Heat Transfer Analysis in MHD Flow of Solid Particles in Non-Newtonian Ree-Eyring Fluid due to Peristaltic Wave in A Channel

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ABSTRACT

In this article, the influence of heat transfer on solid particle motion in MHD Ree-Eyring fluid due to peristaltic wave has been investigated. The governing flow problem is modelled for fluid phase and particle phase with the help of continuity, momentum and energy equations under the assumption of long wavelength and creeping flow regime. The obtained coupled ordinary differential equations have been solved analytically and exact solutions have been presented. The influence of all the physical parameters are discussed for Newtonian and non-Newtonian fluid with the help of graphs. In the present analysis, it is observed that due to influence of Hartmann number, the magnitude of the velocity diminishes.

Keywords: Heat transfer, MHD, particle-fluid suspension, Ree-Eyring fluid.

1. INTRODUCTION

During the past few years, non-Newtonian fluid achieved a major attention due to its application in different physiological and industrial process. Several fluid models can be found in the literature to understand the mechanism of different flows in various geometrical aspects [1-5]. In a living body, various biological fluids propagate due to symmetrical contraction and suspension of smooth muscles. Such type of mechanism is known peristalsis. These phenomena have great importance in the designing of various devices such as dialysis machine, heart lung machine, blood pump machines, roller pumps and finger pumps. This mechanism is also found in various industrial process in pumping the fluids. In a human body, peristaltic mechanism can be observed in urine transport from kidney to bladder, circulation of small blood vessels, ovum in the female fallopian tube, movement of spermatozoa (male reproductive tract), transport of embryo in non-pregnant uterus and bile from gallbladder into duodenum. When the peristaltic wave is non-uniform, it can cause the thrombus formation of blood, infertility in human uterus and pathological transport of bacteria.

On the other hand, heat transfer has significant role in bio-fluid dynamics. The process of heat transfer can be found in radio frequency therapy which is very helpful to treat different diseases such as lung cancer, reflux stomach acid, primary liver cancer and tissue coagulation. Peristaltic flow with heat transfer has also many applications in medical sciences such as blood pumps in heart lung machine sanitary fluid transport, hemodialysis and oxygenation process, therapy to damage the unwanted tissues, reverse osmosis and diffusion of chemical process. In view of aforesaid applications various authors investigated peristaltic

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motion with heat transfer [6-8]. Ramesh and Devakar [9] investigated the peristaltic flow of couple stress fluid under the effects of Magnetohydrodynamics and heat transfer in an inclined asymmetric porous channel. Hayat *et al.* [10] examined the effects of induced magnetic field with heat and mass transfer effects on peristaltic flow. Hayat and Hina [11] explored the effects MHD and wall properties on peristaltic flow of non-Newtonian Maxwell fluid under the influence of heat and mass transfer. Mekheimer [12] analyze the effects of heat and mass transfer on MHD peristaltic flow Newtonian fluid in a vertical annuls. El-Dabe *et al.* [13] examined the MHD peristaltic flow through a porous medium under the influence of heat transfer. Some pertinent studies on peristaltic flow with heat transfer can be found from the refs. [14-16].

In a hydrodynamics of biological system, the continuum theory of mixtures is very helpful to understand various diverse subjects such as the rheology of blood, diffusion of proteins, particle disposition in respiratory tract and swimming of microorganisms. The flow of mixtures is also very helpful to figure out various physical problems in distinct areas of significant and technical importance such as powder technology, lunar ash flows, combustion, atmospheric fallout, aerosol filtration, sedimentation and fluidization. Misra and Pandey [17] investigated the peristaltic motion of particle fluid suspension in cylindrical tube. Later, Srivastava and Saxena [18] analyzed the particulate suspension flow due to peristaltic wave. Srivastava and Srivastava [19] observed the poiseuille flow on particle fluid motion due to peristaltic wave under the effects of wall elasticity. Mekheimer *et al.* [20] studied the peristaltic flow of particulate fluid suspension through a planar channel. Later, Mekheimer and Elmaboud [21] studied the peristaltic flow of particle fluid suspension through a uniform and non-uniform annulus. Gad [22] considered the influence of Hall current on the interaction between peristaltic flow of particle fluid suspension and pulsatile flow. According to the best of author's knowledge, no such attempt has been made on solid particle-fluid of dusty MHD Ree-Eyring fluid under the effects of heat transfer. Some more pertinent studies on particle-fluid with peristaltic flow can found from the available referneces [23-28].

