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THE EFFECT OF VARIABLE MAGNETIC FIELD ON HEAT TRANSFER AND FLOW ANALYSIS OF UNSTEADY SQUEEZING NANOFLUID FLOW BETWEEN PARALLEL PLATES USING GALERKIN METHOD

by

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In this paper, an analytical investigation of nanofluid flow and heat transfer in a rotating system is studied by a semi exact method based on weighted residual called least square method. We used this method to solve the governing nonlinear coupled equations of the described problem and compared it with numerical method (Runge-Kutta 4th order). Comparisons indicate that least square method is so suitable computational process. The results indicate that skin friction parameter increases with augment of Reynolds number and rotation parameter but it decreases with increase of injection parameter. Also it can be found that Nusselt number has a direct relationship with Reynolds number and injection parameter while it has a reverse relationship with rotation parameter, Schmidt number, thermophoretic, and brownian parameter.

Key words: *nanofluid, fluid flow, thermophoresis, particle transport, Schmidt number*

Introduction

Effective cooling techniques are absolutely needed for cooling any sort of high energy device. Common heat transfer fluids such as water, ethylene glycol, and engine oil have limited heat transfer capabilities due to their low heat transfer properties. In contrast, metals thermal conductivities are up to three times higher than the fluids, so it is naturally desirable to combine the two substances to produce a heat transfer medium that behaves like a fluid, but has the thermal conductivity of a metal.

Many studies are investigated about nanofluid recently. Steady MHD free convection boundary-layer flow past a vertical semi-infinite flat plate embedded in water filled with a nanofluid has been theoretically studied by Hamad *et al.* [1]. They found that Cu and Ag nanoparticles proved to have the highest cooling performance for this problem. Several recent studies on the modeling of nanofluid flow and heat transfer have been studied [2, 3].

The incompressible fluid flow and heat transfer over rotating bodies have many industrial and engineering applications such as gas turbine engines and electronic devices. Originally Karman [4] discussed the steady flow of Newtonian fluid over a rotating disk, who introduced an elegant transformation that enabled the Navier-Stokes equations for an isothermal, imper-

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meable rotating disk to be reduced to a system of coupled ODE. Using momentum integral method, he obtained an approximate solution to the ODE. Sibanda and Makinde [5] investigated the hydromagnetic steady flow and heat transfer characteristics of an incompressible viscous electrically conducting fluid past a rotating disk in a porous medium with the ohmic heating and viscous dissipation, they found that magnetic field retards the fluid motion due to the opposing Lorentz force generated by the magnetic field and the magnetic field and Eckert number tend to enhance the heat transfer efficiency.

In most of the available studies, the base fluid is a common fluid with low thermal conductivity. The resulting performances of such thermal systems are poor. A recent way of improving the performance of these systems is to suspend metallic nanoparticles in the base fluid. Rashidi *et al.* [6] considered the analysis of the second law of thermodynamics applied to an electrically conducting incompressible nanofluid flowing over a porous rotating disk. They concluded that using magnetic rotating disk drives has important applications in heat transfer enhancement in renewable energy systems. Ellahi [7] studied the MHD flow of non-Newtonian nanofluid in a pipe. He observed that the MHD parameter decreases the fluid motion and the velocity profile is larger than that of temperature profile even in the presence of variable viscosities.

All the previous studies assumed that there are no slip velocities between nanoparticles and fluid molecules and assumed that the nanoparticle concentration is uniform. Nield and Kuznetsov [8] studied the natural convection in a horizontal layer of a porous medium. Their analysis revealed that for a typical nanofluid (with large Lewis number) the prime effect of the nanofluids is via a buoyancy effect coupled with the conservation of nanoparticles, the contribution of nanoparticles to the thermal energy equation being a secondorder effect. Khan and Pop [9] published a paper on boundary-layer flow of a nanofluid past a stretching sheet. They indicated that the reduced Nusselt number is a decreasing function of higher Prandtl number and a decreasing function of lower Prandtl number for each Lewis, *Nb*, and *Nt* numbers. Recently, there have been published several numerical studies on the modeling of natural convection heat transfer and effect of using nanofluids on heat transfer enhancement [10-13].

