The fluid-flow and heat transfer in a buoyancy-driven microcavity heated from below are numerically investigated. In spite of the fact that microcavities are widely used in micro-electro-mechanical systems, now a day, more interest in the evacuated cavity on Eqn. solar collectors are very common to reduce heat loss from the system. This paper provides a useful information for engineers to estimate heat transfer in low pressure cavities. The finite volume technique was used to solve the governing equations along with temperature jump and slip flow boundary conditions. The simulations are carried out for various cavity aspect ratios (H/L) and different Rayleigh number for both macro- and micro-fluids. The effect of Knudsen number in the rarefied flow regime (micro-fluidic) has also been investigated. It is shown that for both cases the effect of aspect ratios on heat transfer becomes significant at high Rayleigh numbers and when the aspect ratio is below 5. It was also found that increasing Knudsen number reduces the heat transfer. The interaction between Nusselt, Rayleigh, Knudsen numbers, and the aspect ratio was investigated using the design of experiments, results show that no interaction between these parameters. To help engineers to estimate heat transfer in low pressure cavities, widely used in solar energy applications, a correlation for convection heat transfer coefficient is introduced.

Key words: horizontal microcavity, slip flow, temperature jump, Knudsen number.

Introduction

The recent trends of moving towards increasing the share of solar energy in the energy mix have accelerated new technological developments in the field. Due to the need of the presence of transport walls to allow solar radiation to enter the space, the use of conventional insulation materials to minimize the heat loss is impossible. Furthermore, the good understanding of heat transfer and fluid-flow in micro-electro-mechanical systems (MEMS) is the key point of producing high quality and low-cost MEMS.

Natural convection in cavities at atmospheric pressure or higher of various boundary conditions has been extensively studied and documented. Naffouti and Djebali [1] solved dimensionless governing equations to find heat transfer rates by lattice Boltzmann method. This was done considering a 2-D natural convection flow in a square enclosure containing isothermal...
hot source placed asymmetrically at the bottom wall. Results were analyzed and documented [1].

Globe and Dropkin [2] presented an experimental investigation of convection heat transfer between two horizontal plates filled with liquids heated from below. The heat transfer coefficients, as confirmed by the results for all investigated liquids, may be determined from eq. 

\[
\text{Nu} = 0.069 \text{Ra}^{0.2} \text{Pr}^{0.74}
\]

(1)

where the characteristic dimensions for the Nusselt and Rayleigh numbers were the spacing between the copper plates.

Numerical investigation of rarefied gas inside microcavities of a wavy micro-channel flow was done by Alshare et al. [3]. Authors used Navier Stokes Fourier (NSF) equation with temperature jump and velocity slip conditions. They found that the amplitude of the wavy channel affects the heat transfer rates. Al-Kouz et al. [4] used the same formulation in solving the steady 2-D, laminar, natural convective heat transfer for low pressure gaseous flows in the annulus region between two concentric horizontal cylinders. They investigated the effects of Rayleigh and Knudsen numbers on the flow and heat transfer characteristics in the annulus region. Alkhalidi et al. [5] investigated the buoyancy-driven heat transfer a conjugate cavity filled with rarefied gas. The simulations were carried out for different Rayleigh number, conductivity ratios, Knudsen number, and cavity tilt angles. However, they did not investigate in details the role of cavity aspect ratio.

Polikarpov et al. [6] studied heat transfer through the binary gas mixture, confined between two infinite parallel plates. Two monoatomic gas mixtures: Ne-Ar and He-Ar were investigated for transient behavior in heat transfer caused by the sudden change of the plates’ temperatures. Rovenskaya [7] investigated the use of non-linear Shakhov model of the Boltzmann kinetic equation to find the rarefied gas flow caused by a temperature gradient in the direction tangential to a wall through a planar channel of finite length.

