Numerical Study of Heat Transfer in a Porous Medium of Steel Balls

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Abstract

In this study, heat transfer in unidirectional flow through a porous medium with the fluid phase being water is analyzed using the commercial software Comsol©. The aim of the study is to validate the suitability of this package for similar problems regarding heat transfer calculations in unidirectional flow through porous media. The porous medium used in the study is comprised of steed balls of 3 mm in diameter filled in a pipe of 51.4 mm inner diameter. The superficial velocity range is 3-10 mm/s which correspond to a Reynolds number range of 150-500 for an empty pipe. Heat is applied peripherally on the outer surface of the pipe at a rate of ca. 7.5 kW/m2 using electrical ribbon heaters. The numerical results obtained using the commercial software Comsol© are compared with those obtained in the experiments once conducted by the author of this article. Results have shown that Comsol© can generate reliable results in heat transfer problems through porous media, provided all parameters are selected correctly, thus making it unnecessary to prepare expensive experimental setups and spending extensive time to conduct experiments.

Key words: *Porous media, Heat Transfer, Pressure loss, Darcy, Dispersion*

1. Introduction

A porous medium is a material containing pores through which a fluid is allowed to flow. Fluid flow through porous media was first investigated by Henry Darcy, a French hydrologist, about 150 years ago. He designed a water filter for the drinking water system of the city establishing the first theory on fluid flow through porous media. Ergun [1] calculated experimentally the coefficients of permeability and inertial coefficients of porous media which are used for pressure loss calculations.

Due to its complex structure, the fluid passing through a porous medium is mixed violently in a chaotic manner, enhancing considerably the heat transfer between the fluid and solid phases comprising the medium. This advantage makes porous media preferable in many heat transfer engineering areas such as cryogenic coolers, Stirling engines, solid matrix heat exchangers, cooling of electronic equipment and regenerators. Owing to its unrivaled heat transfer properties, porous media have therefore been studied by many researches and still are of great interest. The reason for the heat transfer enhancement in a flow through a porous medium is primarily due to a property called thermal dispersion. Dispersion is a complex phenomenon caused by the tortuous fluid motion within the pores. Tortuosity is the tendency that causes the fluid to move violently (circulations etc.) in a chaotic manner due to the geometry of the porous medium causing the convection heat transfer to occur at much higher rates. Besides, thermal conductivities of both solid particles that constitute porous medium and the fluid decide the amount of heat transferred where an average, or weighted thermal conductivity to present both solid and fluid together is calculated using the porosity which is the volume fraction of the voids (or fluid displacing the voids) and the thermal conductivities of two separate phases. The total conduction heat transfer can thus be calculated by using the effective thermal conductivity that includes also the dispersion effect which is not easy to calculate since dispersion conductivity can be obtained empirically, or using the various correlations found in the literature.