Peristaltic flow with Magnetohydrodynamics (MHD) play a major role in engineering such as in microfluidics MHD is very much helpful in producing a continuous flow and also very much favorable to control the flow. In biomedical engineering peristaltic flow with Magnetohydrodynamics is also applicable in different process such as Magnetic resonance imaging (MRI), vascular diseases, magnetic drug targeting and cancer tumor treatment. Mekheimer [29] analyzed the influence of Magnetic field on peristaltic flow of couple stress fluid. Ellahi *et al.* [30] studied the effects of Magnetohydrodynamics on peristaltic flow of non-Newtonian Jeffrey fluid through a porous rectangular duct and obtained the exact solutions. He analyzed that due to the influence of magnetic field, the velocity of the fluid decreases. Kothandapani and Srinivas [31] examined the influence of magnetic field on peristaltic flow of Jeffrey fluid in an asymmetric channel. Several authors investigated the peristaltic flow various biological fluids under the effects of Magnetic field [32-39].

With the above analysis in mind, the aim of this present study is to analyze the heat transfer effects on particle-fluid of dusty MHD Ree-Eyring fluid [40] induced by peristaltic wave. Such type of flows in the presence of heat transfer are important in a human tissues that consists of a complicated mechanism i.e. heat transfer due to a perfusion of arterial venous blood towards the pores of a tissue, heat conduction process in the tissues, metabolic heat generation and external interactions. The governing flow problem is based on continuity, momentum and energy equations which are simplified with the help of long wavelength and low Reynolds number approximations. The resulting equations for fluid phase and particulate phase are solved analytically and a closed form solution is obtained. This paper is runs as follows: after the introduction in Sec. (1), Sec. (2) describes the formulation of the problem, Sec. (3) illustrate the solution of the problem, while Sec. (4) is devoted to numerical results and discussion.

NOMENCLATURE

$\tilde{U},\tilde{V}$ Velocity components$(\mathrm{m/s})$;
$\tilde{X},\tilde{Y}$ Cartesian coordinate(m);
$\tilde{P}$ Pressure in fixed frame$(\mathrm{N/m^2})$;
$\tilde{a}$ Wave amplitude(m);
$\tilde{b}$ Width of the channel(m);
$\tilde{c}$ Wave velocity$(\mathrm{m/s})$;
Ec Eckert number;
C Particle volume fraction;
Pr Prandtl number;
Re Reynolds number;
$\tilde{t}$ Time(s);
M Hartmann number;
c Specific heat at constant volume$(\mathrm{J/K})$;
B_0 Magnetic field(T);
S Drag force (N);
k Thermal conductivity (W/mK);
T Temperature (K);
$\tilde{\mathbf{V}}$ Velocity field$(\mathrm{m/s})$;
c specific heat;
Q Volume flow rate $\left(\frac{\mathrm{m^3}}{\mathrm{s}}\right)$;

GREEK SYMBOLS

θ Dimensionless temperature;
λ Wavelength(m);
μ_s Viscosity of the fluid$(\mathrm{Ns/m^2})$;
ϕ Amplitude ratio;
τ Stress tensor;
α Fluid parameter;
σ Electrical conductivity$(\mathrm{S/m})$;
ϖ_T Thermal equilibrium time;
ϖ_v Relaxation time of the particle;
ρ Fluid density$(\mathrm{kg/m^3})$;

SUBSCRIPTS

f Fluid phase;
p Particulate phase;

2. MATHEMATICAL FORMULATION

Let us consider the sinusoidal wave motion of solid particles in Ree-Eyring fluid having incompressible, constant density and irrotational properties moving with a velocity $\tilde{c}$. The fluid is electrically conducting by an external magnetic field, while the induced magnetic field is neglected due to small magnetic Reynolds number. We have chosen a Cartesian coordinate system i.e. $\tilde{X}$ −axis is considered along the direction of the flow and $\tilde{Y}$ −axis is taken normal to it as shown in Fig. (1). The geometry of the wall can be written as

$$\tilde{H}\left(\tilde{X},\tilde{t}\right)=\tilde{a}+\tilde{b}\sin\frac{2\pi}{\lambda}\left(\tilde{X}-\tilde{c}\tilde{t}\right). \tag{1}$$