Least square method (LSM), collocation method (CM), and Galerkin method (GM) called the weighted residuals methods. Lately, more attention has been dedicated to these analytical methods for using in the heat transfer problems and other engineering application [14-21]. Recently, LSM is introduced by Aziz and Bouaziz [22] and is applied for prediction of the performance of a longitudinal fin [23]. They found that LSM is simple compared with other analytical methods.

An analytical investigation is applied for unsteady flow of a nanofluid squeezing between two parallel plates studied by Pourmehran *et al.* [24] recently. They applied CM and LSM to solve the governing equations. The results demonstrate that when the two plates move toward together, the Nusselt number has a direct relationship with nanoparticle volume fraction and Eckert number while it has a reverse relationship with the squeeze number. Rahimi-Gorji *et al.*[25] studied an analytical investigation of the heat transfer for the micro-channel heat sink (MCHS) cooled by different nanofluids (Cu, Al_2O_3, Ag, TiO_2 in water and ethylene glycol as base fluids) using porous media approach and the GM. They applied response surface methodology to obtain the desirability of the optimum design of the channel geometry. They found that Ag-water nanofluid has the maximum Nusselt number enhancement. Recently Pourmehran *et al.* [26] presented a thermal and flow analysis of a fin shaped MCHS cooled by different nanofluids (Cu and Al_2O_3 in water) based on *saturated porous medium* and LSM then results are compared with numerical procedure. They proved that Cu-water nanofluid is more lucrative thermally versus Al_2O_3-water nanofluid.

In this work we applied two phase model for simulating nanofluid flow and heat transfer in a rotating system. Also, LSM is used to solve the reduced non-linear ODE. The effects of Reynolds number, rotation parameter, injection parameter, Schmidt number, thermophoretic parameter, and Brownian parameter on flow, heat and mass transfer are studied.

Problem description and governing equations

Consider the steady nanofluid flow between two horizontal parallel plates when the fluid and the plates rotate together around they axis which is normal to the plate switch an angular velocity. A Cartesian co-ordinate system is considered as follows: the x-axis is along the plate, the y-axis is perpendicular to it, and the z-axis is normal to the xy plane, fig. 1.

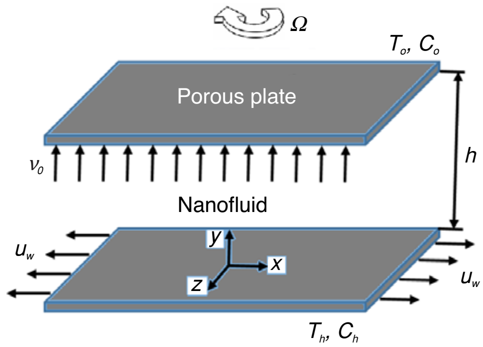

Figure 1. The schematic diagram of the physical model

The upper plate is subjected to a constant wall injection velocity v_0 (>0). The plates are located at $y = 0$ and $y = h$. The lower plate is being stretched by two equal and opposite forces so that the position of the point (0, 0, 0) remains unchanged. The upper plate is subjected to a constant flow injection with a velocity v_0. The governing equations in a rotating frame of reference are:

$$u_x + v_y + w_z = 0 \tag{1}$$

$$\rho_f (uu_x + vu_y + 2\Omega w) = -p_x^* + \mu(u_{xx} + u_{yy}) \tag{2}$$

$$\rho_f (uv_y) = -p_y^* + \mu(v_{xx} + v_{yy}) \tag{3}$$

$$\rho_f (uw_x + vw_y - 2\Omega w) = \mu(w_{xx} + w_{yy}) \tag{4}$$

$$uT_x + vT_y + wT_z = \alpha(T_{xx} + T_{yy} + T_{zz}) + \frac{(\rho C_p)_p}{(\rho C_p)_f}\left\{ D_B \left[C_x T_x + C_y T_y + C_z T_z \right] + \frac{D_T}{T_c}\left[T_{xx}^2 + T_{yy}^2 + T_{zz}^2 \right] \right\} \tag{5}$$

$$uC_x + vC_y + wC_z = D_B (C_{xx} + C_{yy} + C_{zz}) + \left(\frac{D_T}{T_0} \right)\left\{ T_{xx} + T_{yy} + T_{zz} \right\} \tag{6}$$

