

THE STUDY OF NATURAL CONVECTION HEAT TRANSFER OF NANOFLUIDS IN A PARTIALLY POROUS CAVITY BASED ON LATTICE BOLTZMANN METHOD

by

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This article used the lattice Boltzmann method to study the heat transmission of natural convective of nanofluids in a 2-D square cavity partially filled with porous medium. The nanoparticles volume fraction of Al_2O_3 , Cu, and SiO_2 were 0.5%, 1%, 1.5%, 2%, 3%, and 4%, which were mixed with water and 70% of ethylene glycol aqueous solution as the base fluid, and made up six kinds of nanofluids as the research object. Using nanofluids coupled double distribution lattice Boltzmann method model, this paper studied the rules of natural convection heat transfer of different nanofluids with the changing of Rayleigh number and the concentration of the nanoparticles in the 2-D square cavity partially filled with porous medium. The results showed that the average Nusselt number of the hot wall will increase with the increase of Rayleigh number number, and under different heat transfer conditions, there are two different critical Rayleigh numbers. In the case of different concentrations of the same concentration, the critical Rayleigh number is about 10^5 , when $Ra > Ra_c$, the average Nusselt number of water is higher; when $Ra < Ra_c$, the average Nusselt number of, the average Nusselt number of ethylene glycol is higher. In the case of different concentrations of same particles, the critical Rayleigh number is between 10^5 and 10^6 , there was also a critical Rayleigh number ($Ra_c = 10^5$), when the $Ra < Ra_c$, the average Nusselt number of the hot wall will increase with the increasing of concentration; when the $Ra > Ra_c$, there is a slight decreasing in the average Nusselt number with the increasing of concentration. The critical Rayleigh number of water as the base fluid is smaller than that of ethylene glycol as the base fluid.

Key words: porous medium, natural convention, lattice Boltzmann method, nanofluids

Introduction

A porous medium is a multiphase, multifunctional material composed of a solid substance with tiny voids in its skeleton. Such a medium has many outstanding features, such as low density, high porosity, and large specific surface area [1]. Nanofluids have rapidly devel-

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oped into a new type of heat transfer working medium because of their high heat conductivity coefficient and extensive application prospects in strengthening heat transfer [2]. Studies on the heat transfer mechanism of fluids in porous media generally focus on two aspects: the properties of the porous medium frame and nanofluids. Li *et al.* [3] built a test bench to study solid-liquid phase changes strengthening heat transfer in metal foam using paraffin as the phase change material and systematically studied the effect of foam porosity and pore density on temperature change. Results showed that high porosity can accelerate the speed of interface movement. Nnanna *et al.* [4] experiment confirmed that adding nanoparticles in solution can enhance the heat transfer characteristics of the liquid and that the increase in the volume fraction of nanoparticles weakens the effect of nanofluid heat exchange to a certain extent. Li *et al.* [5] analysed the effect of nanoparticles on the heat transfer characteristics of natural convection in a closed cavity by numerical simulation method. Results showed that adding Cu nanoparticles to aqueous liquid can strengthen the heat transfer characteristics of the natural convection of the base solution. The velocity component of the nanofluids also increases with increasing volume fraction, thereby accelerating the energy transmission in the fluid. Khanafer *et al.* [6] numerically studied the heat transfer characteristics of Cu-water nanofluids in rectangular vessels and found that the heat transfer characteristics of natural convection can be improved by increasing the volume fraction of the Cu nanofluids.

Complex porous medium structures cause great complexity in analyzing internal flows. The lattice Boltzmann method (LBM) can accurately obtain the Navier-Stokes (N-S) equations. This equation can solve the real pore geometric structure during treatment of complicated pore geometries without simplifying or modifying the complex pore structure. Thus, LBM has become a very competitive numerical calculation method in studying fluid-flows in porous media [7]. Reports on the use of LBM in simulations of the natural convection heat transfer in nanofluids in a cavity partially filled with a porous medium are rare. In this paper, the mixture of Al_2O_3 -water, Al_2O_3 -ethylene glycol, Cu-water, Cu-ethylene glycol, SiO_2 -water, and SiO_2 -ethylene glycol are selected as study materials, tab. 1 and the coupling double distribution LBM model proposed by Guo and Zheng [8] is employed. Heat transfer in the natural convection of different nanofluids in a 2-D square cavity partially filled with porous media is simulated. The average Nusselt numbers are obtained, and the effects of different Rayleigh number values and volume fractions of nanoparticles on the nanofluids are studied.

