12) with respect to $\Xi_n(x,\sigma)$, we obtain:

$$\delta\,\Xi_{n+1}(x,\sigma)=\delta\,\Xi_n(x,\sigma)+\Phi(\sigma)\delta\left(\mathrm{Y}\{[\mu\,\Xi_n(x,t)-\lambda\Xi_n(x,t)-\eta m(x,\tau)]\}\right)=0 \tag{13}$$

which yields the following equation of the form:

$$\begin{aligned}\delta\,\Xi_{n+1}(x,\sigma)&=1+\Phi(\sigma)\delta\left[\mathrm{Y}\{[\mu\mathrm{Y}^{-1}[\Xi_n(x,\sigma)]+\lambda\,\Xi_n(x,t)-\eta m(x,\tau)]\}\right]=\\&=1+\Phi(\sigma)\delta\{\mu\,\Xi_n(x,t)\}=\\&=1+\frac{1}{\sigma}\Phi(\sigma)=0\end{aligned} \tag{14}$$

Therefore, from eq. (14) we present:

$$\Phi(\sigma)=-\sigma \tag{15}$$

From eq. (11) and eq. (15), the iteration algorithm can be given:

$$\Xi_{n+1}(x,\sigma)=\Xi_n(x,\sigma)-\sigma\mathrm{Y}\left\{\mu\mathrm{Y}^{-1}\left[\Xi_n(x,\sigma)\right]\right\}+\sigma\left[-\lambda\Xi_n(x,\sigma)-m(x,\sigma)\right] \tag{16}$$

With the aid of eq. (16) we have the integral transform solution in the form:

$$\Xi(x,\sigma)=\lim_{n\to\infty}\Xi_n(x,\sigma) \tag{17}$$

which deduces the equation of the form:

$$\Xi(x,t)=\mathrm{Y}^{-1}\left[\lim_{n\to\infty}\Xi_n(x,\sigma)\right]=\lim_{n\to\infty}\mathrm{Y}^{-1}\left[\Xi_n(x,\sigma)\right] \tag{18}$$

Solving the 1-D heat transfer equation with radiative loss

In this section, we consider the 1-D heat transfer equation with radiative loss with the initial value condition.

Let us consider eq. (1) with the initial value condition:

$$m(x,0)=\mathrm{e}^x \tag{19}$$

With the use of eq. (16), the following iterative algorithm can be written:

$$m_{n+1}(x,\sigma)=m_n(x,\sigma)-\sigma\mathrm{Y}\left[\frac{\partial m_n(x,t)}{\partial t}\right]+\sigma\left[\frac{\kappa}{c_p\rho}\frac{\partial^2 m_n(x,\sigma)}{\partial x^2}+m_n(x,\sigma)\right] \tag{20}$$

which leads to:

$$m_{n+1}(x,\sigma)=(1+\sigma)m_n(x,\sigma)+\frac{\kappa\sigma}{c_p\rho}\frac{\partial^2 m_n(x,\sigma)}{\partial x^2}-\sigma\mathrm{Y}\left\{\frac{\partial}{\partial t}\mathrm{Y}^{-1}\left[m_n(x,\sigma)\right]\right\} \tag{21}$$

subject to the initial value condition:

$$m_0(x,\sigma)=\mathrm{Y}\left[\mathrm{e}^x\right]=\sigma\mathrm{e}^x \tag{22}$$

From eqs. (21) and (22), we have:

$$\begin{aligned}m_1(x,\sigma)&=(1+\sigma)m_0(x,\sigma)+\frac{\kappa\sigma}{c_p\rho}\frac{\partial^2 m_0(x,\sigma)}{\partial x^2}-\sigma\mathrm{Y}\left\{\frac{\partial}{\partial t}\mathrm{Y}^{-1}\left[m_0(x,t)\right]\right\}=\\&=(1+\sigma)\sigma\mathrm{e}^x+\frac{\kappa\sigma}{c_p\rho}\frac{\partial^2(\sigma\mathrm{e}^x)}{\partial x^2}=\\&=(1+\sigma)\sigma\mathrm{e}^x+\frac{\kappa\sigma^2}{c_p\rho}\mathrm{e}^x=\\&=\sigma\mathrm{e}^x+\nu\sigma^2\mathrm{e}^x\end{aligned} \tag{23}$$

where $\nu=1+\kappa/(c_p\rho)$.