Here u, v, and w are the velocities in the x-, y-, and z-directions, respectively. Also p^* is the modified fluid pressure. The absence of p_z^* in eq. (4) implies that there is a net cross-flow along the z-axis. The relevant boundary conditions are:

$$\begin{aligned} &y = 0 \quad \rightarrow \quad u = ax \,,\; v = 0 \,,\; w = 0 \,,\; T = T_h \;,\; C = C_h \\ &y = +h \quad \rightarrow \quad u = 0 \,,\; v = v_0 \;,\; w = 0 \,,\; T = T_0 \;,\; C = C_0 \end{aligned} \tag{7}$$

The following non-dimensional variables are introduced:

$$\eta = \frac{y}{h}, \quad u = axf'(\eta), \quad v = -ahf(\eta), \quad w = axg(\eta), \quad \theta(\eta) = \frac{T - T_h}{T_0 - T_h}, \quad \varphi(\eta) = \frac{C - C_h}{C_0 - C_h} \tag{8}$$

where prime denotes differentiation with respect to η. By substituting eq. (8) in eqs. (1)-(4), we have:

$$-\frac{1}{\rho h}p_{\eta}^{*}=a^{2}x\left(f'-ff''-\frac{f'''}{\mathrm{Re}}+\frac{2K_{r}}{\mathrm{Re}}g\right) \tag{9}$$

$$-\frac{1}{\rho h}p_{\eta}^{*}=a^{2}h\left(ff''+\frac{1}{\mathrm{Re}}f''\right) \tag{10}$$

$$g''-\mathrm{Re}(f'g-fg')+2K_{r}f'=0 \tag{11}$$

and the non-dimensional quantities are defined:

$$\mathrm{Re}=\frac{ah^{2}}{\nu},\quad K_{r}=\frac{\Omega h^{2}}{\nu} \tag{12}$$

Equation (9) with the help of eq. (10) can be written:

$$f'''-\mathrm{Re}\left(f'^{2}-ff''\right)-2K_{r}^{2}g=A \tag{13}$$

Differentiation of eq. (13) with respect to η gives:

$$f^{iv}-\mathrm{Re}\left(f'f''-ff'''\right)-2K_{r}^{2}g'=0 \tag{14}$$

Therefore, the governing equations and boundary conditions for this case in non-dimensional form are given by:

$$f^{iv}-\mathrm{Re}\left(f'f''-ff'''\right)-2K_{r}^{2}g'=0 \tag{15}$$

$$g''-\mathrm{Re}(f'g-fg')+2K_{r}f'=0 \tag{16}$$

Also eqs. (5) and (6) turn to:

$$\theta''+\mathrm{Pr}\,\mathrm{Re}\,f\theta'+Nb\phi'\theta'+Nt\theta'^{2}=0 \tag{17}$$

$$\phi''+\mathrm{Re}\,\mathrm{Sc}f\phi'+\frac{Nt}{Nb}\theta''=0 \tag{18}$$

With these boundary conditions:

$$\begin{aligned}&\eta=0\ \rightarrow\ f=0,\ \ f'=1,\ \ g=0,\ \ \theta=1,\ \ \phi=1\\&\eta=1\ \rightarrow\ f=\lambda,\ \ f'=0,\ \ g=0,\ \ \theta=0,\ \ \phi=0\end{aligned} \tag{19}$$

Other non-dimensional quantities are defined:

$$\begin{gathered}\lambda=\frac{v_{0}}{ah},\quad \mathrm{Pr}=\frac{\mu}{\rho_{f}\alpha},\quad \mathrm{Sc}=\frac{\mu}{\rho_{f}D}\\ Nb=(\rho c)_{p}D_{B}\frac{C_{h}}{(\rho c)_{f}\alpha},\quad Nt=(\rho c)_{p}D_{T}\frac{T_{h}}{(\rho c)_{f}\alpha T_{c}}\end{gathered} \tag{20}$$

Skin friction coefficient, C_{f}, along the stretching wall and Nusselt number along the stretching wall are defined:

$$\tilde{C}_{f}=\frac{\mathrm{Re}\,x}{h}C_{f}=f''(0),\quad \mathrm{Nu}=-\theta'(0) \tag{21}$$

