A NEW FRACTIONAL MODEL FOR CONVECTIVE STRAIGHT FINS WITH TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

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The key aim of this work is to present a new non-integer model for convective straight fins with temperature-dependent thermal conductivity associated with Caputo-Fabrizio fractional derivative. The fractional energy balance equation is solved by using homotopy perturbation method coupled with Laplace transform method. The efficiency of straight fin has been derived in terms of thermo-geometric fin parameter. The numerical results derived by the application of suggested scheme are demonstrated graphically. The subsequent correlation equations are very helpful for thermal design scientists and engineers to design straight fins having temperature-dependent thermal conductivity.

Key words*: Fractional energy balance equation, Thermal conductivity, Straight fins, Caputo-Fabrizio fractional derivative, Homotopy perturbation method*

1. Introduction

Most of scientific problems such as heat transfer are modeled by using nonlinear differential equations [1]. The concept of heat transfer is very important and useful in mechanical engineering as it is required in numerous objects. The convective heat transfer rate can be enhanced with the aid of distinct schemes such as enlarging the surface area of heat transfer or coefficient of heat transfer. It is very popular that the heat transfer surface area can enlarged by affixing the fins fabricated of materials having highly conductive on base surface. Fins are designed in such a manner that it increases the heat transfer from the base surface to its environment. Besides the well known utilizations such as heat exchanges, internal combustion engines and compressors, fins also insures efficacious in heat rejection systems in cooling of various types of electric instruments and space vehicles [2,3]. In this connection a detailed report was presented in a monograph by Kern and Kraus [4]. In an attempt Domairry and Fazeli [5] examined the efficiency of convective straight fins with the help of homotopy perturbation method (HPM). Chiu and Chen [6] operated the Adomian's decomposition method (ADM) to investigate convective longitudinal fins having variable thermal conductivity. In another study Chiu and Chen [7] examined the convective-radiative with the help of decomposition technique. In a study authors of [8] analyzed the heat-rejecting system Furthermore, Coskun and Atay [9] employed the variational iteration technique to examine the convective straight and radial fins. Arslanturk [10] used and show the efficiency of ADM to examine the optimum design of space radiators. In another investigation Aziz and Hug [11] employed the classical perturbation scheme to examine the efficiency of convective straight fins. In this sequel Patra and Ray [12] studied the efficiency of convective straight fins possessing temperature-dependent thermal conductivity with the help of HPM by using sumudu transform scheme.

In the last decade derivatives and integrals of fractional orders had notable development as revealed by several monographs dedicated to it (e.g. Hilfer [13], Podlubny [14], Miller and Ross [15], Baleanu et al. [16], Kilbas et al. [17], etc), the plethora of research papers published in scientific journals (e.g. Kumar et al. [18] studied differential-difference equation of fractional order, Singh et al. [19] investigated the local fractional Tricomi equation, Bhrawy et al. [20] examined the fractional Burgers' equations, Area et al. [21] analyzed the Ebola epidemic model of fractional order, Carvalho and Pinto [22] presented a delay mathematical model of fractional order to determine the co-infection of malaria and the human immunodeficiency virus, Srivastava et al. [23] examined a fractional model of vibration equation, Yang et al. [24] studied the fractional Korteweg-de Vries equation involving local fractional derivative, Jafari et al. [25] investigated the differential equations pertaining to local fractional operators, Yang et al. [26] examined the local fractional diffusion and relaxation equations, He at al. [27] studied a new fractional derivative and its application to explanation of polar bear hairs, Wang and Liu [28] showed the applications of He's fractional derivative for non-linear fractional heat transfer equation, Liu eat a. [29] used the He's fractional derivative for heat conduction in a fractal medium arising in silkworm cocoon hierarchy, Sayevand and Pichaghchi [30] studied a nonlinear fractional KdV equation based on He’s fractional derivative,] Hu et al. [31] studied fractal space-time and fractional calculus, Liu et al. [32] presented a fractional model for insulation clothings with cocoon-like porous structure, etc.) and definitions of various derivatives and integrals (e.g. Caputo [33], Yang [34], He [35,36], etc.). In a recent work Caputo and Fabrizio [37] introduced a new fractional derivative. The importance of the newly derivative is due to the requisite of employing a mathematical model explaining the nature of various processes. In fact, the classical Caputo fractional derivative comes into view to be especially suitable for those mechanical processes, associated with plasticity, fatigue, damage and with electromagnetic hysteresis. The physical processes in which these effects absent it seem more suitable to employ the novel definition of fractional derivative. Atangana [38] used the newly fractional derivative to understand the nature of Fisher’s reaction- diffusion equation. In another investigation Atangana and Koca [39] employed this approach to nonlinear Baggs and Freedman model and show the efficiency of the newly fractional derivative. In a series of papers Hristov [40,41] has shown that the newly Caputo –Fabrizio fractional derivative can easily be obtained from the Cattaneo concept of the flux relaxation if the damping function is the Jeffrey memory kernel. Sun et al. [42] examined the fractional relaxation and diffusion problems with non-singular kernels. Yang et al. [43] proposed a new fractional derivative having non-singular kernel and verify its applications in steady heat flow. In another work Atangana and Baleanu [44] suggested a new non-integer order derivative having nonlocal and non-singular kernel. Mirza et al. [45] obtained the solutions of advection-diffusion equation with time-fractional Caputo-Fabrizio derivative by using a combination of Laplace transform and Fourier transform. Atangana and Baleanu [46] show the application of Caputo-Fabrizio derivative in groundwater flow within confined aquifer. Ali et al. [47] used the Caputo-Fabrizio derivative to examine the MHD free convection flow of generalized Walters'-B fluid model. Baleanu et al. [48] presented a comparative study of Caputo and Caputo-Fabrizio derivatives for advection differential equation. Algahtani [49] used the Atangana-Baleanu and Caputo-Fabrizio derivative with fractional order in Allen Cahn model.