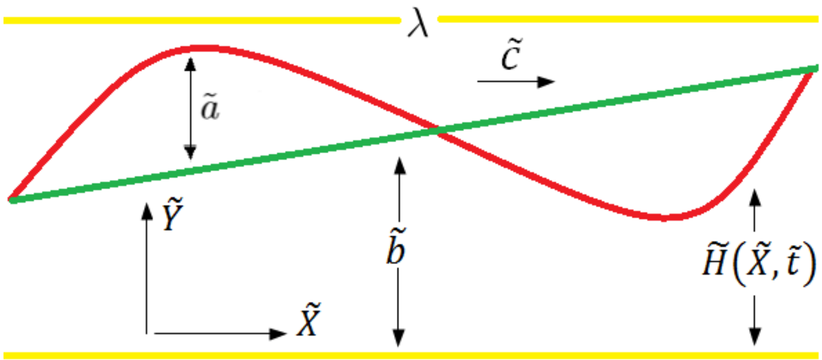

Fig. 1 Geometry of the problem.

The governing equation of continuity and linear momentum equation with the combination of heat transfer for both fluid phase and particulate phase can be described as

(i) Fluid Phase:

$$\frac{\partial \tilde{U}_f}{\partial \tilde{X}}+\frac{\partial \tilde{V}_f}{\partial \tilde{Y}}=0, \tag{2}$$

$$(1-C)\rho_f\left(\frac{\partial \tilde{U}_f}{\partial \tilde{t}}+\tilde{U}_f\frac{\partial \tilde{U}_f}{\partial \tilde{X}}+\tilde{V}_f\frac{\partial \tilde{U}_f}{\partial \tilde{Y}}\right)=-(1-C)\frac{\partial \tilde{P}}{\partial \tilde{X}}+(1-C)\left(\frac{\partial}{\partial \tilde{X}}\tau_{\tilde{X}\tilde{X}}+\frac{\partial}{\partial \tilde{Y}}\tau_{\tilde{X}\tilde{Y}}\right)$$
$$+\frac{CS}{\varpi_v}\left(\tilde{U}_p-\tilde{U}_f\right)-\sigma B_0^2\tilde{U}_f, \tag{3}$$

$$(1-C)\rho_f\left(\frac{\partial \tilde{V}_f}{\partial \tilde{t}}+\tilde{U}_f\frac{\partial \tilde{V}_f}{\partial \tilde{X}}+\tilde{V}_f\frac{\partial \tilde{V}_f}{\partial \tilde{Y}}\right)=-(1-C)\frac{\partial \tilde{P}}{\partial \tilde{Y}}+(1-C)\left(\frac{\partial}{\partial \tilde{X}}\tau_{\tilde{Y}\tilde{X}}+\frac{\partial}{\partial \tilde{Y}}\tau_{\tilde{Y}\tilde{Y}}\right)$$
$$+\frac{CS}{\varpi_v}\left(\tilde{V}_p-\tilde{V}_f\right), \tag{4}$$

$$(1-C)\rho_f c\left(\frac{\partial T_f}{\partial \tilde{t}}+\tilde{U}_f\frac{\partial T_f}{\partial \tilde{X}}+\tilde{V}_f\frac{\partial T_f}{\partial \tilde{Y}}\right)=\mathrm{k}(1-C)\frac{\partial^2 T_f}{\partial \tilde{Y}^2}+\frac{\rho_p cC}{\varpi_T}\left(T_p-T_f\right)$$
$$+\frac{CS}{\varpi_v}\left(\tilde{U}_f-\tilde{U}_p\right)^2+\tau_{\tilde{X}\tilde{Y}}(1-C)\left(\frac{\partial \tilde{U}_f}{\partial \tilde{Y}}\right), \tag{5}$$