Least square method

If the continuous summation of all the squared residuals is minimized, the rationale behind the name can be seen. In other words, a minimum of [24]

$$S = \int R(x)R(x)\mathrm{d}x = \int R^2(x)\mathrm{d}x \tag{22}$$

In order to achieve a minimum of this scalar function, the derivatives of S with respect to all the unknown parameters must be zero. That is:

$$\frac{\partial S}{\partial c_i} = 2\int R(x)\frac{\partial R}{\partial c_i}\mathrm{d}x = 0 \tag{23}$$

Comparing with eq. (22), the weight functions are seen to be [25]

$$W_i = k\frac{\partial R}{\partial c_i}, \quad k = 2 \tag{24}$$

However, the coefficient k can be dropped, since it cancels out in the equation. Therefore, the weight functions for the LSM are just the derivatives of the residual with respect to the unknown constants

$$W_i = \frac{\partial R}{\partial c_i} \tag{25}$$

Results and discussion

In the present paper LSM is applied to obtain an approximate analytical solution of two phases modeling of nanofluid in a rotating system with permeable sheet. A comparison between the LSM and numerical method is investigated. For this aim eqs. (16)-(18) are solved by LSM and comparison with numerical method (NUM) is demonstrated in fig. 2. Presented curves in figs. 2 and 3 confirm that LSM is an accurate and convenient method for solving such nanofluid flow.

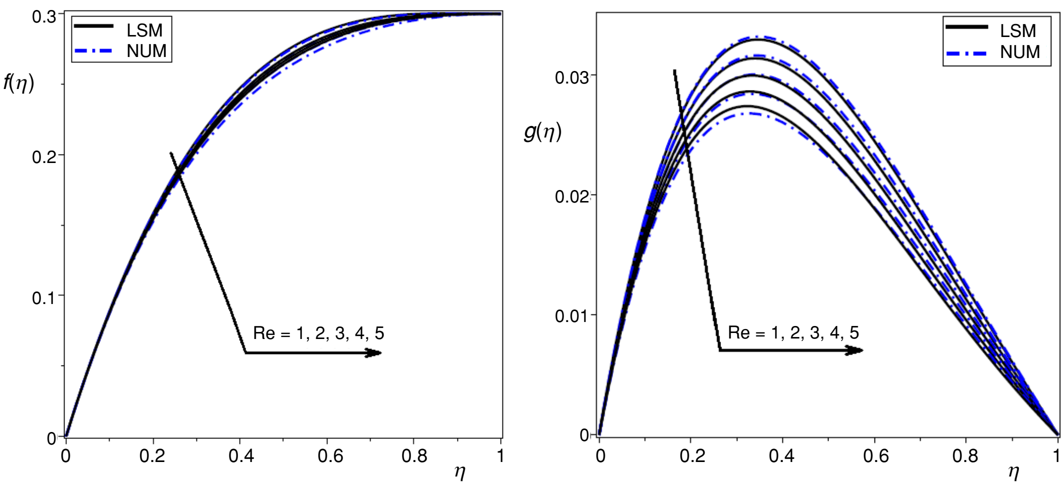

Figure 2. Comparison of velocity profiles (f, g) between LSM and NUM when λ = 0.3, Nt = Nb = 0.1, Pr = 10, Sc = 0.5, and K_r = 0.5 for Re = 1, 2, 3, 4, and 5

The presence of nanoparticles in a fluid (nanofluid) results in the change of physical properties such as thermal conductivity and specific heat that are functions of the nanoparticles concentration. So we can have result as follows.