In view of the great importance of heat transfer and related problems, we present a new fractional model for energy balance equation associated with newly Caputo-Fabrizio fractional derivative to calculate the efficiency of the convective straight fins possessing temperature-dependent thermal conductivity. The efficiency of homotopy perturbation method (HPM) coupled with Laplace transform algorithm is employed to examine the energy balance equation of fractional order. We estimate the non-dimensional temperature distribution and fin tip temperature for the convective straight fins possessing thermal conductivity for the distinct values of various physical parameters. The used method is an extension of HPM by using Laplace transform [50-52] to handle nonlinear partial differential equations associated with newly Caputo-Fabrizio fractional derivative and is a combined form of HPM [53-55], Lapalce transform technique and He's polynomials [56]. The supremacy of applied scheme over the classical analytical techniques is that requires the less computer memory and reduces the computation time. The plan of the remaining part of this article is as follows: The primary definitions of Caputo-Fabrizio derivative are presented in Section 2. In section 3, we present the non-integer model of the problem. In Section 4, we demonstrate the basic plan of HPM coupled with Laplace transform algorithm. Section 5 reports the application of HPM coupled with Laplace transform scheme to solve the fractional energy balance equation. Section 6 contains the fin efficiency. In section 7, we interpret the numerical results and discussion. Finally the Section 8 is dedicated to the conclusions.

2. Preliminaries

Definition 1. Let us consider that $\theta \in H^1(a,b), b > a, \beta \in [0,1],$ then the Caputo-Fabrizio derivative of fractional order discovered by Caputo and Fabrizio [37] is written in the following manner

$$D_\xi^\beta[\theta(\xi)] = \frac{M(\beta)}{1-\beta}\int_a^\xi \theta'(\tau)\exp\left[-\beta\frac{\xi-\tau}{1-\beta}\right]d\tau. \tag{1}$$

In the Eq. (1) $M(\beta)$ is denoting normalization function, which holds the property $M(0) = M(1) = 1$[37]. If $\theta \notin H^1(a,b)$ then the Caputo-Fabrizio fractional derivative can be re-expressed as

$$D_\xi^\beta[\theta(\xi)] = \frac{\beta M(\beta)}{1-\beta}\int_a^\xi (\theta(\xi)-\theta(\tau))\exp\left[-\beta\frac{\xi-\tau}{1-\beta}\right]d\tau. \tag{2}$$

The Eq. (2) also reduces in the following similar equation

$$D_\xi^\gamma[\theta(\xi)] = \frac{N(\gamma)}{\gamma}\int_a^t \theta'(\tau)\exp\left[-\frac{\xi-\tau}{\gamma}\right]d\tau,\ N(0) = N(\infty) = 1, \tag{3}$$

if $\gamma = \dfrac{1-\beta}{\beta} \in [0,\infty), \beta = \dfrac{1}{1+\gamma} \in [0,1]$ as presented by the Atangana [38].

Moreover,

$$\lim_{\gamma\to 0}\frac{1}{\gamma}\exp\left[-\frac{\xi-\tau}{\gamma}\right] = \delta(\tau-\xi). \tag{4}$$

The associate integral of the Caputo-Fabrizio fractional derivative was introduced by Losada and Nieto [57] expressed in the following manner.