$$m_2(x,\sigma)=(1+\sigma)m_1(x,\sigma)+\frac{\kappa\sigma}{c_p\rho}\frac{\partial^2 m_1(x,\sigma)}{\partial x^2}-\sigma\mathrm{Y}\left\{\frac{\partial}{\partial t}\mathrm{Y}^{-1}\left[m_1(x,t)\right]\right\}=$$

$$=\left(1+\sigma\right)\left(1+\nu\sigma\right)\sigma\mathrm{e}^{x}+\frac{\kappa\sigma^{2}}{c_{p}\rho}\left(1+\nu\sigma\right)\mathrm{e}^{x}-\nu\sigma\mathrm{e}^{x}=$$
$$=\left(1-\nu\right)\sigma\mathrm{e}^{x}+2\nu\sigma^{2}\mathrm{e}^{x}+\nu^{2}\sigma^{3}\mathrm{e}^{x} \tag{24}$$

$$m_{3}\left(x,\sigma\right)=\left(1+\sigma\right)m_{2}\left(x,\sigma\right)+\frac{\kappa\sigma}{c_{p}\rho}\frac{\partial^{2}m_{2}\left(x,\sigma\right)}{\partial x^{2}}-\sigma Y\left\{\frac{\partial}{\partial t}Y^{-1}\left[m_{2}\left(x,t\right)\right]\right\}=$$
$$=\left(1+\sigma\right)\left[\left(1-\nu\right)\sigma e^{x}+2\nu\sigma^{2}e^{x}+\nu^{2}\sigma^{3}e^{x}\right]-\left(2\nu\sigma^{2}e^{x}+\nu^{2}\sigma^{3}e^{x}\right)+$$
$$+\frac{\kappa\sigma}{c_{p}\rho}\left[\left(1-\nu\right)\sigma e^{x}+2\nu\sigma^{2}e^{x}+\nu^{2}\sigma^{3}e^{x}\right]=$$
$$=\left[\left(1-\nu\right)\sigma e^{x}+2\nu\sigma^{2}e^{x}+\nu^{2}\sigma^{3}e^{x}\right]-\left(2\nu\sigma^{2}e^{x}+\nu^{2}\sigma^{3}e^{x}\right)+$$
$$+\nu\sigma\left[\left(1-\nu\right)\sigma e^{x}+2\nu\sigma^{2}e^{x}+\nu^{2}\sigma^{3}e^{x}\right]=$$
$$=\left(1-\nu\right)\sigma e^{x}+\nu\left(1-\nu\right)\sigma^{2}e^{x}+2\nu^{2}\sigma^{3}e^{x}+\nu^{3}\sigma^{4}e^{x} \tag{25}$$