Boiling and condensation had been investigated in microcavity by Zhou et al. [8], They numerically investigated the heat transfer characteristics of the aqueous n-butanol solution in the thin film region of the closed microcavity, based on an enhanced Young-Laplace equation that includes the contribution of disjoining pressure for self-rewetting binary fluids. Li et al. [9] developed a micro-channel configuration that enables more efficient utilization of the coolant through integrating multiple micro-scale nozzles connected to auxiliary channels as well as micro-scale reentry cavities on sidewalls of main microchannels.

Tatsios et al. [10] used the non-linear Shakhov kinetic model and the direct simulation Monte-Carlo (DSMC) model to simulate the fluid-flow in a square cavity. He found that both models gave close results in the investigated ranges. Rana et al. [11] utilized the R13 equations and the NSF equations to investigate moderately rarefied gas-flow and heat transfer in square cavity heated from below. The aim of the investigation was to improve the understanding of the effect of slip flow on the thermal flow characteristics in early transition regime.

Benard problem was investigated for a rarefied gas in literature. For example, the instability of a stationary stratified gas in the 2-D rectangular domain was studied by Sone et al. [12] using numerical methods. The longtime behavior (final state) of the Rayleigh-Benard flow of a rarefied monatomic gas was investigated by Stefanov et al. [13]. The investigation was done for a set of the non-dimensional Knudsen and Froude numbers and results were documented. Stefanov et al. [14] investigated 3-D geometry with different aspect ratios Rayleigh-Benard flow problem, the investigation was for a set of different Froude and Knudsen numbers at a fixed temperature ratio. Although authors [10-14] investigated rarefied flow inside cavities, they did not investigate the heat transfer associated with natural convection inside the cavity. That is, their
main investigation was focused on fluid-flow and the temperature distribution inside the cavity under different boundary conditions.

The 2-D, steady analysis of buoyancy-driven flow in uniform wall temperature microcavity is investigated in this work. At the micro level, the surface effects become more important than the volume effects. This fact may lead to an enhancement of the thermal transport. The flow is assumed laminar. Numerical methods were used to solve the continuum governing equations along with Maxwell slip [15], and temperature jump [16] boundary conditions. The effects of the aspect ratio and Knudsen number on the Nusselt number is investigated with respect to a range of Rayleigh number.

Problem statement and solution methodology

The schematic diagram is shown in fig. 1 shows the geometry of the cavity considered in this study. It consists of a top isothermal cold wall, bottom isothermal hot wall, and insulated sidewalls.

The buoyancy-driven flow is assumed compressible (the density was calculated by Boussinesq approximation), steady, 2-D, and laminar, the thermo physical properties are assumed to be constant. The continuum governing equations are utilized in conjunction with the slip velocity and temperature jump boundary conditions. The continuity, momentum and energy equations governing the flow and heat transfer in the cavity under consideration in Cartesian co-ordinates are:

Continuity:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(2)

\[ \rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial P}{\partial x} + \mu \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \]  

(3)

\[ \rho \left( u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial P}{\partial y} + \mu \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right] \]  

(4)

\[ \rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] \]  

(5)

The boundary conditions for the problem are:

(i) the finite temperature at the outer surfaces of the bottom and top walls,
(ii) Colin [17] reported the slip-condition and temperature jump at the interfaces between the solid walls and the fluid inside the cavity:

- The velocity slip at all walls is:
  \[ u_s - u_w = \left( \frac{2 - \sigma_s}{\sigma_w} \right) \frac{\partial u}{\partial y} \]  \(\text{(8)}\)

- Since the walls are stationary (\(u_w = 0\)), therefore, eq. (8) becomes,
  \[ u_s = \left( \frac{2 - \sigma_s}{\sigma_w} \right) \frac{\partial u}{\partial y} \]  \(\text{(9)}\)