Definition 2. The integral operator of fractional order for the function $\theta(\xi)$ of order $\beta, 0 < \beta < 1$ is written as [57]

$$I_{\xi}^{\beta}\left[\theta(\xi)\right]=\frac{2(1-\beta)}{(2-\beta)M(\beta)}\theta(\xi)+\frac{2\beta}{(2-\beta)M(\beta)}\int_{0}^{\xi}\theta(s)ds,\xi\geq 0. \tag{5}$$

From Eq. (5), the following result can be found

$$\frac{2(1-\beta)}{(2-\beta)M(\beta)}+\frac{2\beta}{(2-\beta)M(\beta)}=1. \tag{6}$$

It yields the following result

$$M(\beta)=\frac{2}{(2-\beta)},0\leq\beta<1. \tag{7}$$

Further Losada Nieto [57] showed that the Caputo-Fabrizio derivative of order $0<\beta<1$ can be redefined by using above discussed expression as

$$D_{\xi}^{\beta}\left[\theta(\xi)\right]=\frac{1}{1-\beta}\int_{a}^{\xi}\theta^{'}(\tau)\exp\left[-\beta\frac{\xi-\tau}{1-\beta}\right]d\tau. \tag{8}$$

Now, we present the following important theorem for the newly Caputo-Fabrizio fractional derivative.

Definition 3. If ${}_{0}^{CF}D_{\xi}^{\beta+1}\theta(\xi)$ is the Caputo-Fabrizio fractional derivative of a function $\theta(\xi)$ then its Laplace transform formula is expressed as [37]

$$L\left[{}_{0}^{CF}D_{\xi}^{\beta+1}\theta(\xi)\right]=\frac{s^{2}\bar{\theta}(s)-s\theta(0)-\theta^{'}(0)}{s+\beta(1-s)}, \tag{9}$$

where $\bar{\theta}(s)$ indicates the Laplace transform of the function $\theta(\xi)$.

3. Mathematical model of the problem

The schematic diagram of straight fin problem possessing the arbitrary cross-sectional area A_c, perimeter P, and length b is presented in Fig. 1. The fin is joined with the base surface having the temperature T_b and extends into fluid having temperature T_a and its tip is insulated.

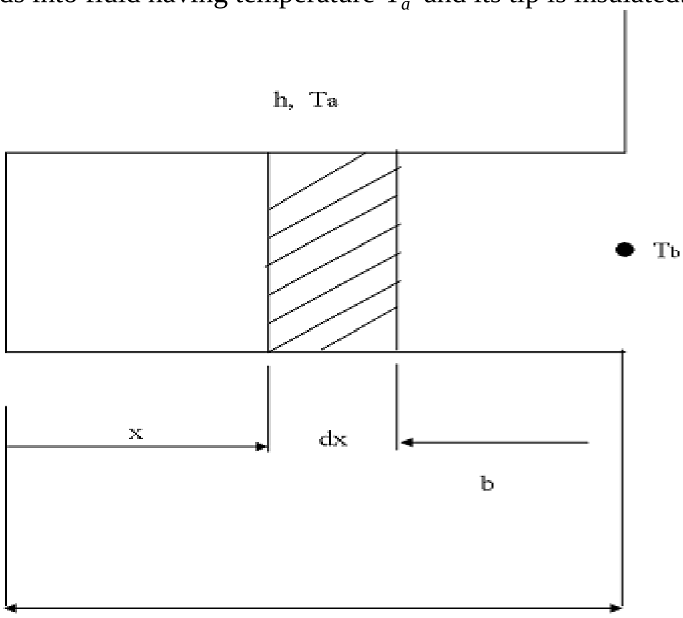

Fig. 1 Schematic diagram of the problem.

Then the energy balance equation is written as [9,10]

$$A_c\frac{d}{dx}\left[k(T)\frac{dT}{dx}\right]-Ph(T-T_a)=0. \tag{10}$$

In the Eq. (10) $k(T)$ indicates the temperature-dependent thermal conductivity and h represents the coefficient of heat transfer. It is considered that the thermal conductivity for the fin material is expressed as

$$k(T) = k_b\left[1 + \lambda(T - T_b)\right]. \tag{11}$$

In the above expression (11) k_b represents the thermal conductivity at the ambient fluid temperature of the fin and λ is standing for the variation of the thermal conductivity.

Using the non-dimensional variables

$$\theta = \frac{T - T_a}{T_b - T_a}, \xi = \frac{x}{b}, \alpha = \lambda(T_b - T_a) \text{ and } \psi = \left(\frac{hPb^2}{k_a A_c}\right)^{1/2}. \tag{12}$$

Consequently the Eq. (10) reduces in the following form

$$\frac{d^2\theta}{d\xi^2} + \alpha\theta\frac{d^2\theta}{d\xi^2} + \alpha\left(\frac{d\theta}{d\xi}\right)^2 - \psi^2\theta, 0 \le \xi \le 1 \tag{13}$$

with the boundary conditions

$$\left.\frac{d\theta}{d\xi}\right|_{\xi=0} = 0 \text{ and } \left.\theta\right|_{\xi=1} = 1. \tag{14}$$

Since we know that the integer order derivatives are in local in nature, so these derivatives cannot describe the problem accurately. The Caputo-Fabrizio fractional derivative is more suitable to describe to natural phenomena because its kernel is non-local and non-singular. Therefore, we replace the second order derivative $\frac{d^2\theta}{d\xi^2}$ in Eq. (13) by the newly Caputo-Fabrizio fractional derivative and Eq. (13) converts to a fractional model of energy balance equation expressed as

$${}^{CF}D_\xi^{\beta+1}\theta + \alpha\theta\frac{d^2\theta}{d\xi^2} + \alpha\left(\frac{d\theta}{d\xi}\right)^2 - \psi^2\theta, 0 < \beta \le 1 \text{ and } \quad 0 \le \xi \le 1 \tag{15}$$

along with the boundary conditions (14).