Özdemir [2], in his thesis study, conducted experiments of pressure loss and heat transfer in porous media comprised of wire meshes. He established correlations of Nusselt number as a function of Peclet number taking into consideration the temperature gradient perpendicular to flow direction and dispersion phenomenon. Özdemir and Özgüç [3] found a correlation for thermal entry length for porous media of wire meshes. Shivakumara [4] found that in a porous channel between parallel plates, thermal equilibrium is reached only when both (lower and upper) boundary conditions are of same type, otherwise system becomes thermally destabilized even at very small flow rates, depending on the Prandtl number and porosity. Chen and Hadim [5] investigated hydro-mechanical and heat transfer parameters numerically (3-D) for a non-Darcy flow through a porous medium in a square cross-sectioned channel considering the influence of the channel effect and dispersion using available models. Pulsifer and Raffray [6] found that local heat transfer coefficient is much higher at the entrance region and that solid's thermal conductivity plays an important role in heat transfer for media with lower porosities. Çelik and Kurtbaş [7] found that in a channel in which a porous medium of aluminum foam is inserted, Nusselt number may be 8 times more than that of an empty channel, depending on the Reynolds number. They concluded that smaller pores within the aluminum foam cause a definite increase in heat transfer, however also a higher pressure drop, therefore pumping cost. Nield and Kuznetsov [8] found that Nusselt number varies as the ratio of two velocities that take place when the porous medium is comprised of two different particle sizes (bi-disperse), suggesting to consider two separate temperature and velocity fields. Jiang et al. [9] used lump-capacitance and one dimensional numerical analysis methods for determining the convection coefficient at the solid-fluid interface in porous media and concluded that the first method produced more satisfactory results despite its simplicity. Hooman and Gurgenci [10] investigated the effects of viscous dissipation and boundary conditions on heat transfer using 3 different dissipation models and concluded that the shape factor of the channel (*W/H*) is responsible for the reliability of the approach. Yang and Nakayama [11] used an averaging theorem to find the stagnant and dispersion conductivities, both radial and axial, for porous media of metal foams, wire meshes and balls, comparing the experimental data with theory. Pamuk [12] conducted experiments of heat transfer and fluid flow through porous media of steel balls, both for unidirectional and oscillating flows. He found Nusselt number correlations for oscillating flow with parameters being Reynolds number, flow displacement and oscillation frequency. Dukhan et al. [13] conducted heat transfer experiments for a commercial open-cell aluminum foam cylinder heated at the wall by a constant heat flux and cooled by water flow. They identified a thermal entrance region and a thermally fully-developed region along the conduit. The thermal entry length was determined and was seen to be significant. It was found to be higher for higher flow velocities. Bağcı et al. [14] investigated the heat transfer through porous media of aluminum foams where the fluid phase is water. They found that the cycle-averaged Nusselt number was higher for higher flow displacement amplitudes for the case of higher flow frequencies. Sayehvand et al. [15] investigated numerically the effect of porous media where $(\rho c)_m$ and k_m are weighted thermal capacitance and weighted thermal conductivity of the medium, respectively.

2. Porous Medium

A schematic diagram of the porous medium used as the test chamber of the experiments mentioned above is shown in Fig. 1. The test chamber is made of stainless steel pipe of 305 mm long with an inner diameter of 51.4 mm. The volume is filled with 3 mm stainless steel balls comprising of the porous medium. Flow rate is regulated with valves installed on the water feeding line. Pressure loss along the pipe is calculated using the signals obtained from the pressure transducers mounted at the both ends of the test chamber that are then collected by the data acquisition system shown. Similarly, temperature data obtained from thermocouples shown in Fig. 2 are used for heat transfer calculations.

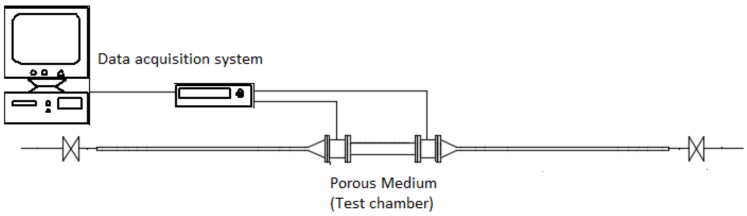

Figure 1: Test chamber [12].

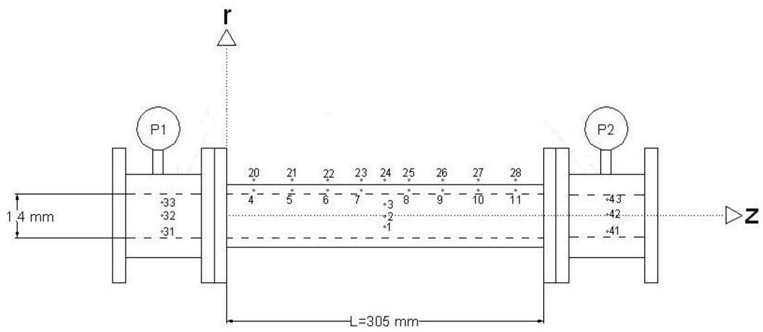

Figure 2: Thermocouple locations on the test chamber [12].