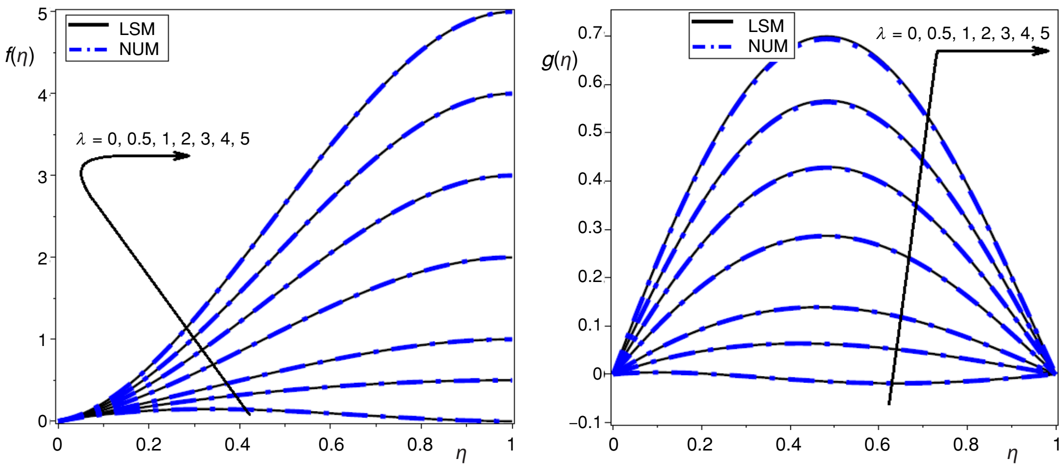

Figure 3. Comparison of velocity profiles (*f*, *g*) between LSM and NUM when Re = 0.1, *Nt* = *Nb* = 0.1, Pr = 10, Sc = 0.5, and K_r = 0.5 for λ = 0, 0.5, 1, 2, 3, 4, and 5

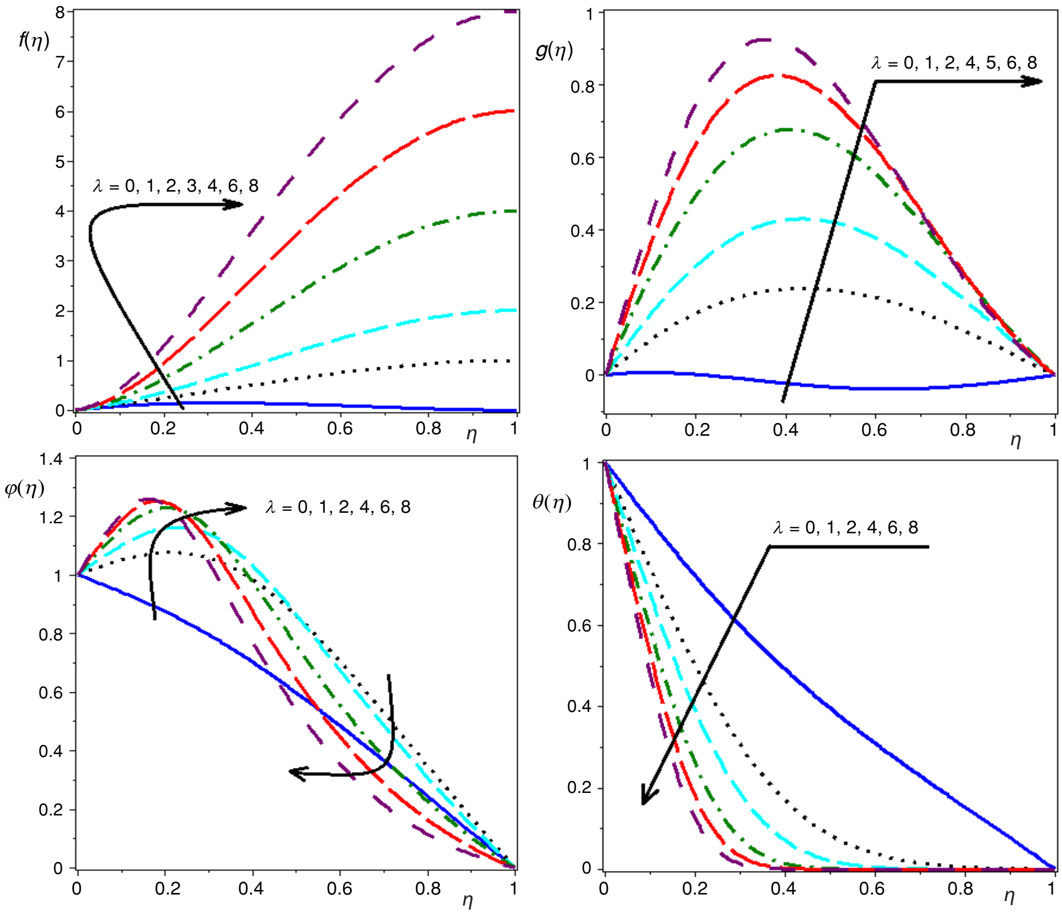

Figure 4. The effect of injection parameter on velocity profiles (*f*, *g*), temperature profile (θ), and concentration profile (φ) when Re = 1, K_r = 1, Sc = 0.5, *Nb* = *Nt* = 0.1, and Pr = 10

Effect of injection parameter on velocity, temperature, concentration profiles is shown in fig. 4. As injection parameter increases, velocity profiles increase. Temperature profile decreases with increase of injection parameter while opposite trend is observed for concentration profile, it physically means than when the temperature profile decrease, the temperature gradient increases so we can have better heat transfer performance in this manner.