(ii) Particulate Phase:

$$\frac{\partial \tilde{U}_p}{\partial \tilde{X}}+\frac{\partial \tilde{V}_p}{\partial \tilde{Y}}=0, \tag{6}$$

$$C\rho_p\left(\frac{\partial \tilde{U}_p}{\partial \tilde{t}}+\tilde{U}_p\frac{\partial \tilde{U}_p}{\partial \tilde{X}}+\tilde{V}_p\frac{\partial \tilde{U}_p}{\partial \tilde{Y}}\right)=-C\frac{\partial \tilde{P}}{\partial \tilde{X}}+\frac{CS}{\varpi_v}\left(\tilde{U}_f-\tilde{U}_p\right), \tag{7}$$

$$C\rho_p\left(\frac{\partial \tilde{V}_p}{\partial \tilde{t}}+\tilde{U}_p\frac{\partial \tilde{V}_p}{\partial \tilde{X}}+\tilde{V}_p\frac{\partial \tilde{V}_p}{\partial \tilde{Y}}\right)=-C\frac{\partial \tilde{P}}{\partial \tilde{Y}}+\frac{CS}{\varpi_v}\left(\tilde{V}_f-\tilde{V}_p\right), \tag{8}$$

$$\rho_p Cc\left(\frac{\partial T_p}{\partial \tilde{t}}+\tilde{U}_p\frac{\partial T_p}{\partial \tilde{X}}+\tilde{V}_p\frac{\partial T_p}{\partial \tilde{Y}}\right)=\frac{\rho_p Cc}{\varpi_T}\left(T_f-T_p\right), \tag{9}$$

The mathematical expression for the drag coefficient and the empirical relation for the viscosity of the suspension can be described as

$$S=\frac{9\mu_0}{2a^2}\tilde{\lambda}(C),\tilde{\lambda}(C)=\frac{4+3\sqrt{8C-3C^2}+3C}{\left(2-3C\right)^2},\mu_s=\frac{\mu_0}{1-\chi C},\chi=0.07e^{\left[2.49C+\frac{1107}{T}e^{-1.69C}\right]}. \tag{10}$$

The stress tensor of Ree-Eyring fluid model is defined as [40]

$$\boldsymbol{\tau}_{ij}=\mu_s\frac{\partial \tilde{V}_i}{\partial \tilde{x}_j}+\frac{1}{\bar{B}}\sinh^{-1}\left(\frac{1}{\bar{C}}\frac{\partial \tilde{V}_i}{\partial \tilde{x}_j}\right). \tag{11}$$

Since $\sinh^{-1}x\approx x$ of $|x|\leq 1$ then [40]

$$\boldsymbol{\tau}_{ij}=\mu_s\frac{\partial \tilde{V}_i}{\partial \tilde{x}_j}+\frac{1}{\bar{B}}\left(\frac{1}{\bar{C}}\frac{\partial \tilde{V}_i}{\partial \tilde{x}_j}\right). \tag{12}$$

Let us define the transformation variable from fixed frame to wave frame, we have

$$\tilde{x}=\tilde{X}-\tilde{c}\tilde{t},\tilde{y}=\tilde{Y},\tilde{u}_{f,p}=\tilde{U}_{f,p}-\tilde{c},\tilde{v}_{f,p}=\tilde{V}_{f,p},\tilde{p}=\tilde{P}. \tag{13}$$

Introducing the following non-dimensional quantities

$$\begin{gathered}\tilde{x}=\frac{x}{\lambda},\tilde{y}=\frac{y}{\tilde{a}},\tilde{u}_{f,p}=\frac{u_{f,p}}{\tilde{c}},\tilde{v}_{f,p}=\frac{v_{f,p}}{\tilde{c}\delta},h=\frac{\tilde{H}}{\tilde{a}},\phi=\frac{\tilde{b}}{\tilde{a}},p=\frac{\tilde{a}^2}{\lambda\tilde{c}\mu_s}\tilde{p},\mathrm{Re}=\frac{\rho\tilde{a}\tilde{c}}{\mu_s},\theta_{f,p}=\frac{T_{f,p}-T_0}{T_1-T_0},\\ \mathrm{Pr}=\frac{\mu_s c}{\mathrm{k}},\mathrm{Ec}=\frac{\tilde{c}^2}{c\left(T_1-T_0\right)},N=\frac{S\tilde{a}^2}{\varpi_v\mu_s},M=\sqrt{\frac{\mathrm{B}_0^2\tilde{a}^2\sigma}{\mu_s}},\alpha=\frac{1}{\mu_s\bar{B}\bar{C}}\end{gathered} \tag{14}$$