3. Theory

Using the momentum equation shown in Eq. 1, which is an extension of the equation in Darcy's law that states pressure loss is proportional to the dynamic viscosity and the superficial fluid velocity, and inversely proportional to a property called permeability, parameters of the porous medium can be calculated.

$$\frac{\Delta P}{L} = \frac{\mu}{K}U + \frac{\rho F}{\sqrt{K}}U^2 \qquad (1)$$

where, the left hand side of the equation is called the pressure gradient, or pressure loss per unit length in the flow direction. The first term on the right hand side is the Darcy term which represents the pressure drop per unit length which is proportional to the flow rate. Here μ is the dynamic viscosity, the only source of fluid friction in lower flow rates and K is the permeability, representing the availability of flow paths. It is a function of porosity, ε which stands for the percentage of volume of the void in the porous matrix, or in other words the volume of the fluid flowing through the medium. The second term on the right hand side is called the Forchheimer term which is proportional to the square of the flow rate. Other terms in the Forchheimer term are ρ and F, density and Forchheimer coefficient which represents the friction due to geometry. The porosity, ε, is calculated by measuring the volume of void in the porous medium. The permeability, K and the Forchheimer coefficient, F are obtained using least squares method based on the pressure loss-flow rate data from the numerous hydrodynamics experiments conductance previously by the author. The coefficients of the fitting curve (second order polynomial) are used to calculate K and F. The experimentally obtained coefficients, ε=0.369, K=8.87x10^{-9} m^2, F=0.580, are used also in Comsol© in order to validate the numerical model for hydrodynamic aspects porous matrix.

As opposed to the relatively straight forward pressure loss calculation as outlined above, heat transfer in porous media is a much more complex phenomenon that requires to take into considerations the concepts such as weighted thermal conductivity, k_m weighted thermal capacitance, $(\rho c)_m$ and dispersion conductivity, k_d. Sometimes the sum of k_m and k_d may be referred to as k_e, or effective thermal conductivity.

Energy equation in a flow through a porous medium is given in Eq. 2. First term on the left hand side represents the transient heat transfer within the medium whereas the second one is the convective heat transfer. The right hand side of the equation represents the conduction heat transfer, taking into consideration fluid and solid phases as well as dispersion.

$$(\rho c)_m \frac{\partial \langle T \rangle}{\partial t} + (\rho c)_f \langle \mathrm{v} \rangle \cdot \nabla \langle T \rangle = \nabla \cdot (k_m + k_d) \nabla \langle T \rangle \qquad (2)$$

where $(\rho c)_m$ and k_m are weighted thermal capacitance and weighted thermal conductivity of the medium, respectively, as given in Eqn.3 and 4 [15].

$$(\rho c)_m = (1-\varepsilon)(\rho c)_s + \varepsilon (\rho c)_f \qquad (3)$$

$$k_m = (1-\varepsilon) k_s + \varepsilon k_f \qquad (4)$$

As mentioned in the introduction, dispersion conductivity, k_d is a parameter much more difficult to evaluate. There are numerous correlations for calculating dispersion conductivity. As an example, following correlation suggested by Yang and Nakayama [11] can be used in analytical studies.

$$k_d = 0.0075 k_f \frac{Pe_d^2}{2 + 1.1 Pe_d^{0.6} / \mathrm{Pr}^{0.35}} \quad \text{(laminar flow)} \qquad (5)$$

Where $Pe_d = Ud/\alpha$, $\alpha = k_f / (\rho c)_f$ (6)

4. Numerical Procedure

Due to the geometry of the flow domain, axisymmetric case is chosen. An extra fine physics-controled mesh with 20248 elements is preferred for higher precision. Meshing is even finer near the wall (right boundary) as shown in Figure 3. Coarser mesh sizes have also been tested to see the mesh dependence of the problem. Although similar results have been obtained in less computational time, owing to the simplicity of the geometry, it is recommended to utilize a finer mesh for more complex problems if computational time does not become very critical.The width shown is half the pipe radius (25.7 mm) with the left boundary being the centerline of the pipe and the height being about 1/10th of pipe length.