Figure 5 shows the effects of Reynolds number on velocity, temperature, and concentration profiles. It is worth to mention that the Reynolds number indicates the relative significance of the inertia effect compared to the viscous effect. Thus, both velocity and temperature boundary-layer thicknesses decrease with the increase of Reynolds number and in turn increasing Reynolds number leads to the increase in the magnitude of the skin friction coefficient and Nusselt number also in physics of boundary-layer it is obvious that by decreasing the temperature boundary-layer thickness lead to increasing the temperature gradient so we can have the more heat transfer rate in performance. It can be seen that concentration profile increases with augment of Reynolds number. Also it can be seen that Nusselt number increases with the increase of injection parameter. With increasing rotation parameter, the transverse velocity increases. As rotation parameter increases thermal boundary-layer thickness decreases and in turn Nusselt number increases with the increase of K_r.

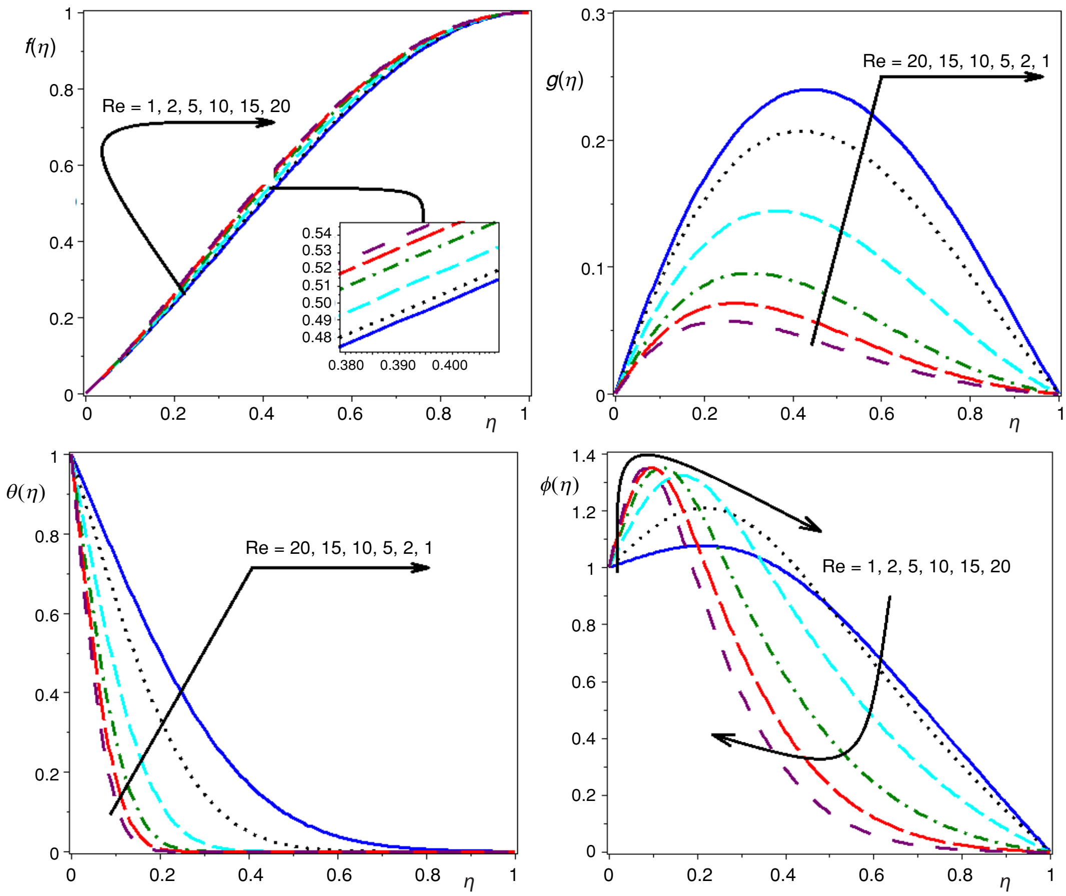

Figure 5. The effect of Reynolds number on velocity profiles (*f*, *g*), temperature profile (*θ*) and concentration profile (*φ*) when *λ* = 1, K_r = 1, Sc = 0.5, *Nb* = *Nt* = 0.1, and Pr = 10