- The temperature jump condition is applied at all walls,
  \[ T_s - T_w = \left( \frac{2 - \sigma_s}{\sigma_w} \right) \frac{2\gamma}{\gamma + 1} \frac{k}{\mu c_v} \frac{\partial T}{\partial y} \]  \(\text{(10)}\)

- For the top wall, \(T = T_s\), Thus, eq. (8) becomes,
  \[ T_s - T_w = \left( \frac{2 - \sigma_s}{\sigma_w} \right) \frac{2\gamma}{\gamma + 1} \frac{k}{\mu c_v} \frac{\partial T}{\partial y} \]  \(\text{(11)}\)

- Similarly, the temperature of the gas at the bottom wall is given by,
  \[ T_s - T_w = \left( \frac{2 - \sigma_s}{\sigma_w} \right) \frac{2\gamma}{\gamma + 1} \frac{k}{\mu c_v} \frac{\partial T}{\partial y} \]  \(\text{(12)}\)

- The mean free path length is specified [18],
  \[ \lambda = \frac{k T}{\sqrt{2 \pi d}} \]  \(\text{(13)}\)

where, \(k_s = 1.38066 \times 10^{-2} [\text{JK}^{-1}]\) is the Boltzmann constant, \(T\) [K] the temperature, \(P\) [Pa] the pressure, and \(d\) – the molecular diameter. Knudsen number is defined:

\[ \text{Kn} = \frac{\lambda}{L} \]  \(\text{(14)}\)

The spacing between hot and cold walls, shown in fig. 1, is noted by \(L\). fig. 1(b) shows the boundary conditions. Fourier’s law of conduction was used to calculate the local heat flux at the bottom wall:

\[ q^÷ = -k_f \frac{\partial T}{\partial y} \bigg|_{y=0} \]  \(\text{(15)}\)

The problem solved in this paper is steady, therefore, the heat transfer was calculated by integrating the local heat flux along the bottom wall of the cavity:

\[ Q = \int q^÷ dA \]  \(\text{(16)}\)

The average heat transfer coefficient along the bottom wall of the cavity is calculated from:

\[ \overline{h} = \frac{Q}{(T_s - T_f) A} \]  \(\text{(17)}\)

The Nusselt number is calculated from:

\[ \text{Nu} = \frac{\overline{h} L}{k_f} \]  \(\text{(18)}\)
Solution method

The governing equations, dimensional-form, along with the boundary conditions are solved in primitive variables terms. Finite volume method was used to obtain the solution, by utilizing commercial, FLUENT 18, software. The CAD modeler and a meshing tool were used to create the computational domain, shown in fig. 1. For $H/L = 1$, the mesh was increased in sizes, $(40 \times 40), (50 \times 50) \ldots$ until $(100 \times 100)$ to select an optimal mesh size. Nusselt number was used to depict the effect of mesh. The effect was found marginal after mesh $(90 \times 90)$. To ensure the convergence of the results, a $(100 \times 100)$ mesh is used and presented. This mesh size is utilized for all simulations considered in this study.

Governing equations are discretized using second-order upwind scheme. The SIMPLE algorithm was used to tackle pressure-velocity decoupling, and the pressure discretization was performed via the PRESTO algorithm [18]. Simulations assumed totally diffuse walls, the momentum, and thermal accommodation coefficients, $\sigma$ and $\sigma_v$, respectively, are given a value of $v T$ one in all simulations. Knudsen number was varied within the slip flow regime in the range of $0.01$ to $0.1$. The values of the mean free path, $\lambda$, were calculated according to Knudsen number and $L$, see eq. (12).

The solution is considered to be converged when the mass and velocity residuals are $-3 -6$ less than $10^3$ and when the energy residual is less than $10^3$.