Using Eq. (13) and Eq. (14) in Eq. (2) to Eq. (12), and taking the approximation of long wavelength and creeping flow regime, after some simplification we get the resulting equations for fluid phase as

$$\left(1+\alpha\right)\frac{\partial^2 u_f}{\partial y^2}-M^2u_f-\frac{1}{1-C}\frac{dp}{dx}=M^2, \tag{15}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2\theta_f}{\partial y^2}+\mathrm{Ec}\left(1+\alpha\right)\left(\frac{\partial u_f}{\partial y}\right)^2=-\frac{\mathrm{Ec}}{N\left(1-C\right)}\left(\frac{dp}{dx}\right)^2, \tag{16}$$

and for particulate phase it can be written as

$$\frac{dp}{dx}-N\left(u_f-u_p\right)=0, \tag{17}$$

$$\theta_f=\theta_p. \tag{18}$$

and their corresponding non-dimensional boundary conditions are

$$u_f' = \theta_f = 0, \text{ at } y = 0 \text{ and } u_f = -1, \theta_f = 1 \text{ at } y = h. \tag{19}$$

3. SOLUTION OF THE PROBLEM

The exact solution for velocity profile and temperature profile in simplified form can be written as

$$u_f = \frac{(1-C)M^2 - \frac{dp}{dx} + \frac{dp}{dx}\cosh\frac{yM}{\sqrt{1+\alpha}}\operatorname{sech}\frac{hM}{\sqrt{1+\alpha}}}{(1-C)M^2}, \tag{20}$$

$$u_p = \frac{(1-C)M^2 - \frac{dp}{dx} + \frac{dp}{dx}\cosh\frac{yM}{\sqrt{1+\alpha}}\operatorname{sech}\frac{hM}{\sqrt{1+\alpha}}}{(1-C)M^2} - \frac{1}{N}\frac{dp}{dx}, \tag{21}$$

$$\theta_{f,p} = C_1\sinh^2\left[\frac{my}{2\sqrt{1+\alpha}}\right] + C_2\sinh^2\left[\frac{my}{2\sqrt{1+\alpha}}\right]\cosh\left[\frac{my}{\sqrt{1+\alpha}}\right] + C_3 y + C_3 y^2, \tag{22}$$

The constants appearing in above equation can be found using routine calculations.
The volume flow rate for fluid phase and particulate phase is given by

$$Q = (1-C)\int_0^h u_f \,\mathrm{dy}, + C\int_0^h u_p \,\mathrm{dy}, \tag{23}$$

$$Q = \frac{hM\left(N\frac{dp}{dx} + (1-C)M^2(N+Cp)\right) - \sqrt{1+\alpha}N\frac{dp}{dx}\tanh\left[\frac{hM}{\sqrt{1+\alpha}}\right]}{(-1+C)M^3 N}, \tag{24}$$

The pressure gradient dp/dx is obtained after solving Eqs

$$\frac{dp}{dx} = -\frac{(-1+C)M^3 N(h+Q)}{(-1+C)ChM^3 - hMN + \sqrt{1+\alpha}\,N\tanh\left[\frac{hM}{\sqrt{1+\alpha}}\right]}, \tag{25}$$

The non-dimensional pressure rise $\Delta\mathrm{P}$ is evaluated numerically by using the following expression

$$\Delta\mathrm{P} = \int_0^1 \frac{dp}{dx}\,\mathrm{dx}. \tag{26}$$

4. NUMERICAL RESULTS AND DISCUSSION

This section deals with the numerical and graphical results of all the emerging parameters arises in the governing flow problem. Computational software Mathematica has been used to analyze the novelties of all the parameters such as Hartmann number, particle volume fraction, Prandtl number and Eckert number. In particular, we discuss their effects on velocity profile, temperature profile and pressure rise. With the help of computational software "Mathematica", the expression for pressure rise in Eq. (26) has been evaluated numerically. For this purpose Fig. (2) to Fig. (9) are sketched for Newtonian $(\alpha = 0)$ and non-Newtonian $(\alpha \neq 0)$ fluid cases. Fig. (2) shows that velocity distribution for different values of Hartmann number (M). It is evident from this figure that velocity of the fluid decelerate due to the influence of magnetic field. When the magnetic field is applied to the fluid, it creates a Lorentz force which retards the movement of the fluid. It depicts from Fig. (3) that when the particle volume fraction (C) increases then the magnitude of the velocity profile decreases and the same behavior has been observed when the fluid is Newtonian. Infect, when the amount of particles increases in the fluid then it enhances the fluid viscosity which in results the velocity of the fluid decreases.