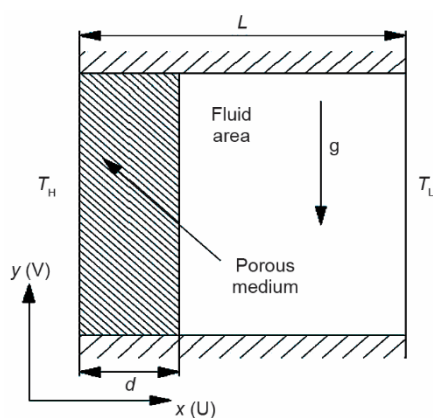


Figure 1. Schematic of the physical model of enclosure cavity

Physical model and governing equations

Physical model

In this paper, the physical model is a 2-D square cavity of length, L , that partially filled with vertical porous layer, as shown in fig. 1. For the model, the vertical surfaces are held at constant temperature, T_L and T_H , respectively. The two horizontal walls are adiabatic. The actual width of the porous layer is d , which is in the left of the cavity, and the dimensionless width of the layer is $D(d/L)$. In this paper, d is 20 units. The origin of co-ordinate system is the lower left corner of the cavity. The

model takes horizontal direction as the x -direction and the opposite direction of gravity as the y -direction.

Table 1. The physical parameters of selected material

Material	Water	Ethylene glycol aqueous solution	Al ₂ O ₃	Cu	SiO ₂
ρ [kgm ⁻³]	997.1	1093.24	3970	8933	2200
C_p [J ⁻¹ kg ⁻¹ K ⁻¹]	4179	2927	765	385	703
K [Wmk ⁻¹]	0.613	0.327	25	400	1.2
$\beta \cdot 10^{-5}$ [K ⁻¹]	21	21	0.85	1.67	0.65

The initial conditions and boundary conditions are $t = 0$, $u = v = 0$, $T = 0$, $x = 0$, $u = v = 0$, $T = T_H$, $x = L$, $u = v = 0$, $T = T_L$, $y = 0$, $u = v = 0$, $\partial T/\partial n = 0$, $y = L$, $u = v = 0$, $\partial T/\partial n = 0$.

Calculation of properties of nanofluids

The volume fraction of Al₂O₃-water and Al₂O₃-ethylene glycol, Cu-water and Cu-ethylene glycol, SiO₂-water and SiO₂-ethylene glycol nanofluids this paper chose were 0.5%, 1%, 1.5%, 2%, 3%, and 4%.

The density of nanofluids is expressed [9]:

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_p \quad (1)$$

where φ is the volume fraction of nanofluids, ρ_f – the density of base fluids, and ρ_p – the density of nanoparticle.

The capacity of nanofluids is expressed [9]:

$$\rho_{nf} C_{p,nf} = (1 - \varphi)\rho_f C_{p,f} + \varphi\rho_p C_{p,p} \quad (2)$$

where $C_{p,f}$ and $C_{p,p}$ are the capacities of base fluids and nanoparticle.

The viscosity of nanofluids is expressed [10]:

$$\mu_{nf} = \mu_f (1 + 39.11\varphi + 533.9\varphi^2) \quad (3)$$

where μ_f is the viscosity of base fluids.

The coefficient of thermal conductivity of nanofluids is expressed as [10]:

$$\frac{k_{nf}}{k_f} = \frac{k_p + 2k_f - 2\varphi(k_f - k_p)}{k_p + 2k_f + \varphi(k_f - k_p)} \quad (4)$$

where k_p and k_f are the thermal conductivity of nanoparticle and base fluids.

The thermal diffusion coefficient of nanofluids is expressed [10]:

$$a_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (5)$$

Governing equations

In this paper we studied the heat transmission of unsteady natural convective of nanofluids in a 2-D square cavity partially filled with porous medium, so we assume that the

configuration of porous media in the cavity is homogeneous, rigid and isotropic. We also assume that the nanofluids are considered as two different homogeneous components of single-phase medium mixed and the fluid is incompressible and viscous fluid flow can be described by Brinkman-Forchheimer model and it meet the Boussinesq hypothesis [11]. This hypothesis consists of three parts: the viscous heat dissipation in the flow is negligible, other parameters except density is constant, and for density, the density of the rest items is constant, considering only the volume dependent force in the momentum equation. At this point, the flow continuity equation and Brinkman-Forchheimer equation can be written:

$$\nabla \mathbf{u} = 0 \quad (6)$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \left(\frac{\mathbf{u}}{\varepsilon} \right) = -\frac{1}{\rho} \nabla(\varepsilon p) + \nu_e \nabla^2 \mathbf{u} + \mathbf{F} \quad (7)$$

where ε is the porosity of porous media, ρ – the density of fluid, \mathbf{u} and p – the average speed of fluid volume and pressure, respectively, ν_e – the effective kinematic viscosity coefficient, and \mathbf{F} – the total body force due to the presence of porous media and other external force fields, which can be written:

$$\mathbf{F} = -\frac{\varepsilon \nu}{K} \mathbf{u} - \frac{\varepsilon F_\varepsilon}{\sqrt{K}} |\mathbf{u}| \mathbf{u} + \varepsilon \mathbf{G} \quad (8)$$