$$m_{4}\left(x,\sigma\right)=\left(1+\sigma\right)m_{3}\left(x,\sigma\right)+\frac{\kappa\sigma}{c_{p}\rho}\frac{\partial^{2}m_{3}\left(x,\sigma\right)}{\partial x^{2}}-\sigma\mathrm{Y}\left\{\frac{\partial}{\partial t}\mathrm{Y}^{-1}\left[m_{3}\left(x,t\right)\right]\right\}=$$
$$=\left(1+\sigma\right)\left[\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+2\nu^{2}\sigma^{3}\mathrm{e}^{x}+\nu^{3}\sigma^{4}\mathrm{e}^{x}\right]-$$
$$-\left[\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+2\nu^{2}\sigma^{3}\mathrm{e}^{x}+\nu^{3}\sigma^{4}\mathrm{e}^{x}\right]+$$
$$+\frac{\kappa\sigma}{c_{p}\rho}\left[\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+2\nu^{2}\sigma^{3}\mathrm{e}^{x}+\nu^{3}\sigma^{4}\mathrm{e}^{x}\right]=$$
$$=\left[\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+2\nu^{2}\sigma^{3}\mathrm{e}^{x}+\nu^{3}\sigma^{4}\mathrm{e}^{x}\right]+$$
$$+\nu\sigma\left[\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+2\nu^{2}\sigma^{3}\mathrm{e}^{x}+\nu^{3}\sigma^{4}\mathrm{e}^{x}\right]-$$
$$-\left[\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+2\nu^{2}\sigma^{3}\mathrm{e}^{x}+\nu^{3}\sigma^{4}\mathrm{e}^{x}\right]=$$
$$=\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+\nu^{2}\left(1-\nu\right)\sigma^{3}\mathrm{e}^{x}+2\nu^{3}\sigma^{4}\mathrm{e}^{x}+\nu^{4}\sigma^{5}\mathrm{e}^{x} \tag{26}$$

$$m_{5}\left(x,\sigma\right)=\left(1+\sigma\right)m_{4}\left(x,\sigma\right)+\frac{\kappa\sigma}{c_{p}\rho}\frac{\partial^{2}m_{4}\left(x,\sigma\right)}{\partial x^{2}}-\sigma\mathrm{Y}\left\{\frac{\partial}{\partial t}\mathrm{Y}^{-1}\left[m_{4}\left(x,t\right)\right]\right\}=$$
$$=\left(1+\sigma\right)\left[\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+\nu^{2}\left(1-\nu\right)\sigma^{3}\mathrm{e}^{x}+2\nu^{3}\sigma^{4}\mathrm{e}^{x}+\nu^{4}\sigma^{5}\mathrm{e}^{x}\right]+$$
$$+\frac{\kappa\sigma}{c_{p}\rho}\left[\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+\nu^{2}\left(1-\nu\right)\sigma^{3}\mathrm{e}^{x}+2\nu^{3}\sigma^{4}\mathrm{e}^{x}+\nu^{4}\sigma^{5}\mathrm{e}^{x}\right]-$$
$$-\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+\nu^{2}\left(1-\nu\right)\sigma^{3}\mathrm{e}^{x}+2\nu^{3}\sigma^{4}\mathrm{e}^{x}+\nu^{4}\sigma^{5}\mathrm{e}^{x}=$$
$$=\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\sigma\left[\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+\nu^{2}\left(1-\nu\right)\sigma^{3}\mathrm{e}^{x}+2\nu^{3}\sigma^{4}\mathrm{e}^{x}+\nu^{4}\sigma^{5}\mathrm{e}^{x}\right]=$$
$$=\left(1-\nu\right)\sigma\mathrm{e}^{x}+\nu\left(1-\nu\right)\sigma^{2}\mathrm{e}^{x}+\nu^{2}\left(1-\nu\right)\sigma^{3}\mathrm{e}^{x}+\nu^{3}\left(1-\nu\right)\sigma^{4}\mathrm{e}^{x}+2\nu^{4}\sigma^{5}\mathrm{e}^{x}+\nu^{5}\sigma^{6}\mathrm{e}^{x} \tag{27}$$