4. Basic idea of HPM coupled with Laplace transform method

In order to demonstrate the fundamental plan of HPM coupled with Laplace transform algorithm, we take a nonlinear differential equation involving Caputo-Fabrizio fractional derivative written as:

$${}^{CF}D_\xi^{\beta+1}\theta(\xi) + R\theta(\xi) + N\theta(\xi) = g(\xi), 0 < \beta \le 1 \tag{16}$$

with the initial conditions

$$\theta(0) = a,\ \theta'(0) = b, \tag{17}$$

where ${}^{CF}D_\xi^{\beta+1}\theta(\xi)$ represents the Caputo-Fabrizio derivative of non-integer order for the function $\theta(\xi)$, R indicates the linear differential operator, N stands for the nonlinear differential operator of general nature and $g(\xi)$ denotes the term due to the source.

Firstly we operate the Laplace transform on fractional Eq. (16), it yields

$$L[\theta(\xi)] = \frac{a}{s} + \frac{b}{s^2} + \left(\frac{s+\beta(1-s)}{s^2}\right) L[g(\xi)] - \left(\frac{s+\beta(1-s)}{s^2}\right) L[R\theta(\xi) + N\,\theta(\xi)]. \tag{18}$$

On operating with the inverse of Laplace transform on Eq. (18), it gives

$$\theta(\xi) = G(\xi) - L^{-1}\left[\left(\frac{s+\beta(1-s)}{s^2}\right) L[R\theta(\xi) + N\,\theta(\xi)]\right]. \tag{19}$$

In the Eq. (19) $G(\xi)$ indicates the term occurring due to the source term and the initial conditions. Next we use the classical HPM as

$$\theta(\xi) = \sum_{n=0}^{\infty} p^n \theta_n(\xi), \tag{20}$$

and the nonlinear term $N\,\theta(\xi)$ can be deformed in the following manner

$$N\,\theta(\xi) = \sum_{n=0}^{\infty} p^n H_n(\theta), \tag{21}$$

where $H_n(\theta)$ are the He's polynomials [56] that are presented as

$$H_n(\theta_0, \theta_1, \ldots, \theta_n) = \frac{1}{n!}\frac{\partial^n}{\partial p^n}\left[N\left(\sum_{i=0}^{\infty} p^i \theta_i\right)\right]_{p=0}, \text{ n = 0, 1, 2, 3…} \tag{22}$$

On using the Eqs. (20) and (21) in (19), it yields

$$\sum_{n=0}^{\infty} p^n \theta_n(\xi) = G(\xi) - p\left(L^{-1}\left[\left(\frac{s+\beta(1-s)}{s^2}\right) L\left[R\sum_{n=0}^{\infty} p^n \theta_n(\xi) + \sum_{n=0}^{\infty} p^n H_n(\theta)\right]\right]\right). \tag{23}$$

The above result is the combination of the standard Laplace transform scheme and the classical HPM employing the He's polynomials. On equating the coefficients of same powers of p, we have

$$\begin{aligned}
p^0 &: \theta_0(\xi) = G(\xi),\\
p^1 &: \theta_1(\xi) = -L^{-1}\left[\left(\frac{s+\beta(1-s)}{s^2}\right) L\left[R\theta_0(\xi) + H_0(\theta)\right]\right],\\
p^2 &: \theta_2(\xi) = -L^{-1}\left[\left(\frac{s+\beta(1-s)}{s^2}\right) L\left[R\theta_1(x,t) + H_1(\theta)\right]\right],\\
p^3 &: \theta_3(\xi) = -L^{-1}\left[\left(\frac{s+\beta(1-s)}{s^2}\right) L\left[R\theta_2(\xi) + H_2(\theta)\right]\right],\\
&\vdots
\end{aligned} \tag{24}$$

Using the same way, the remaining the iterates $\theta_n(\xi)$ can obtained. Finally, the FHPTM solution $\theta(\xi)$ is expressed as

$$\theta(\xi) = \lim_{N\to\infty} \sum_{n=1}^{N} \theta_n(\xi). \tag{25}$$

5. HPM coupled with Laplace transform method for nonlinear energy balance equation of fractional order

First of all, we operate the Laplace transform on Eq. (15), it yields

$$L[\theta(\xi)] = \frac{K}{s} - \frac{s+\beta(1-s)}{s^2} L\left[\alpha\theta \frac{d^2\theta}{d\xi^2} + \alpha\left(\frac{d\theta}{d\xi}\right)^2 - \psi^2\theta\right], \tag{26}$$

where $K = \theta(0)$.