On the right side of eq. (8), the first item is the frictional resistance of fluid and porous media skeleton, the second one is the inertia due to the presence of porous medium, ν – the kinematic viscosity of the fluid, K and F_ε represent permeability and geometric function, respectively, and \mathbf{G} is the volume force caused by external forces. If \mathbf{G} is caused only by gravity, the influence of gravity can be expressed as eq. (9) under the Boussinesq hypothesis:

$$\mathbf{G} = -g\beta(T - T_m) \quad (9)$$

where g is the gravitational acceleration, β – the thermal expansion coefficient, T_m – the average temperature of the system. The geometric function, F_ε , and the permeability, K , have relationship with the porosity, ε , respectively. For the porous media that is made of solid particles, F_ε and K can be expressed based on Ergun's empirical formula [9]:

$$F_\varepsilon = \frac{1.75}{\sqrt{150\varepsilon^3}}, \quad K = \frac{\varepsilon^3 d_p^2}{150(1-\varepsilon)^2} \quad (10)$$

where d_p is the diameter of the solid particles. In eq. (7), the generalized N-S eq. (7) will be degraded to the standard N-S equation when the porosity $\varepsilon \rightarrow 1$, namely, in the absence of the porous media.

The heat transfer problem always involves in fluid-flow in actual applications. If we ignore the compression work and viscous heat dissipation, it can meet local thermodynamic equilibrium condition between the fluid and solid, and then the energy equation of convection heat transfer in the porous media can be expressed:

$$\sigma \frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \nabla(\alpha_m \nabla T) \quad (11)$$

where T is the average volume temperature of fluid. The formula $\sigma = \varepsilon + (1-\varepsilon)\rho_s c_{ps} / \rho_f c_{pf}$ represents the ratio between the heat capacities of the solid and fluid phase, ρ_s , ρ_f , c_{ps} , and c_{pf} ,

are the density and capacity of the solid and fluid phase, respectively, α_m denotes the effective thermal diffusivity.

In order to represent the characters of natural convection heat transfer in porous media, we can introduce several dimensionless numbers: the Darcy number, $Da = K/L^2$, the viscosity ratio, $J = \nu_e/\nu$, the Prandtl number, $Pr = \nu/\alpha_m$, and the Rayleigh number $Ra = g\beta\Delta TL^3/\nu/\alpha_m$, where L is the cavity length, $\Delta T = T_H - T_L$ – the temperature difference between the hot and cold side walls.

Lattice Boltzmann model and boundary condition

Nanofluids double distribution of lattice Boltzmann model

For the natural convection heat transfer problem of nanofluids in partial filled with porous media cavity in this paper, we use the double distribution function model to study the fluid flow field and temperature field. Meanwhile, the D2Q9 model is employed and the lattice Boltzmann evolution equations [11] can be expressed:

$$f_i(\mathbf{x} + \mathbf{e}_i\delta t, t + \delta_t) - f_i(\mathbf{x}, t) = -\frac{f_i(\mathbf{x}, t) - f_i^{\text{eq}}(\mathbf{x}, t)}{\tau_v} + \delta_t F_i \quad (12)$$

$$g_i(\mathbf{x} + \mathbf{e}_i\delta t, t + \delta_t) - g_i(\mathbf{x}, t) = -\frac{g_i(\mathbf{x}, t) - g_i^{\text{eq}}(\mathbf{x}, t)}{\tau_t} \quad (13)$$

where $i = 0 \sim 8$, f_i the distribution function, f_i^{eq} – the corresponding equilibrium distribution function, g_i – the temperature distribution function, g_i^{eq} – the equilibrium temperature distribution function, τ_v and τ_t are the velocity non-dimensional relaxation time and the temperature relaxation time, respectively. Equation (12) recovers the continuity and the momentum eqs. (6) and (7). Equation (13) describes the evolution of the internal energy and leads to eq. (11).

Usually the speed configuration of D2Q9 model is defined:

$$\mathbf{e}_i = \begin{cases} (0, 0) & i = 0 \\ c \left\{ \cos \left[(i-1) \frac{\pi}{2} \right], \sin \left[(i-1) \frac{\pi}{2} \right] \right\} & i = 1, 2, 3, 4 \\ \sqrt{2} \left\{ \cos \left[(2i-1) \frac{\pi}{4} \right], \sin \left[(2i-1) \frac{\pi}{4} \right] \right\} & i = 5, 6, 7, 8 \end{cases} \quad (14)$$

where lattice speed $c = \delta_x/\delta_t$, δ_x and δ_t are time step and the grid step, respectively. Generally the grid spaces on the directions of x and y are the same $\delta_x = \delta_y$.