Model validation

Model validation was performed by reproducing experimental results found by Glope and Dropkin [2]. The current software code was able to predict the Nusselt number given by eq. (1) with a maximum error of approximately $11.5\%$ for the cases of $H/L = 5$ and above. More deviation in results between eq. (1) and CFD was found for cases with $H/L$ less than 5. This could be attributed to the fact that eq. (1) has conditions set by [2] that could be used only for high $H/L$ ratios. More investigation about aspect ratio effect will be done in the current study to find the threshold for eq.(1). It should be noted that at $Kn = 0$, the no-slip condition holds true, and this corresponds to the case of the microcavity. Model validation for slip and jump boundary conditions was done by the same authors and presented in [5]. It is concluded that the current simulation technique is capable of predicting the thermal performance within an acceptable accuracy.

Discussion of results

To establish a benchmark case for the no-slip boundary condition of the problem under consideration, the effect of changing the aspect ratio is studied and presented. To do so, three different aspect ratios, 3, 5, and 7 are analyzed. Figure 3 shows marginal effect of the aspect ratio on Nusselt number for geometries with $H/L = 5$, and 7 and about $14\%$ difference for the case of $H/L = 3$ compared to the case with $H/L = 5$, and 7 at $Ra = 3.66 \times 10^4$. This difference diminishes below $Ra = 1 \cdot 10^4$.

Furthermore, fig. 2 shows that below $Ra = 3.66 \times 10^4$, conduction is the dominant mode of heat transfer for all aspect ratios, while, the convection becomes more significant at higher Rayleigh numbers. The effect of the aspect ratio of the average Nusselt number was correlated and presented in eq. (19):

$$\overline{Nu} = 0.186 \, Ra^{1/3} \, (H/L)^{1/5}$$

(19)

where $Pr = 0.7$

$10^2 < Ra < 10^4$

$1 < (H/L) < 14$
Figure 3 shows the velocity streamlines inside cavities of different aspect ratios for the same Rayleigh number. It is clear that the number of rolling cells described by Glope and Dropkin [2] increases as the aspect ratio increases. These rolling cells characterize the advection in a horizontal fluid layer in a heated from below cavity. Results found in this work and presented in fig. 3, show a similar pattern to that of the rolling cells flow inside the cavity described by [2].

Moreover, it was found, in this work, that number and size of rolling cells have its fingerprint on the heat transfer in the cavity; the number and, therefore, the size of rolling cells differs for different aspect ratios and it is constant for all Rayleigh number range for the same aspect ratio. At high aspect ratios, a large number of rolling cells was noticed, reducing the aspect ratio reduces the number of rolling cells inside the cavity, see fig. 3. The number, size, and strength of rolling cells control the heat transfer inside the cavity; the heat transfer increases as the number of rolling cells increases as observed for $H/L = 3, 5,$ and $7$ which were discussed earlier. Secondly, for the same aspect ratios, Rayleigh number increases, the strength of the circulation of the rolling cell increases.

Figure 4 shows the velocity magnitude across a horizontal line located at the mid-plane of the cavity for different Rayleigh number numbers. It is clear that the increase of Rayleigh number did not have any influence on the number of the rolling cells, however, the intensity of the velocity increases in each rolling cell. Moreover, for $H/L = 5,$ and $7,$ the average Nusselt number is almost identical at $Ra = 3.66 \times 10^6,$ (see fig. 2) then it is expected to have stronger rolling cells for $H/L = 5$ than that for $H/L = 7$ case as noticed in fig. 5.