On operating the inverse of Laplace transform on Eq. (26), it gives

$$\theta(\xi) = K - L^{-1}\left[\frac{s+\beta(1-s)}{s^2} L\left[\alpha\theta \frac{d^2\theta}{d\xi^2} + \alpha\left(\frac{d\theta}{d\xi}\right)^2 - \psi^2\theta\right]\right]. \tag{27}$$

On applying the HPM, it yields

$$\sum_{n=0}^{\infty} p^n \theta_n = K - p\left(L^{-1}\left[\frac{s+\beta(1-s)}{s^2} L\left[\alpha\sum_{n=0}^{\infty} p^n H_n + \alpha\sum_{n=0}^{\infty} p^n H_n^{'} - \psi^2 \sum_{n=0}^{\infty} p^n \theta_n\right]\right]\right). \tag{28}$$

In the Eq. (28) $H_n(\theta)$ and $H_n^{'}(\theta)$ are He's polynomials given in the following manner

$$\sum_{n=0}^{\infty} p^n H_n(\theta) = \theta \frac{d^2\theta}{d\xi^2}. \tag{29}$$

The initial few, components $H_0(\theta),\ H_1(\theta),\ H_2(\theta),\cdots$ are expressed as

$$H_0(\theta) = \theta_0(\xi)\frac{d^2\theta_0(\xi)}{d\xi^2},$$

$$H_1(\theta) = \theta_0(\xi)\frac{d^2\theta_1(\xi)}{d\xi^2} + \theta_1(\xi)\frac{d^2\theta_0(\xi)}{d\xi^2}, \tag{30}$$

$$H_2(\theta) = \theta_0(\xi)\frac{d^2\theta_2(\xi)}{d\xi^2} + \theta_1(\xi)\frac{d^2\theta_1(\xi)}{d\xi^2} + \theta_2(\xi)\frac{d^2\theta_0(\xi)}{d\xi^2},$$

$$\vdots$$

for $H_n^{'}(\theta)$, we find that

$$\sum_{n=0}^{\infty} p^n H_n^{'}(\theta) = \left(\frac{d\theta}{d\xi}\right)^2,$$

$$H_0^{'}(\theta) = \left(\frac{d\theta_0(\xi)}{d\xi}\right)^2,$$

$$H_1^{'}(\theta) = 2\frac{d\theta_0(\xi)}{d\xi}\frac{d\theta_1(\xi)}{d\xi}, \tag{31}$$

$$H_2^{'}(\theta) = 2\frac{d\theta_0(\xi)}{d\xi}\frac{d\theta_2(\xi)}{d\xi} + \left(\frac{d\theta_1(\xi)}{d\xi}\right)^2,$$

$$\vdots$$

On equating the coefficients of the same powers of p, it yields

$$p^0 : \theta_0(\xi) = K,$$

$$p^1 : \theta_1(\xi) = \frac{K\psi^2}{2}\left[2(1-\beta)\xi + \beta\xi^2\right], \tag{32}$$

$$\vdots$$

Using the same way, the remaining iterates $\theta_n(\xi)$ can be completely obtained. Therefore the solution is expressed as

$$\theta(\xi) = \theta_0(\xi) + \theta_1(\xi) + \theta_2(\xi) + \theta_3(\xi) + \theta_4(\xi) + \cdots. \tag{33}$$

In the above Eq. (29) K indicates the temperature at the fin tip and $K \in [0,1].$ The value of K can be easily obtained by using the boundary condition $\theta\big|_{\xi=1} = 1.$

6. Fin efficiency

The rate of heat transfer from the straight fins is derived by employing the Newton's law of cooling

$$Q = \int_0^b P(T - T_a)dx. \tag{34}$$

The most significant property of the fins is the fin efficiency. It is investigated in the problems of heat and mass transfer's. The fin efficiency is the ratio of the actual heat transferred by the fin and the heat transfer if the fin is entirely present in base temperature:

$$\rho = \frac{Q}{Q_{ideal}} = \frac{\int_0^b P(T - T_a)dx}{Pb(T_b - T_a)} = \int_0^1 \theta(\eta)d\eta. \tag{35}$$

Now substituting the value of θ from Eq. (33) in Eq. (35), we calculate the dimensionless fin efficiency for the straight fins.