On the basis of the continuous Boltzmann equation, we can get the equilibrium distribution function according to discrete the time and space. It is defined:

$$f_i^{\text{eq}} = \omega_i \rho \left[1 + \frac{\mathbf{e}_i \mathbf{u}}{c_s^2} + \frac{\mathbf{u} \mathbf{u} : (\mathbf{e}_i \mathbf{e}_i - c_s^2 \mathbf{I})}{2\epsilon c_s^4} \right] \quad (15)$$

$$g_i^{\text{eq}} = \omega_i T \left(\sigma + \frac{\mathbf{e}_i \mathbf{u}}{c_s^2} \right) \quad (16)$$

where $c_s = c/\sqrt{3}$ is the speed of sound and the values of the weight are given by $\omega_0 = 4/9$, $\omega_i = 1/9 (i=1 \sim 4)$, $\omega_i = 1/36 (i=5 \sim 8)$.

In eq. (12), the forcing term can be given by:

$$F_i = \omega_i \rho \left(1 - \frac{1}{2\tau_v} \right) \left[\frac{\mathbf{e}_i \mathbf{F}}{c_s^2} + \frac{\mathbf{u} \mathbf{F} : (\mathbf{e}_i \mathbf{e}_i - c_s^2 \mathbf{I})}{\varepsilon c_s^4} \right] \quad (17)$$

The corresponding effective viscosity and the effective thermal conductivity in macro equation are given by:

$$\nu_e = c_s^2 \left(\tau_v - \frac{1}{2} \right) \delta_t, \quad \alpha_m = \sigma c_s^2 \left(\tau_t - \frac{1}{2} \right) \delta_t \quad (18)$$

The macroscopic quantities, fluid density and internal energy are defined:

$$\rho = \sum_i f_i, \quad T = \sum_i \frac{g_i}{\sigma} \quad (19)$$

The speed of the fluid is calculated by using a temporary speed, which can be written:

$$\mathbf{u} = \frac{\mathbf{v}}{c_0 + \sqrt{c_0^2 + c_1 |\mathbf{v}|}} \quad (20)$$

where parameters c_0 and c_1 are given by:

$$c_0 = \frac{1}{2} \left(1 + \varepsilon \frac{\delta_t}{K} \frac{\nu}{K} \right), \quad c_1 = \varepsilon \frac{\delta_t}{2} \frac{F_\varepsilon}{\sqrt{K}} \quad (21)$$

Both the equilibrium distribution of lattice Boltzmann model and the forcing term contain porosity. When the porosity is equal to one, they can become the standard form. Therefore, in this model, not only the effect of porous media but also the feature of free flow of fluid are considered. Both strictly obey the momentum transfer in different regions. The interface between porous medium and free fluid can automatically satisfy the continuity conditions, which avoids the problem of interface slip.

Boundary processing and criterion

After choosing the governing equation, we need to determine the distribution function on the boundary node according to the known macroscopic boundary conditions. There are lots of boundary conditions, such as Heuristic scheme, extrapolation scheme, Surface boundary scheme and pressure boundary. In this paper we used the non-equilibrium extrapolation scheme, the basic thought is to divide the distribution function on the boundary nodes into two parts: equilibrium and none equilibrium distribution function, in which the equilibrium distribution function was defined according to the boundary conditions, and the non-equilibrium distribution function was determined according to the non-equilibrium extrapolation.

The non-equilibrium extrapolation scheme divides the particle distribution functions on the boundary into two classes: equilibrium and non-equilibrium. The non-equilibrium extrapolation scheme has high accuracy and feasibility. Therefore, the range of the scheme is

wider than that of other boundary conditions, and the calculation is simple and easy to implement [8].

As shown in fig. 2, after the completion of fluid particle migration in time, t , distribution function f_2, f_5 , and f_6 on boundary lattice, \mathbf{x}_b , is unknown, in order to determine the distribution function on the boundary, it is divided into two parts of equilibrium and non-equilibrium state, i. e.:

$$f_i(\mathbf{x}_b, t) = f_i^{(eq)}(\mathbf{x}_b, t) + f_i^{(ne)}(\mathbf{x}_b, t), \quad i = 2, 5, 6 \quad (22)$$

where $f_i^{(ne)}$ is non-equilibrium parts.

For the velocity boundary, the velocity \mathbf{u}_w at node \mathbf{x}_b is known, while the density ρ_w is unknown. In the non-equilibrium extrapolation scheme, the modified equilibrium distribution function F number is:

$$\bar{f}_i^{(eq)}(\mathbf{x}_b, t) = f_i^{(eq)}[\rho(\mathbf{x}_f, t), \mathbf{u}_w], \quad i = 2, 5, 6 \quad (23)$$

where \mathbf{x}_f and \mathbf{x}_b are adjacent fluid lattices, where the density replaces the wall density, and the non-equilibrium distribution function is:

$$f_i^{(ne)}(\mathbf{x}_b, t) = f_i^{(ne)}(\mathbf{x}_f, t) = f_i(\mathbf{x}_f, t) - f_i^{(eq)}(\mathbf{x}_f, t), \quad i = 2, 5, 6 \quad (24)$$