**Effect of Knudsen number**

Knudsen number was used to characterize rarefied, micro and nanoscale flows. Knudsen number measures the degree of rarefaction of the flow, it is described by the ratio of the mean free path, $\lambda,$ of the characteristic length, $L,$ of the geometry of interest. Flow regimes inside cavities are classified based on the Knudsen number into four types [19, 20]. The continuum regime, $Kn < 0.01,$ is in which the Navier-Stokes equations are used to describe the flow. The slip regime, $0.01 < Kn < 0.1,$ where Navier-Stokes equations are used to describe the flow but with velocity slip and temperature jump boundary conditions. The transitional regime range $0.1 < Kn < 10.$ Finally, the free molecular regime $10 < Kn,$ where the collision between particles is very rare. The boundary slip effect on heat transfer was investigated at various Knudsen number numbers varying from $Kn = 0,$ which represent the no-slip boundary condition, up to $Kn = 0.1,$ which cover the rarefied flow slip regime. As shown in fig. (6), a large reduction (up to 73%) in the heat transfer is noticed due to the effect of Knudsen number number.
To investigate the heat transfer inside the cavity at low pressure conditions (microfluidic), results are plotted in figs. 6–8, the heat transfer indicator was the average Nusselt number, and it was calculated based on the temperature difference between the top and the bottom surfaces of the cavity. The cavity height is used as the characteristic length.

Figure 6 shows the effect of Rayleigh number on the heat transfer for different Knudsen numbers. Varying Knudsen number between 0.01 and 0.1, a significant drop in heat transfer occurs. The rate at which Nusselt number decreases varies between 60 to 80% when compared to the macro-fluidic case, for which Kn = 0. Heat transfer reduction due to the increase in Knudsen number could be attributed to the boundary effects (velocity slip and temperature jump). Figure 7 shows that the temperature jump at the wall results in reducing the temperature of the gas inside the cavity. This will result in reducing the driving buoyancy force (which is due to the temperature difference across the cavity) to move the fluid inside the cavity.

Figure 8 shows that as Knudsen number increases, the velocity of the air across the cavity decreases, which will reduce the advection effects, and thus the convective heat transfer is decreased. Results presented in fig. 9 show the variation of average Nusselt number with Rayleigh number for different aspect ratios and the whole range of Knudsen number. It is clear...
that the effect of changing the aspect ratio of the Nusselt number is negligible, as it does not exceed 2% difference, for the case of $H/L = 3$ compared to $H/L = 7$ case. This effect was correlated in eq. (20):

$$\text{Nu} = 0.0777 \times Ra^{0.182} \times Kn^{0.404} \times H/L^{0.0177}$$  \hspace{1cm} (20)$$

where

$$\begin{cases} 
0.01 < Kn < 0.1 \\
10^1 < Ra < 10^7 \\
1 < H/L < 14
\end{cases}$$

As a special case of eq. (20), for the aspect ratio $H/L < 7$, the Nusselt number can be calculated using eq. (21):

$$\text{Nu} = 0.0846 \times Ra^{0.182} \times Kn^{3.404}$$  \hspace{1cm} (21)$$

where

$$\begin{cases} 
0.01 < Kn < 0.1 \\
10^1 < Ra < 10^7 \\
H/L < 7
\end{cases}$$

Rather than using the relation, it will be more convenient for engineers to use a simplified form for a special case of the low pressure cavity filled with air, eq. (14) was utilized to estimate Knudsen number. Using the following data $k_a = 1.38066 \times 10^{-25} [\text{JK}^{-1}]$ and $\sigma = 3.71 [\text{Å}]$, the convective heat transfer coefficient was correlated to the following:

$$h = 7.785 \left( \frac{T_c - T}{L_c} \right)^{0.134} \frac{P^{0.15}}{T_{avg}}$$  \hspace{1cm} (22)$$

where

$$\begin{cases} 
0 < \frac{T_{avg}}{T_L} < 4400
\end{cases}$$

where $P$ is in Pascal and $T$ is in Kelvin [21, 22].