7. Numerical results and discussions

In this portion, we compute the non-dimensional temperature distribution $\theta(\xi)$ and non-dimensional fin tip temperature K. The comparison between the results derived for integer order derivative, classical Caputo fractional derivative and newly Caputo-Fabrizio fractional derivative are presented in Table 1. It can be perceived from Table 1 that the numerical results associated with Caputo-Fabrizio fractional derivative show new characteristics compared to standard derivative and classical Caputo fractional derivative. The comparison between the numerical simulations for non-dimensional temperature distribution obtained with aid of distinct schemes is discussed in Table 2. From Table 2, it can be easily noticed that the outcomes of the suggested scheme are perfectly agree with the results available in the literature. The numerical results for non-dimensional temperature distribution $\theta(\xi)$ for various values of values of β,ψ and α are displayed through the Figs. 1-3 respectively. It can be detected from the Fig. 1 that as the order of time-fractional derivative increases, it leads the increases in the value of the temperature $\theta(\xi)$. The Fig. 2 reveals that the temperature $\theta(\xi)$ is decreased by increasing the value of ψ. In can be observed from Fig. 3 that the temperature $\theta(\xi)$ is increased by increasing the value of α. The numerical results obtained by using FHPTM for non-dimensional fin tip temperature K for various values of order of non-integer derivative and α are displayed in Figs. 4-5 respectively. The Fig. 4 shows that as β increases, it leads to the corresponding increase in the value of fin tip temperature K. Fig. 5 reveals that the value of K increases by increasing the value of α.

Table 1. Comparative study between integer order derivative, classical Caputo fractional derivative and newly fractional derivative due to Caputo and Fabrizio for the non-dimensional temperature distribution within the fin at $\alpha = 0.3$ and $\psi = 1$.

ξ	Integer order derivative ($\beta = 1$)	Caputo fractional derivative ($\beta = 0.75$)	Caputo-Fabrizio fractional derivative ($\beta = 0.75$)
0.0	0.7004658984	0.6557094016	0.6336851793
0.1	0.7033558030	0.6606701636	0.6493352360
0.2	0.7120421010	0.6735652420	0.6693067431
0.3	0.7265733504	0.6933393015	0.6937271779
0.4	0.7470265036	0.7195597031	0.7227314777
0.5	0.7735009521	0.7519699773	0.7564565254
0.6	0.8061102147	0.7903792329	0.7950350175
0.7	0.8449712927	0.8346092686	0.8385887508
0.8	0.8901917235	0.8844616181	0.8872213648
0.9	0.9418543709	0.9396937045	0.9410105835
1.0	1.000000000	1.000000000	1.000000000

Table 2. The numerical results obtained by using HPM [3], VIM [9] and present method for the non-dimensional temperature distribution within the fin at $\alpha = 0, \psi = 0.2$ and $\beta = 1$.

ξ	HPM [3]	VIM [9]	HPM coupled with Laplace transform method
0.0	0.9803	0.9803	0.9803279973
0.1	0.9805	0.9805	0.9805240694
0.2	0.9805	0.9811	0.9811123643
0.3	0.9821	0.9820	0.9820931172
0.4	0.9835	0.9834	0.9834667204
0.5	0.9852	0.9852	0.9852337234
0.6	0.9874	0.9873	0.9873948330
0.7	0.9900	0.9899	0.9899509138
0.8	0.9929	0.9929	0.9929029880
0.9	0.9963	0.9962	0.9962522366
1.0	1.0000	1.0000	1.0000000000

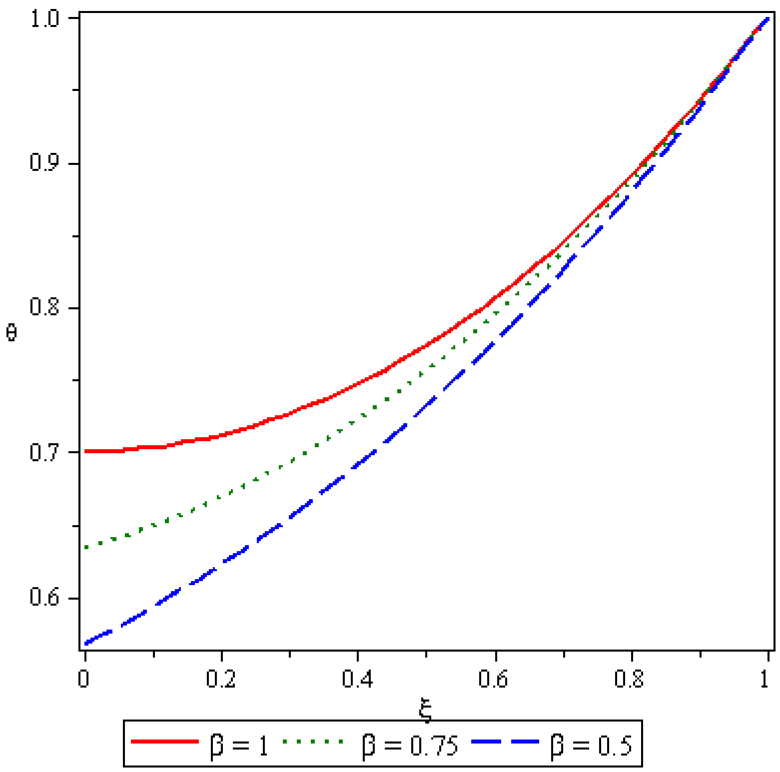

Fig. 1. Response of dimensionless temperature distribution $\theta(\xi)$ for convective straight fins vs. ξ for distinct values of β at $\alpha = 0.3$ and $\psi = 1$.