Then the non-equilibrium extrapolation scheme can be written:

$$f_i(\mathbf{x}_b, t) = f_i^{(eq)}[\rho(\mathbf{x}_b, t), \mathbf{u}_w] + [f_i(\mathbf{x}_f, t) - f_i^{(eq)}(\mathbf{x}_f, t)], \quad i = 2, 5, 6 \quad (25)$$

In this paper, the convergence criterion was:

$$\frac{\sum \|\mathbf{u}(\mathbf{x}, t + \Delta t) - \mathbf{u}(\mathbf{x}, t)\|}{\sum \|\mathbf{u}(\mathbf{x}, t)\|} \leq 1.0e - 6 \quad (26)$$

$$\frac{\sum \|T(\mathbf{x}, t + \Delta t) - T(\mathbf{x}, t)\|}{\sum \|T(\mathbf{x}, t)\|} \leq 1.0e - 6 \quad (27)$$

Results and analysis

Method validation

To test the reliability of the model and the method, LBM was used to simulate natural convection in a 2-D square cavity partially filled with a porous medium. The vertical surfaces of the cavity were held at constant high or low temperatures. The horizontal walls were adiabatic and impermeable. To evaluate the calculation results, the average Nusselt numbers on the hot wall were compared with existing literature data and LBM findings, and the results are tabulated in tab. 2. and fig. 3. Good quantitative agreement between results was obtained.

Effects of different Rayleigh numbers on the nanofluids in porous media

To study the effect of different Rayleigh numbers on nanofluids in cavities partially filled with porous media, Al_2O_3 -water and Al_2O_3 -ethylene glycol, Cu-water and Cu-ethylene glycol, and SiO_2 -water and SiO_2 -ethylene glycol nanofluids were chosen as study materials.

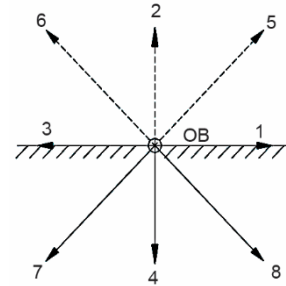
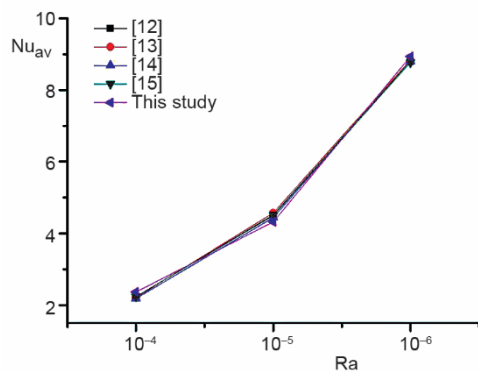
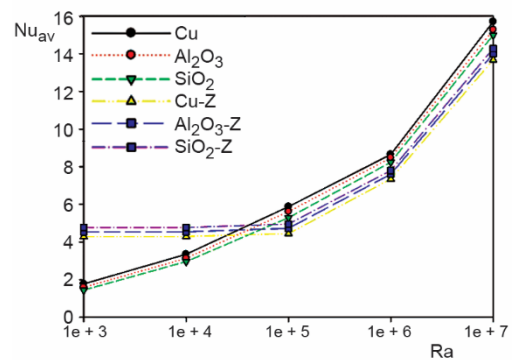


Figure 2. Boundary points distribution function

Table 2. Comparison of the average Nusselt numbers on the hot wall with other numerical results

Ra	ε	Da	Pr	[12]	[13]	[14]	[15]	This study	Maximum relative error
10^4	0.6	10^{-2}	11.40	2.243	2.223	2.195	2.235	2.374	8%
10^5	0.6	10^{-2}	11.40	4.515	4.572	4.450	4.504	4.330	5%
10^6	0.6	10^{-2}	11.40	8.803	8.837	8.803	8.767	8.932	2%

**Figure 3. Comparison of the average Nusselt numbers on the hot wall with other numerical results****Figure 4. Nusselt number as a function of the Rayleigh number of different nanofluids**

The concentration of all of the solutions was 3%; other parameters were set: $D = 0.1$, $\varepsilon = 0.6$, $Da = 10^{-2}$, and $J = 1$.

Figure 4 shows the Nusselt number as a function of the Rayleigh numbers of the six nanofluids. Findings reveal the following.

When the nanoparticles are the same, the average Nusselt numbers of two base fluids in the corresponding nanofluid groups increase with increasing Rayleigh number. A critical Rayleigh number, Ra_c , was also found for each type of nanofluid. When $Ra < Ra_c$, the average Nusselt numbers of ethylene glycol are higher than those of water. It means that, at low Rayleigh number, the heat transfer effect of ethylene glycol as a base fluid is better than that of water. When $Ra > Ra_c$, water is better than ethylene glycol. This finding is attributed to the fact that, at low Rayleigh number, heat transmission in the porous medium cavity is mainly due to the heat conduction of the skeleton and nanofluids. As the thermal diffusion coefficient of ethylene glycol is high, its effect is better than that of water. At high Rayleigh number, natural convection is dominant and the viscosity of water is low. Thus, the buoyant force received by the nanoparticles is more than that received by ethylene glycol. Movement becomes more vigorous, and fluid disturbance is more intense; thus the heat exchange effect of water is better than that of ethylene glycol.