Conclusions

The geometrical aspect ratios effect on heat transfer was investigated inside a no-slip boundary conditions cavity heated from below with insulated sidewalls. It was found that the change in the aspect ratio below $H/L = 5$ has a high effect on the Nusselt number, while it has a marginal effect for ($H/L \geq 5$). It was found that flow pattern inside cavities is represented by the number, size, and the strength of stable rolling cells affects the heat transfer inside the cavity. The micro-fluidic cavity has been investigated as well, and it was found that increasing Knudsen number in the rarefied flow regime reduces the heat transfer in the cavity for all aspect...
ratios due to the reduction in the velocity and temperature in the stable rolling cells. Furthermore, it was found that the effect of the aspect ratio is in general low, however, it becomes marginal by increasing Knudsen number. Correlation of Nusselt number among Rayleigh number, Knudsen number, and the aspect ratio is proposed. The correlation is valid for Prandtl number equals to 0.7 and within the investigated conditions, which reflects rarefied flow.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>area, [m$^2$]</td>
</tr>
<tr>
<td>$C_p$</td>
<td>specific heat capacity at constant pressure, [Jkg$^{-1}$K$^{-1}$]</td>
</tr>
<tr>
<td>$C_v$</td>
<td>specific heat capacity at constant volume, [Jkg$^{-1}$K$^{-1}$]</td>
</tr>
<tr>
<td>$d$</td>
<td>diameter of gas molecules, [m]</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration, [m/s$^2$]</td>
</tr>
<tr>
<td>$h$</td>
<td>average heat transfer coefficient, [Wm$^{-2}$K$^{-1}$]</td>
</tr>
<tr>
<td>$k_n$</td>
<td>Knudsen number ($= \lambda/L$), [-]</td>
</tr>
<tr>
<td>$k_b$</td>
<td>Boltzmann constant, [JK$^{-1}$]</td>
</tr>
<tr>
<td>$k_t$</td>
<td>thermal conductivity of air, [Wm$^{-1}$K$^{-1}$]</td>
</tr>
<tr>
<td>$L$</td>
<td>length of cavity of the computational domain, [m]</td>
</tr>
<tr>
<td>$Nu$</td>
<td>average Nusselt number ($= hL/k_b$), [-]</td>
</tr>
<tr>
<td>$P$</td>
<td>pressure, [Pa]</td>
</tr>
<tr>
<td>$Pr$</td>
<td>prandtl number ($= \nu/\alpha$), [-]</td>
</tr>
<tr>
<td>$Q$</td>
<td>heat transfer rate, [W]</td>
</tr>
<tr>
<td>$R$</td>
<td>universal gas constant</td>
</tr>
<tr>
<td>$Ra$</td>
<td>Rayleigh number ($= g\beta TLc^3/\nu \alpha$), [-]</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature, [K]</td>
</tr>
<tr>
<td>$T_{av}$</td>
<td>average temperature ($= (T_s + T_t)/2$), [K]</td>
</tr>
<tr>
<td>$T_c$</td>
<td>cavity wall temperature, [K]</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity in $x$-axis, [m/s]</td>
</tr>
<tr>
<td>$v$</td>
<td>velocity in $y$-axis, [m/s]</td>
</tr>
<tr>
<td>$x$</td>
<td>axial co-ordinate, [m]</td>
</tr>
<tr>
<td>$y$</td>
<td>vertical co-ordinate, [m]</td>
</tr>
<tr>
<td>$\beta$</td>
<td>thermal expansion, [1K$^{-1}$]</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>ratio of the specific heat ($= c/p\alpha$), [-]</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>molecular mean free path, [m]</td>
</tr>
<tr>
<td>$\nu$</td>
<td>dinamic viscosity, [kgm$^{-1}$s$^{-1}$]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>kinematic viscosity, [m$^2$s$^{-1}$]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of air, given by ideal gas equation ($= P/R\gamma$), [kg/m$^3$]</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Lennard-Jones characteristic length, [Å]</td>
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<tr>
<td>$\sigma_f$</td>
<td>thermal accommodation coefficient</td>
</tr>
<tr>
<td>$\sigma_v$</td>
<td>momentum accommodation coefficient</td>
</tr>
</tbody>
</table>

Acronyms

- DSMC: direct simulation Monte-Carlo
- NSF: naviere Stokes equation

References


[18] ****, Ansys, Fluent Documentation.


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