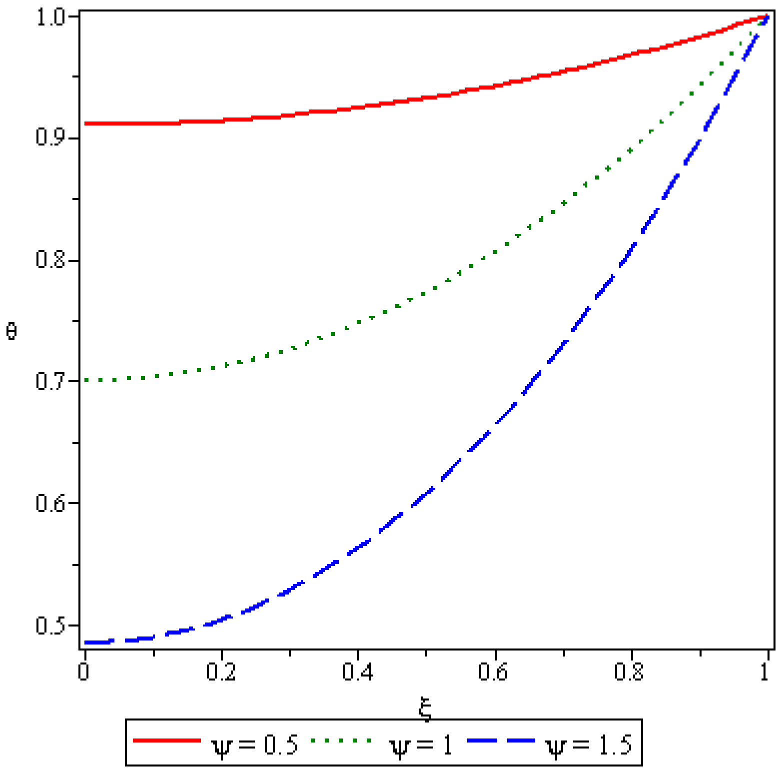

Fig. 2. Behavior of non-dimensional temperature distribution $\theta(\xi)$ for convective straight fins corresponding to ξ for distinct values of ψ at $\alpha = 0.3$ and $\beta = 1$.

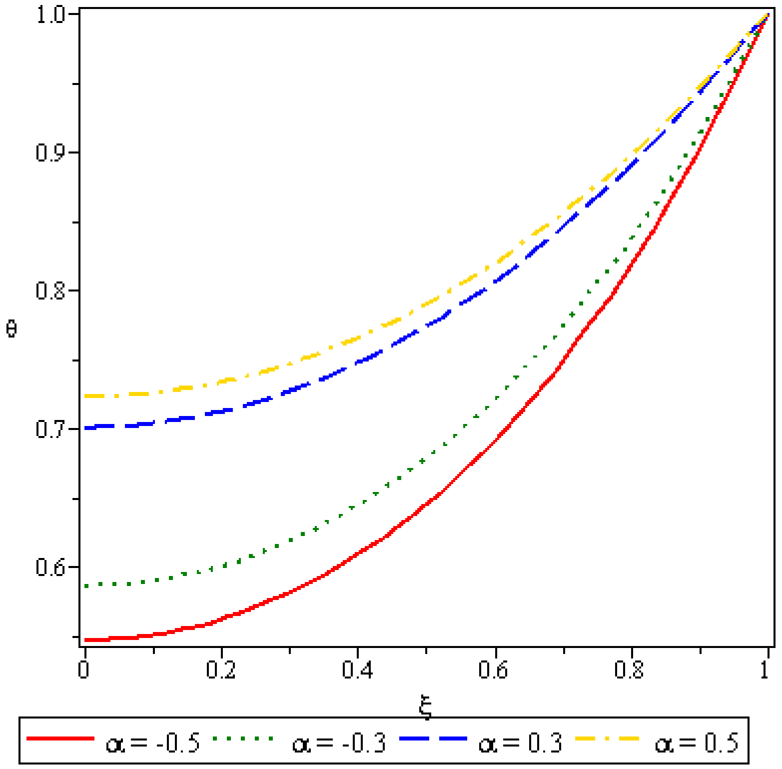

Fig. 3. Nature of non-dimensional temperature distribution $\theta(\xi)$ for convective straight fins vs. ξ for distinct values of α at $\psi = 1$ and $\beta = 1$.