Effects of different nanoparticle concentrations on the nanofluids

To study the effect of the concentration of the same nanoparticle with different Rayleigh numbers, 3% Al_2O_3 -water was chosen as a study material. Other computational parameters were set as follows: $D = 0.1$, $\varepsilon = 0.6$, $Da = 10^{-2}$, and $J = 1$.

Figure 5 clearly shows that when $Ra < 10^5$, the average Nusselt numbers increase with increasing concentration, when $Ra = 10^5$, minimal effects are observed and when $Ra > 10^5$ the average Nusselt numbers decrease with increasing concentration. A critical Rayleigh number, Ra'_c , was also found, the effect of heat exchange on the same nanofluid in the cavity is limited by Rayleigh number. When $Ra < Ra'_c$, the heat exchange increases with increasing concentration, otherwise, heat exchange decreases. At low Rayleigh number ($Ra < Ra'_c$), heat transmission in the porous medium cavity is dominated by heat conduction of the skeleton and nanofluids. The thermal diffusion coefficients increase because of the nanoparticles, so the effect is enhanced with the increase in concentration. With increasing Rayleigh number, the buoyant force of the nanoparticles is strengthened, the irregular motion of the nanoparticles becomes more intense, and the disturbance of the nanofluids increases. All of these phenomena intensifying the effect of heat exchange in the porous medium cavity. However, the increase in concentration also increases the viscosity, and high viscosity restrains the disturbance of nanoparticles and weakens the heat transfer enhancement effect of the nanofluids. Hence, when $Ra > Ra'_c$, the effect decreases even though the effect of heat exchange is dominated by natural convection.

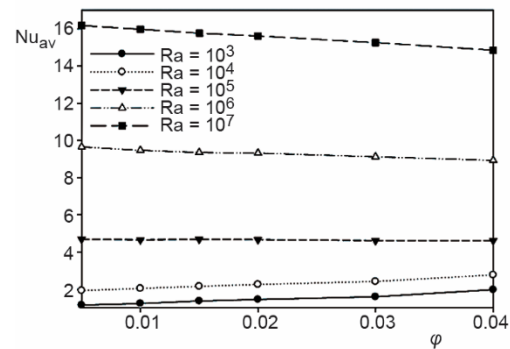


Figure 5. Nusselt number as a function of concentration for the same nanofluid

Effects of different base fluids and nanoparticles on the nanofluids

To study the effect of different base fluids and nanoparticles on the nanofluids in the porous medium cavity, Al_2O_3 -water and Al_2O_3 -ethylene glycol, Cu-water and Cu-ethylene glycol, SiO_2 -water and SiO_2 -ethylene glycol nanofluids were chosen as study materials. Other computational parameters were set as follows: $D = 0.1$, $\varepsilon = 0.6$, $Da = 10^{-2}$, and $J = 1$.

Figures 6(a)-6(d) show the Nusselt number as a function of concentration of different nanofluids when the Rayleigh numbers are 10^4 , 10^5 , 10^6 , and 10^7 . The results clearly reveal:

- When $Ra = 10^4$, the average Nusselt numbers of the nanofluids with water and ethylene glycol increase with increasing concentration. When $Ra = 10^5$, the average Nusselt numbers of the nanofluids with water decrease slightly, whereas those with ethylene glycol still increase. When $Ra = 10^6$, the Nusselt numbers of the nanofluids with water decrease with increasing concentration, whereas those with ethylene glycol change slightly. When $Ra = 10^7$, the average Nusselt numbers of the nanofluids with water and ethylene glycol decrease with increasing concentration. Therefore, the critical Rayleigh numbers for water and ethylene glycol as base fluids are 10^5 and 10^6 , respectively. The critical Rayleigh number of water is less than that of ethylene glycol, which is only related with base fluids.
- For the different base fluids, when $Ra < Ra'_c$, even though the average Nusselt numbers of the three groups of nanofluids increase, the greatest increase occurs in the SiO_2 -ethylene glycol system, followed by the Al_2O_3 -ethylene glycol system. The smallest increase occurs in the Cu-ethylene glycol system when ethylene glycol is used as the base fluid.