Same ε, K anf F parameters obtained in the experiments are also used in Comsol© in which *Brinkman equations* and *Heat Transfer in Porous Media* are chosen as physics.

Hydrodynamic boundary conditions are: A constant inlet velocity as $u=U$ and zero absolute pressure at the outlet, symmetry on the left boundary and no-slip on the right boundary.

Thermal boundary conditions are: A constant inlet temperature as $T=T_{in}$ and a constant wall heat flux applied at the right boundary as $q=Q/A_p$ where $A_p=\pi DL$, the peripheral area of the porous medium.

The computation is performed on a Intel(R) Core(TM) i3-2350M CPU @ 2.30GHz, 2 cores computer using Windows 8 operating system.

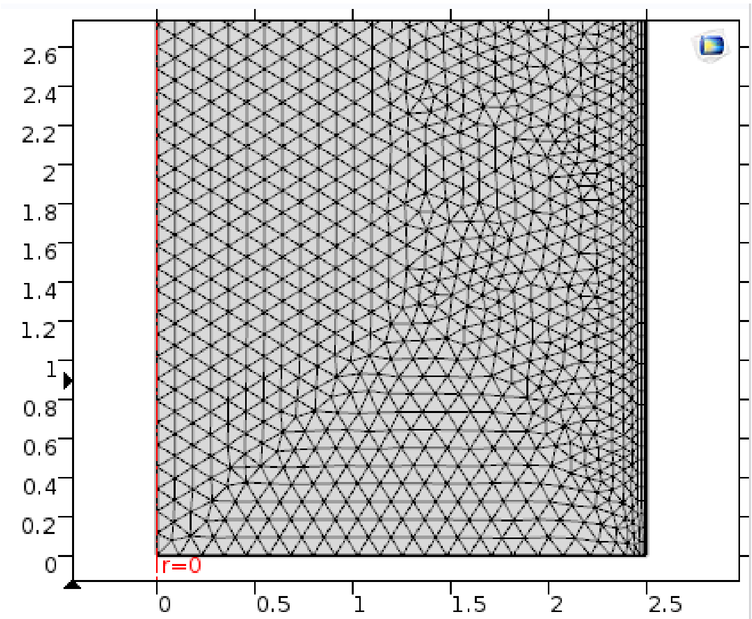

Figure 3: Extra fine physics-controled mesh near the wall.

5. Results and Discussion

The operating parameters of the problem are as follows:

ρ= 1000 kg/m^3, μ= 0.001 Pa.m, L= 0.305 m, D= 0.0514 m, k_s=16.3 W/m.C, k_f=0.613 W/m.C

Eqn. 1 gives pressure loss values 131 Pa and 553 Pa for each flow rate, respectively. The same values can also be seen in Fig. 4, the longitudinal pressure distribution along the pipe, validating the porous model of Comsol© in terms of hydrodynamic aspects. The *channel effect* near the wall can be observed in radial velocity distribution (Fig. 5). It is caused by the increased porosity near the wall, causing the flow to have a higher local velocity which becomes zero at the wall due to no-slip condition.Fig. 6 shows the longitudinal wall temperature variation along the porous medium, generated by Comsol©. Fig. 7 shows the comparison of longitudinal wall temperature distribution with those of experiments. It is also interesting to view the temperature distribution within the domain as given in Figs. 8 and 9. It is also worth checking the radial temperature gradient at the wall (Fig. 10) in the middle of the test section in order to estimate the heat flux given in Eqn. 7:

$$q = -k_e(\Delta T / \Delta r) \tag{7}$$

Where $\Delta T / \Delta r$ is the radial temperature gradient at the wall. If the temperature gradient near the wall for the lower flow rate is taken to be $650\ ^oC/m$ and $550\ ^oC/m$ for lower and higher flow rates respectively, using Fig. 10, k_e, the effective thermal conductivity is then calculated to be *11.5 W/m.C* and *13.6 W/m.C*. As a matter of fact, a similar value can be calculated using Eqns. 4 and 5 for k_m and k_d to obtain $k_e = k_m + k_d$ where $k_m = 10.6$ *W/m.C and* $k_d = 1\text{-}4$ *W/m.C*, depending on Pe_d. Finally, Nusselt number is calculated as:

$$Nu = hD / k_f \tag{8}$$

Where $h = q / \Delta T$ with $\Delta T = T_w - T_{in}$.

Fig. 11 shows the longitudinal Nusselt number variation along the porous medium, generated by Comsol©. Fig. 12 shows the comparison of this Nusselt number variation with those of experiments.

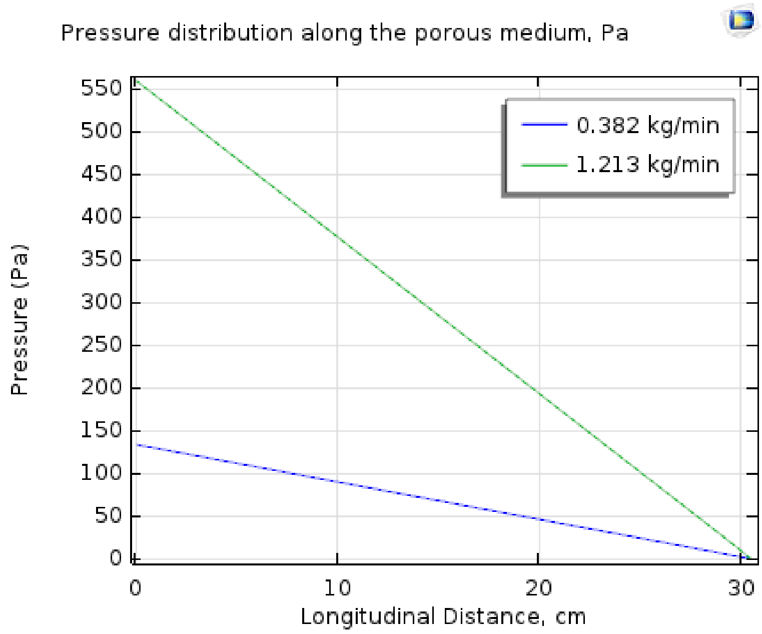

Figure 5: Longitudinal pressure distribution along the pipe.

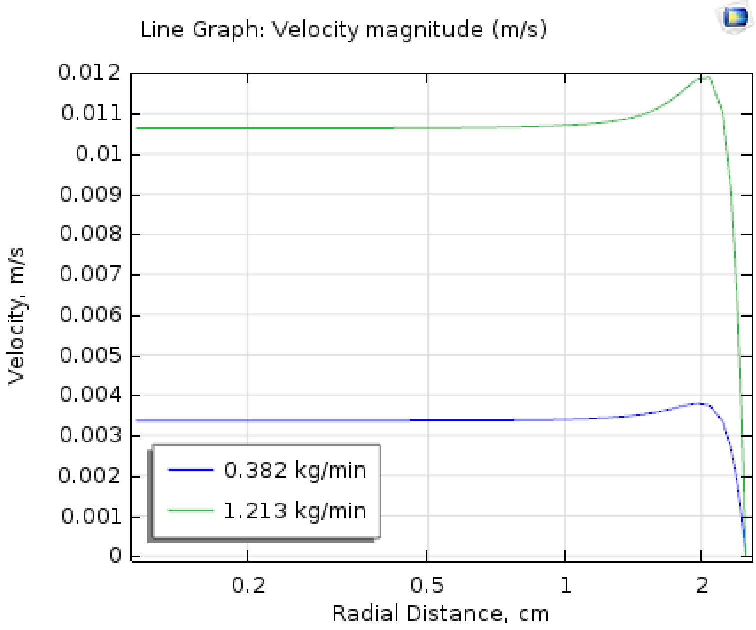

Figure 5: Radial velocity distribution at the pipe exit.