Fig. (4) and Fig. (7) are sketched for temperature distribution against Hartmann number (M), Prandtl number (Pr), Eckert number (Ec) and particle volume fraction (C). It can be observe from Fig. (4) that when the Prandtl number (Pr) increases then the temperature profile increases. It can also observe here that when the Prandtl number is small heat diffuses more quickly as compared to the velocity. It can be examined from Fig. (5) that when the Eckert number (Ec) increases then the temperature distribution increases while the magnitude of the temperature profile is lower when the fluid is Newtonian. In Fig. (6) we can see that temperature profile behaves as decreasing function when the particle volume fraction (C) increases. It can be notice from Fig. (7) that when the Hartmann number (M) increases then temperature profile increases significantly.

Peristaltic pumping is very important and helpful mechanism in transporting various biological fluids in a human body. To analyse the pumping characteristics, Fig. (8) and Fig. (9) are sketched against Hartmann number (M) and particle volume fraction (C). It depicts from Fig. (8) that when the particle volume fraction (C) increases then the pumping rate increases in co-pumping region $(\Delta \mathrm{P} < 0, Q > 0)$ and free pumping region $(\Delta \mathrm{P} < 0, Q < 0)$ while its behaviour is observed opposite in retrograde pumping region $(\Delta \mathrm{P} > 0, Q < 0)$. It can be scrutinize from Fig. (9) that when the Hartmann number (M) increases then pumping rate decreases in co-pumping region $(\Delta \mathrm{P} < 0, Q > 0)$ and free pumping region $(\Delta \mathrm{P} < 0, Q < 0)$ whereas it increases in retrograde pumping region.

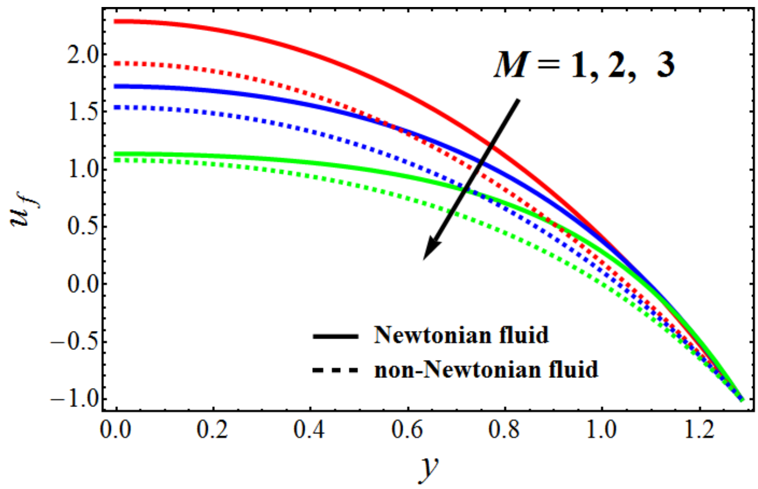

Fig. 2 Velocity profile for various values of M.

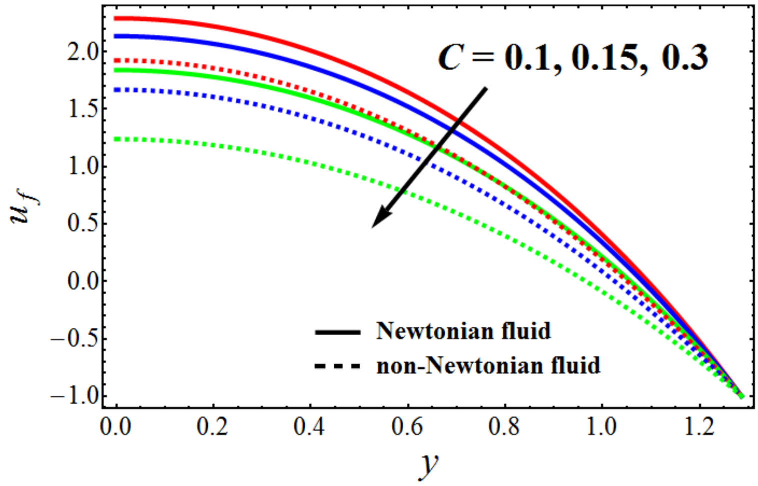

Fig. 3 Velocity profile for various values of C.