By contrast, when $Ra > Ra'_c$, the average Nusselt numbers of the three groups of nanofluids decrease, but the decrease in the SiO_2 -ethylene glycol system is larger than those in the two other systems. Similarly, when water is the base fluid and $Ra < Ra'_c$, the average Nusselt numbers of the three groups of nanofluids increase, and the increase in the Al_2O_3 -water system is larger than those in the two other systems. When $Ra > Ra'_c$, the average Nusselt numbers decrease with concentration, but the effect of Al_2O_3 -water heat exchange is better than that of other heat exchange systems. These findings indicate that different nanoparticles mixed with different base fluids influence the heat transfer of natural convection in a 2-D cavity partly filled with porous medium.

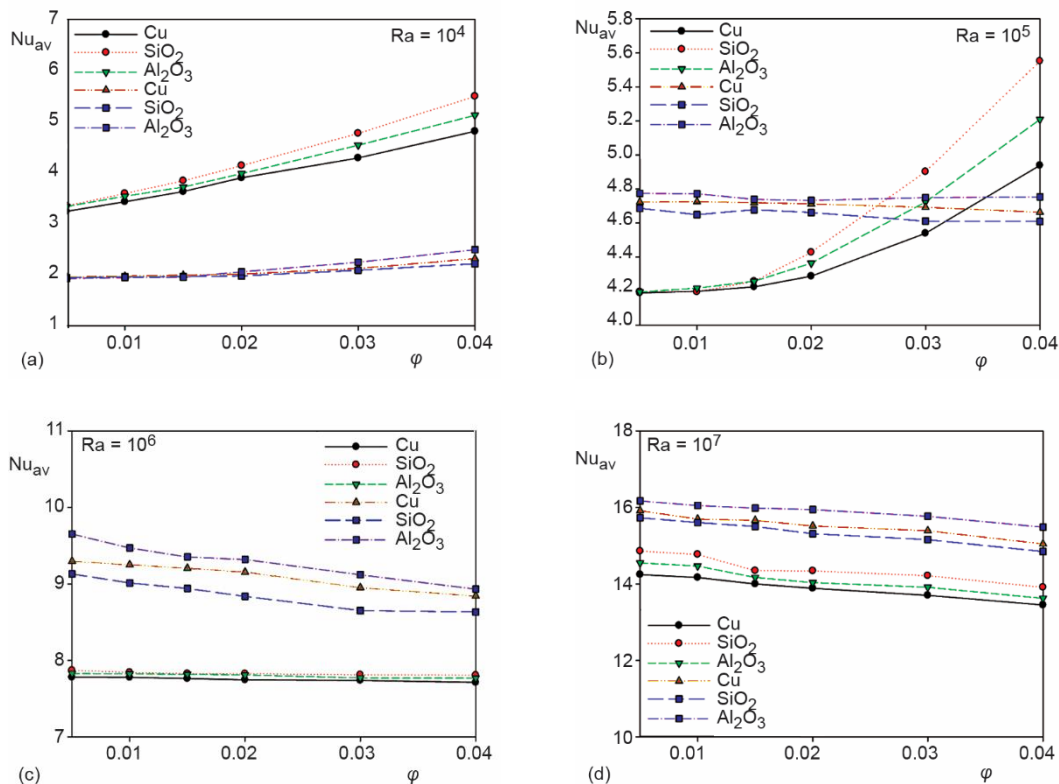
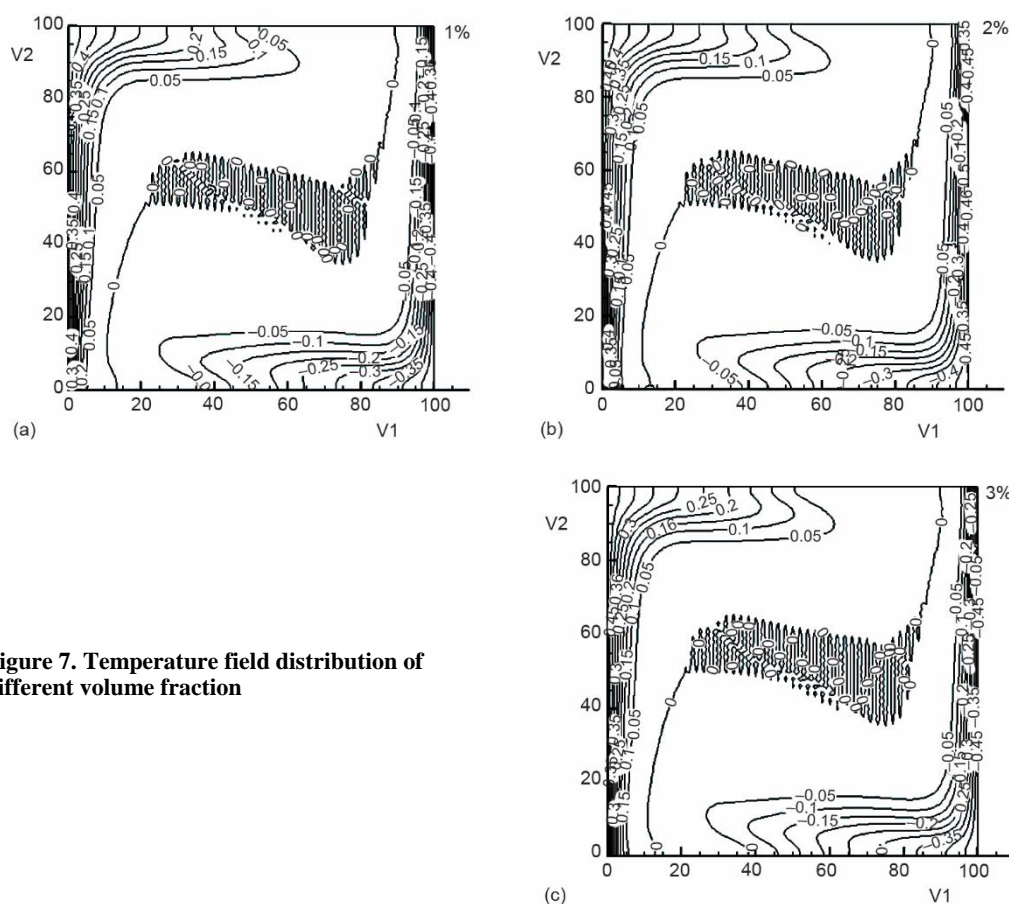