$$m_6(x,\sigma)=(1+\sigma)m_5(x,\sigma)+\frac{\kappa\sigma}{c_p\rho}\frac{\partial^2 m_5(x,\sigma)}{\partial x^2}-\sigma\mathrm{Y}\left\{\frac{\partial}{\partial t}\mathrm{Y}^{-1}\left[m_5(x,t)\right]\right\}=$$
$$=(1+\sigma)\left[(1-\nu)\sigma\mathrm{e}^x+\nu(1-\nu)\sigma^2\mathrm{e}^x+\nu^2(1-\nu)\sigma^3\mathrm{e}^x+2\nu^3\sigma^4\mathrm{e}^x+\nu^4\sigma^5\mathrm{e}^x\right]+$$
$$+\frac{\kappa\sigma}{c_p\rho}\left[(1-\nu)\sigma\mathrm{e}^x+\nu(1-\nu)\sigma^2\mathrm{e}^x+\nu^2(1-\nu)\sigma^3\mathrm{e}^x+2\nu^3\sigma^4\mathrm{e}^x+\nu^4\sigma^5\mathrm{e}^x\right]-$$
$$-\nu(1-\nu)\sigma^2\mathrm{e}^x+\nu^2(1-\nu)\sigma^3\mathrm{e}^x+2\nu^3\sigma^4\mathrm{e}^x+\nu^4\sigma^5\mathrm{e}^x=$$
$$=(1-\nu)\sigma\mathrm{e}^x+\nu(1-\nu)\sigma^2\mathrm{e}^x+\nu^2(1-\nu)\sigma^3\mathrm{e}^x+\nu^3(1-\nu)\sigma^4\mathrm{e}^x+$$
$$+\nu^4(1-\nu)\sigma^5\mathrm{e}^x+2\nu^5\sigma^6\mathrm{e}^x+\nu^6\sigma^7\mathrm{e}^x \tag{28}$$

and so on.

Thus, with the help of the inverse Sumudu type integral transform method, we have:

$$m_0(x,\sigma)=\mathrm{Y}\left[\mathrm{e}^x\right]=\sigma\mathrm{e}^x \tag{29}$$

$$m_1(x,\sigma)=\sigma\mathrm{e}^x+\nu\sigma^2\mathrm{e}^x \tag{30}$$

$$m_2(x,\sigma)=(1-\nu)\sigma\mathrm{e}^x+2\nu\sigma^2\mathrm{e}^x+\nu^2\sigma^3\mathrm{e}^x \tag{31}$$

$$m_3(x,\sigma)=(1-\nu)\sigma\mathrm{e}^x(1+\nu\sigma)+2\nu^2\sigma^3\mathrm{e}^x+\nu^3\sigma^4\mathrm{e}^x \tag{32}$$

$$m_4(x,\sigma)=(1-\nu)\sigma\mathrm{e}^x\left(1+\nu\sigma+\nu^2\sigma^2\right)+2\nu^3\sigma^4\mathrm{e}^x+\nu^4\sigma^5\mathrm{e}^x \tag{33}$$

$$m_5(x,\sigma)=(1-\nu)\sigma\mathrm{e}^x\left(1+\nu\sigma+\nu^2\sigma^2+\nu^3\sigma^3\right)+2\nu^4\sigma^5\mathrm{e}^x+\nu^5\sigma^6\mathrm{e}^x \tag{34}$$

$$m_6(x,\sigma)=(1-\nu)\sigma\mathrm{e}^x\left(1+\nu\sigma+\nu^2\sigma^2+\nu^3\sigma^3+\nu^4\sigma^4\right)+2\nu^5\sigma^6\mathrm{e}^x+\nu^6\sigma^7\mathrm{e}^x \tag{35}$$

such that:

$$m_0(x,t)=\mathrm{e}^x \tag{36}$$

$$m_1(x,t)=\mathrm{e}^x+\nu t\mathrm{e}^x \tag{37}$$

$$m_2(x,t)=(1-\nu)\mathrm{e}^x+2\nu t\mathrm{e}^x+\frac{\nu^2t^2}{2!}\mathrm{e}^x \tag{38}$$

$$m_3(x,t)=(1-\nu)\mathrm{e}^x(1+\nu t)+\frac{2\nu^2t^2}{2!}\mathrm{e}^x+\frac{\nu^3t^3}{3!}\mathrm{e}^x \tag{39}$$

$$m_4(x,t)=(1-\nu)\mathrm{e}^x\left(1+\nu t+\frac{\nu^2t^2}{2!}\right)+\frac{2\nu^3t^3}{3!}\mathrm{e}^x+\frac{\nu^4t^4}{4!}\mathrm{e}^x \tag{40}$$