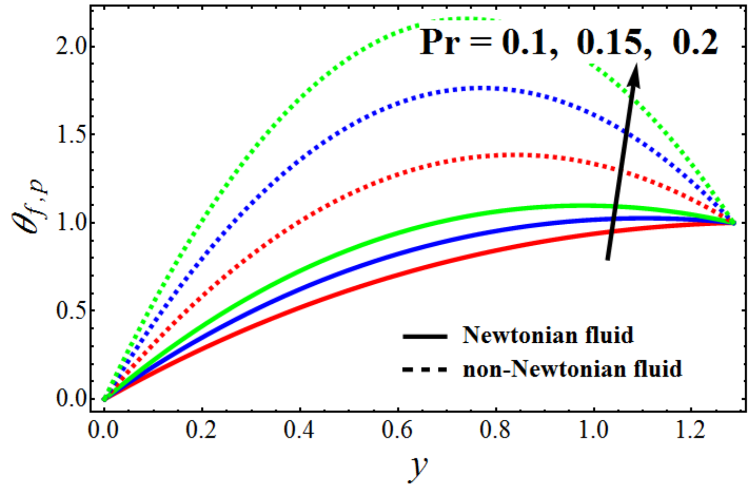

Fig. 4 Temperature profile for various values of $\Pr$.

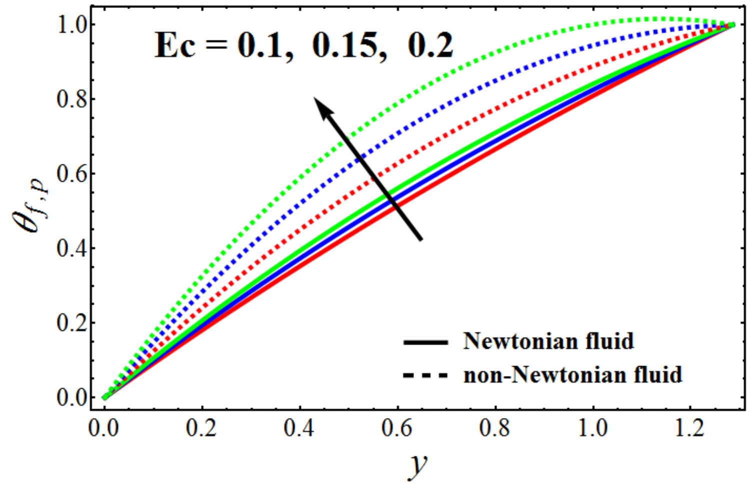

Fig. 5 Temperature profile for various values of Ec.

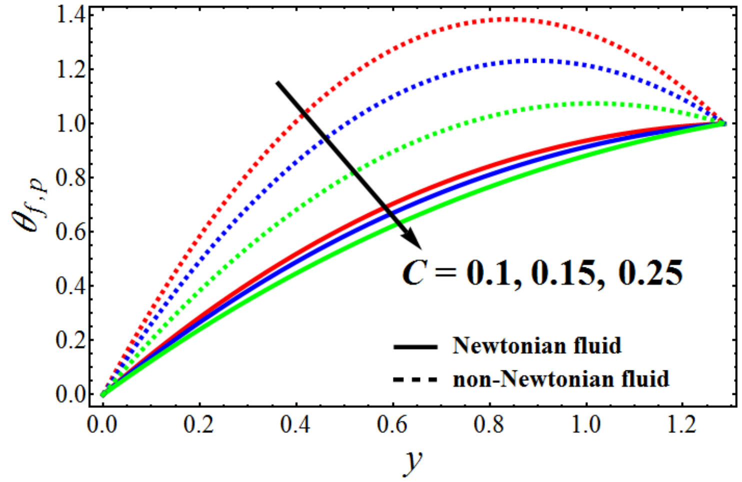

Fig. 6 Temperature profile for various values of C.