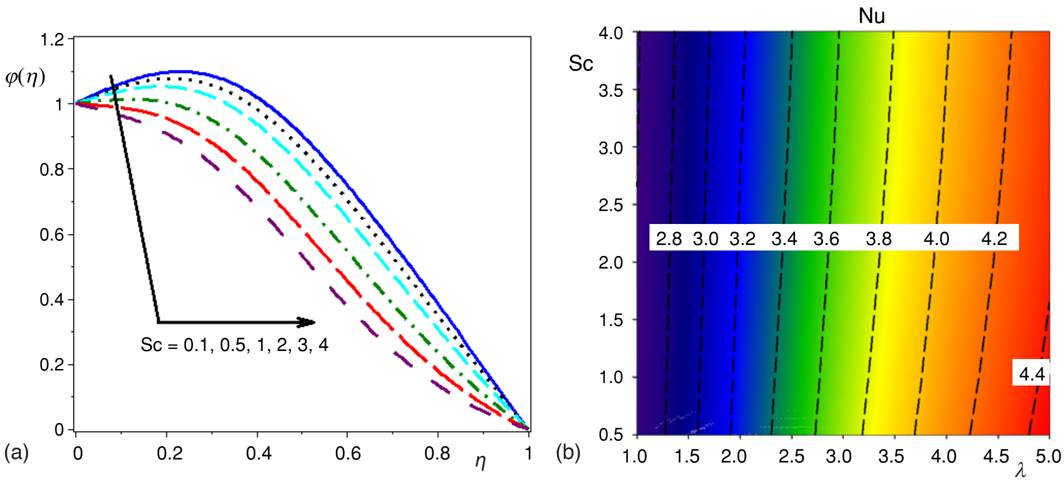

Figure 6. The effect of Schmidt number on concentration profile and Nusselt number when; (a) Re = 1, λ = 1, K_r = 0.5, Nb = Nt = 0.1, and Pr = 10; (b) Re = 1, K_r = 1, Nb = Nt = 0.1, and Pr = 10