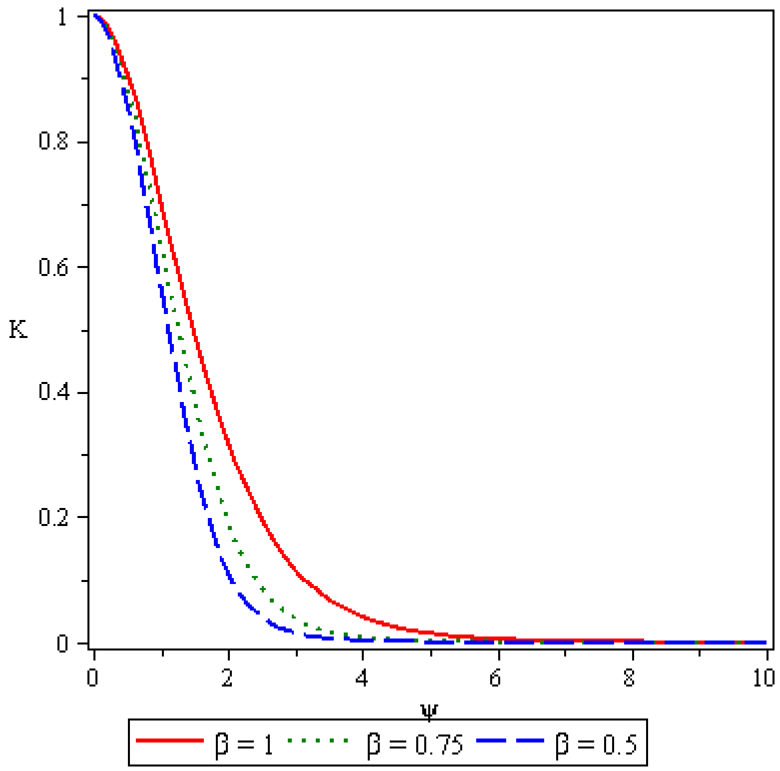

Fig. 4. The nature of fin tip temperature K with respect to ψ for various values of β at $\alpha = 0.2$.

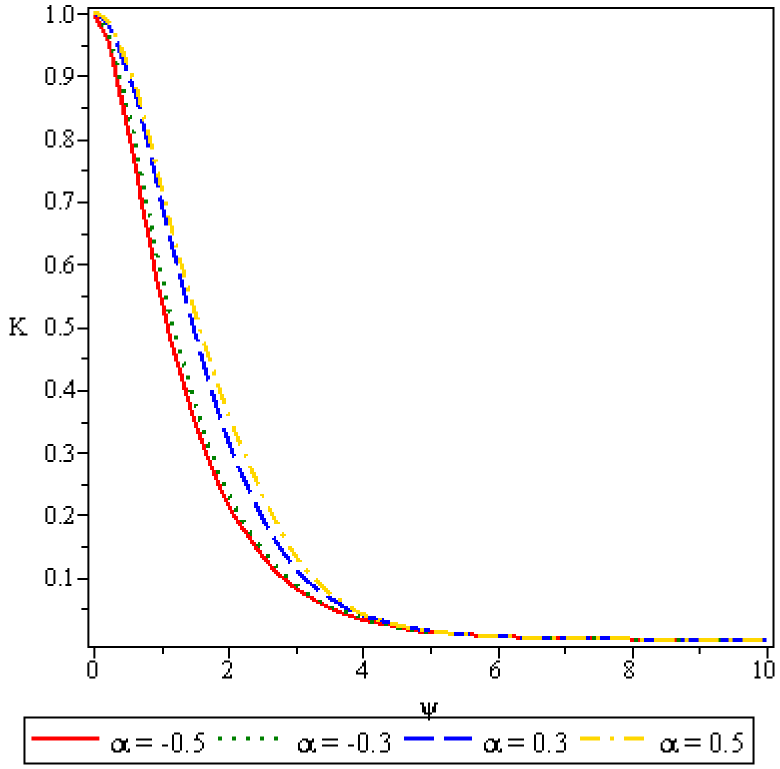

Fig. 5. The nature of fin tip temperature K with respect to ψ for various values of α at $\beta = 1$.

8. Conclusions

In this article, a new non-integer model for convective straight fins possessing temperature-dependent thermal conductivity involving Caputo-Fabrizio fractional derivative is considered. The HPM coupled with Laplace transform method is successfully used to solve the energy balance equation of arbitrary order. The HPM coupled with Laplace transform scheme is specially design to examine the nonlinear differential equations pertaining to Caputo-Fabrizio fractional derivative. The most important part of this study is to use the newly Caputo-Fabrizio fractional derivative to investigate the convective straight fins possessing the temperature-dependent thermal conductivity. The numerical simulation is performed for non-dimensional temperature distribution and fin tip temperature that reveal that the selection of the order of fractional derivative remarkably influence the outcomes. The results of this investigation are very helpful for engineers dealing with the heat conduction problems of strongly nonlinear nature. Thus, we can conclude that the use of newly Caputo-Fabrizio fractional derivative in modeling of the real world problems is very interesting, gives very fruitful consequences and HPM coupled with Laplace transform method is very powerful and efficient scheme to study such type of nonlinear problems of fractional order.

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