Figure 6. Nusselt number as a function of concentration of different nanofluids

The previous analysis shows that a critical Rayleigh number, Ra'_c , can be obtained for various types of nanofluid with different base fluids and in particular, the Ra'_c of ethylene glycol is larger than that of water. When $Ra < Ra'_c$, the average Nusselt numbers increase with increasing concentration because heat exchange in the porous medium cavity is dominated by the heat conduction of nanofluids and the skeleton at low Rayleigh number. When $Ra > Ra'_c$, the average Nusselt numbers decrease slightly with increasing concentration because the viscosity of nanofluids increases with the nanoparticle concentration at high Rayleigh number. However, high viscosity restrains the heat transfer enhancement effect of nanofluids, and heat exchange is still dominated by natural convection.

Figures 7(a)-7(c) show that the temperature field distribution of different volume fraction. In this case, the Rayleigh numbers are 10^6 , other computational parameters were set as follows: $D = 0.1$, $\varepsilon = 0.6$, $Da = 10^{-2}$, and $J = 1$. From figs. 7(a)-7(c), we can see that the temperature boundary layer of 3% volume fraction are thinner than that of 2% and 1%. It shows that when Rayleigh number is certain, in the selected concentration range, with the increase of concentration, the effect of heat exchange is better. This is consistent with the conclusion drawn from article [16].



- Average Nusselt numbers increase with increasing Rayleigh numbers when different nanofluids transfer heat by natural convection in the porous medium. Each nanofluid has a critical Rayleigh number, Ra'_c . When $Ra > Ra'_c$, the average Nusselt numbers of water as a base fluid are large and the effect of natural convection is better than that of ethylene glycol. When $Ra < Ra'_c$, the average Nusselt numbers of ethylene glycol as a base fluid are larger than those of water.
- A critical Rayleigh number, Ra'_c , is also found for different nanoparticles with different base fluids, and Ra'_c is only related to the base fluid; Ra'_c is small when water is used as the base fluid. When $Ra < Ra'_c$, the average Nusselt numbers increase with increasing concentration. When $Ra > Ra'_c$, the average Nusselt numbers decrease slightly with increasing concentration.
- When ethylene glycol is used as the base fluid, the average Nusselt numbers of SiO_2 -ethylene glycol are larger than those of Al_2O_3 -ethylene glycol and Cu-ethylene glycol nanofluids and the effect of heat exchange is the best. When water is used as the base fluid, the average Nusselt numbers of Al_2O_3 -water are larger than those of Cu-water and SiO_2 -water and the effect of heat exchange is the best.

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Nomenclature

c	– sound velocity
d	– diameter
D	– dimensionless thick
F	– mean Nusselt number
g	– gravitational acceleration, [ms^{-2}]
G	– volume force
J	– viscous coefficient ratio
K	– total force of porous medium resistance and other external forces
L	– cavity length and the width
Nu	– local Nusselt number
T	– dimensionless temperature
t	– total step
u, v, w	– velocities, [ms^{-1}]
x, y, z	– co-ordinates, [m]

Greek symbols

α	– thermal diffusivity, [m^2s^{-1}]
β	– thermal expansion coefficient

μ	– dynamic viscosity, [$kgm^{-1}s^{-1}$]
ϕ	– nanoparticles volume fraction
ν	– kinematic viscosity, [m^2s^{-1}]
ρ	– density
σ	– different components

Subscript

c	– critical
f	– fluid
e	– effective
H	– high
L	– low
nf	– nanofluid
p	– solid particle
s	– solid
ε	– effective
θ	– phase change rates

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