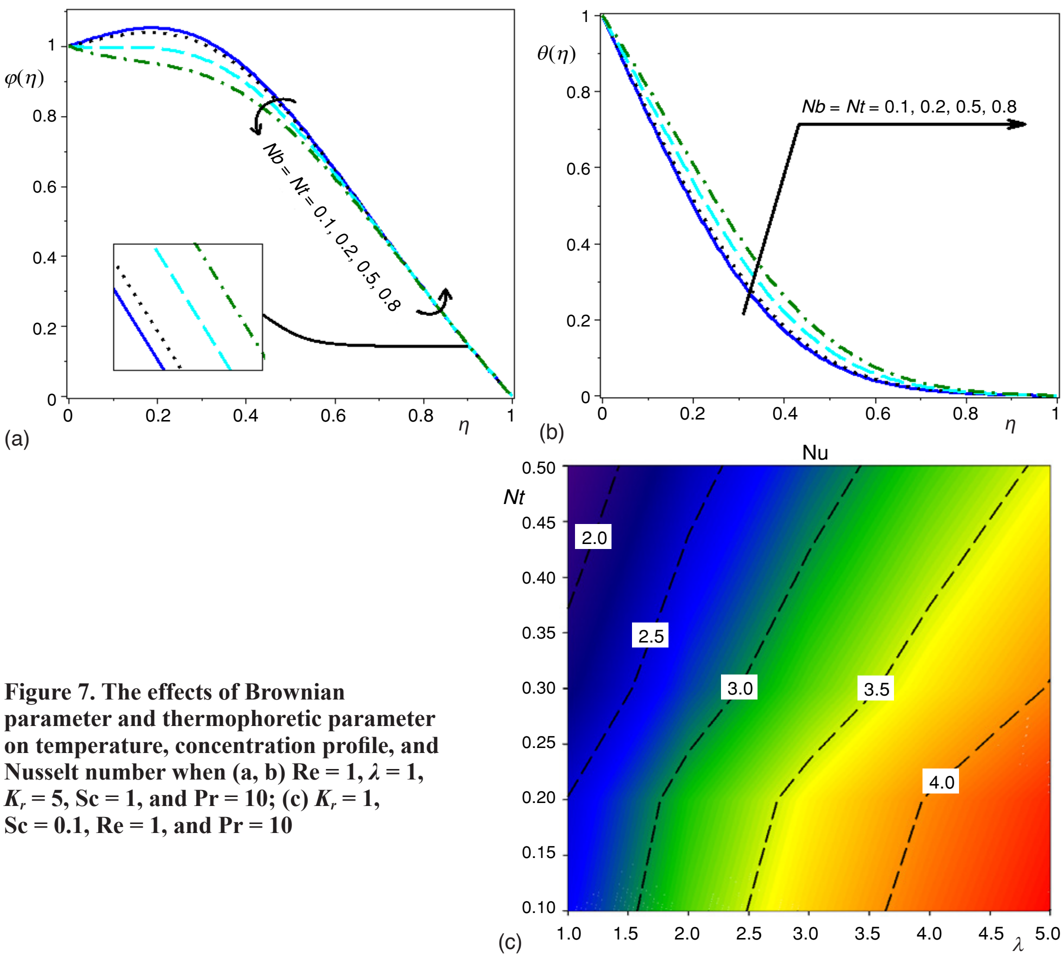

Figure 7. The effects of Brownian parameter and thermophoretic parameter on temperature, concentration profile, and Nusselt number when (a, b) Re = 1, λ = 1, K_r = 5, Sc = 1, and Pr = 10; (c) K_r = 1, Sc = 0.1, Re = 1, and Pr = 10

Figure 6 shows the effect of Schmidt number on concentration profile and Nusselt number. Schmidt number is a dimensionless number defined as the ratio of momentum diffusivity (viscosity) and mass diffusivity. Concentration profile decreases as Schmidt number increases. Also it can be concluded that increasing Schmidt number causes a slight decrease in the rate of heat transfer.

Effects of Brownian parameter and thermophoretic parameter on temperature, concentration profiles, and Nusselt number are shown in fig. 7. These active parameters have similar effects on heat and mass transfer characteristics. It means that temperature boundary-layer thickness increases with their increase while opposite trend is observed for concentration boundary-layer thickness. Nusselt number is a decreasing function of thermophoretic parameter and Brownian parameter. According to previous explanation it is obvious that LSM is a powerful method that can solve such complicated problem especially for high order of Reynolds number and λ number.

Conclusion

The aim of this paper is to employ the LSM to study the strongly non-linear four coupled differential equations that arises from nanofluid flow and heat transfer between two horizontal parallel plates in which plates rotate together. By using the appropriate transformation for the velocity, temperature and concentration [27], the basic equations governing the flow, heat and mass transfer were reduced to a set of ODE. The current method was applied without any discretization, restrictive assumptions, or transformation and is free from the round-off errors. Further, this technique can be used to develop valid solutions even to problems that are highly non-linear and may be considered as an important and significant refinement of the formerly developed methods. As the results are compared with NUM, it is clear that LSM has a good agreement with NUM and provides highly accurate analytical solutions for non-linear problems and markedly reducing the extent of calculations required. Moreover, effects of active parameters on flow, heat and mass transfer are considered. The important effects of Brownian motion and thermophoresis have been included in the model of nanofluid. The results show that concentration boundary-layer thickness decreases with the increase of thermophoretic parameter and Brownian parameter.

Nomenclature

C – nanofluid concentration
C_f, $\tilde{C}_f$ – skin friction coefficients
C_p – specific heat at constant pressure
D_B – diffusion coefficient
D_T – thermophoresis coefficient
h – distance between the place
K_r – rotation parameter
Nb – Brownian motion parameter
Nt – thermophoretic parameter
Nu – Nusselt number
P^* – modified fluid pressure
Pr – Prandtl number
Re – Reynolds number
Sc – Schmidt number
T – temperature

Greek symbols

α – thermal diffusivity
θ – dimensionless temperature
λ – dimensionless suction/injection parameter
μ – dynamic viscosity
ν – kinematic viscosity
ρ – fluid density
φ – dimensionless concentration
Ω – constant rotational velocity

Subscripts

f – base fluid
h – hot
o – cold
p – nanoparticle

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