$$m_5(x,t)=(1-\nu)\mathrm{e}^x\left(1+\nu t+\frac{\nu^2t^2}{2!}+\frac{\nu^3t^3}{3!}\right)+\frac{2\nu^4t^4}{4!}\mathrm{e}^x+\frac{\nu^5t^5}{5!}\mathrm{e}^x \tag{41}$$

$$m_6(x,t)=(1-\nu)\mathrm{e}^x\left(1+\nu t+\frac{\nu^2t^2}{2!}+\frac{\nu^3t^3}{3!}+\frac{\nu^4t^4}{4!}\right)+\frac{2\nu^5t^5}{5!}\mathrm{e}^x+\frac{\nu^6t^6}{6!}\mathrm{e}^x \tag{42}$$

Conclusion

In the present work, the variational iterative Sumudu type integral transform was adopted to handle the 1-D heat transfer equation with radiative loss compared with the surroundings. The analytical solution for the heat transfer problem was discussed in detail. The proposed method is efficient and accurate for finding the analytical solutions for the heat transfer equations.

Nomenclature

c_p – specific heat capacity, [$\mathrm{Jkg^{-1}K^{-1}}$]
$m(x, t)$ – temperature, [K]
t – time, [s]
x – space co-ordinate, [m]

Greek symbols

κ – thermal conductivity, [$\mathrm{Wm^{-1}K^{-1}}$]
ρ – mass density, [$\mathrm{kgm^{-3}}$]

References

[1] Howell, J. R., *et al.*, *Thermal Radiation Heat Transfer*, CRC Press, New York, USA, 2010
[2] Modest, M. F., *Radiative Heat Transfer*, Academic Press, New York, USA, 2013
[3] Chui, E. H., Raithby, G. D., Computation of Radiant Heat Transfer on a Nonorthogonal Mesh Using the Finite-Volume Method, *Numerical Heat Transfer*, *23* (1993), 3, pp. 269-288
[4] Abbasbandy, S., The Application of Homotopy Analysis Method to Nonlinear Equations Arising in Heat Transfer, *Physics Letters A*, *360* (2006), 1, pp. 109-113
[5] Ganji, D. D., Sadighi, A., Application of Homotopy-Perturbation and Variational Iteration Methods to Nonlinear Heat Transfer and Porous Media Equations, *Journal of Computational and Applied Mathematics*, *207* (2007), 1, pp. 24-34
[6] Belhadj, A., *et al.*, Boubaker Polynomials Expansion Scheme-Related Heat Transfer Investigation Inside Keyhole Model, *Journal of Thermophysics and Heat Transfer*, *23* (2009), 3, pp. 639-640
[7] Yang, X. J., *et al.*, On Local Fractional Operators View of Computational Complexity, *Thermal Science*, *20* (2016), Suppl. 3, pp. S723-S727
[8] Yang, X. J., Srivastava, *et al.*, A New Fractional Derivative without Singular Kernel: Application to the Modelling of the Steady Heat Flow, *Thermal Science*, 20 (2016), 2, pp. 753-756
[9] Yang, X. J., A New Integral Transform Operator for Solving the Heat-Diffusion Problem, *Applied Mathematics Letters*, *64* (2017), Feb., pp. 193-197
[10] Yang, X. J., A New Integral Transform Method for Solving Steady Heat Transfer Problem, *Thermal Science*, *20* (2016), Suppl. 3, pp. S639-S642
[11] Weerakoon, S., Complex Inversion Formula for Sumudu Transform, *International Journal of Mathematical Education in Science and Technology*, *29* (1998), 4, pp. 618-620
[12] Yang, X. J., Gao, F., A New Technology for Solving Diffusion and Heat Equations, *Thermal Science*, *21* (2017), 1A, pp. 133-140
[13] Gao, L., *et al.*, Analytical Solutions of Linear Diffusion and Wave Equations in Semi-Infinite Domains by Using a New Integral Transform, *Thermal Science*, *21* (2017), Suppl. 1, pp. S71-S78, (in this issue)

Paper submitted: March 10, 2017
Paper revised: May 1, 2017
Paper accepted: May 10, 2017