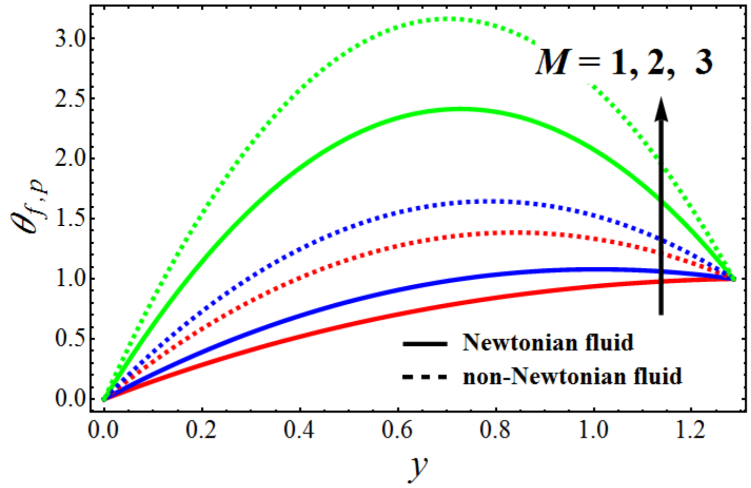

Fig. 7 Temperature profile for various values of M.

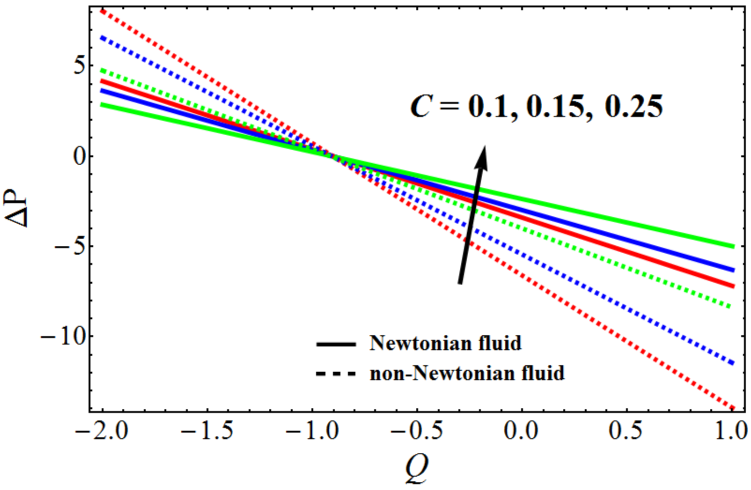

Fig. 8 Pressure rise for various values of C.

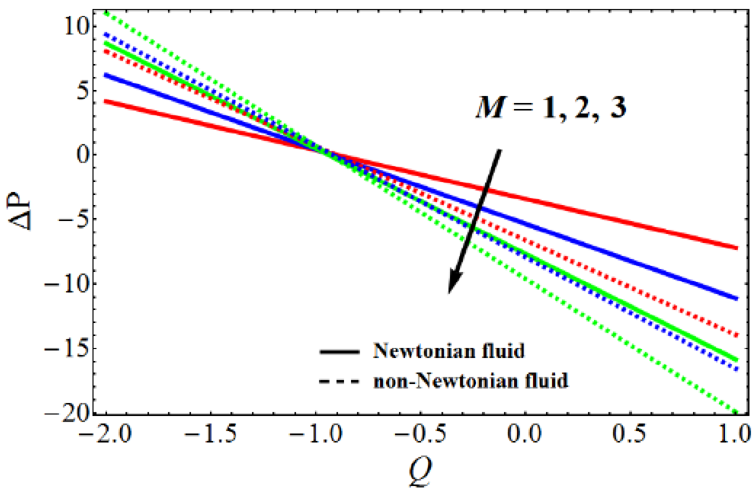

Fig. 9 Pressure rise for various values of M.

5. CONCLUSION

In this article, Heat transfer effects on solid particle motion in MHD Ree-Eyring fluid due to peristaltic wave has been examined. The governing flow problem for fluid phase and particulate phase are simplified under the assumption of long wavelength and creeping flow regime. The resulting coupled ordinary differential equations are solved analytically and a closed form solution are presented. The major outcomes for the present analysis are summarized below:

- The magnitude of velocity distribution for Newtonian fluid is smaller as compared to non-Newtonian fluid.
- Velocity distribution decreases due to the influence of magnetic field and particle volume fraction.
- Temperature profile increases due to Prandtl number and Hartmann number while its behavior is opposite due to the impact of particle volume fraction.
- Pressure rise also behaves as an increasing function when the fluid is Newtonian.
- The present analysis can also be reduced to Newtonian fluid by taking $\alpha = 0$, as a special case of our study.

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