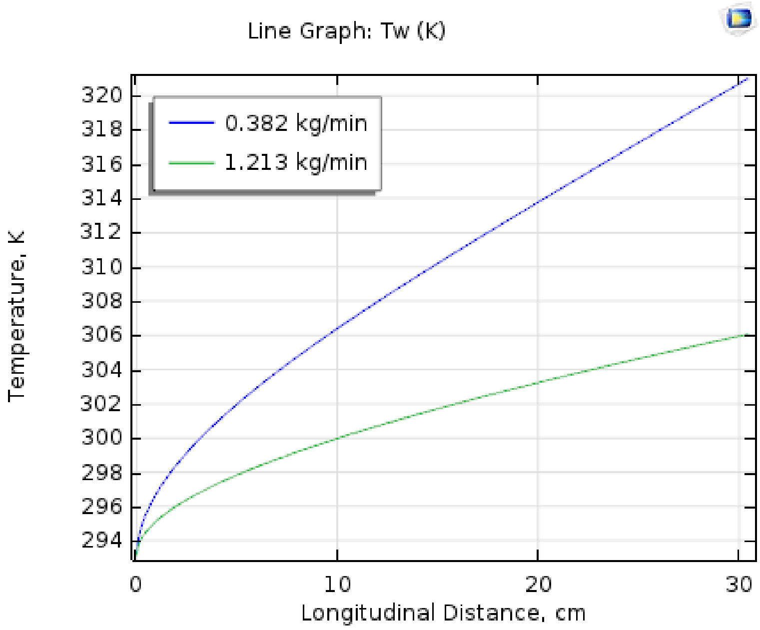

Figure 6: Longitudinal temperature distribution along the pipe.

Figs. 7 and 12 show the comparison of the experimental heat transfer results with those of Comsol© for the flow through a porous medium. It is clearly seen that the both results are very close to each other, proving the reliability of Comsol© solutions in heat transfer and flow through porous media problems.

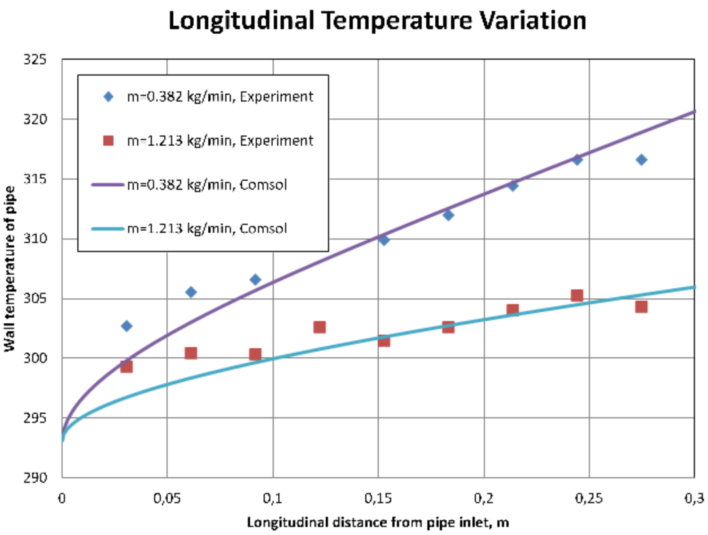

Figure 7: Comparison of longitudinal temperature distributions.

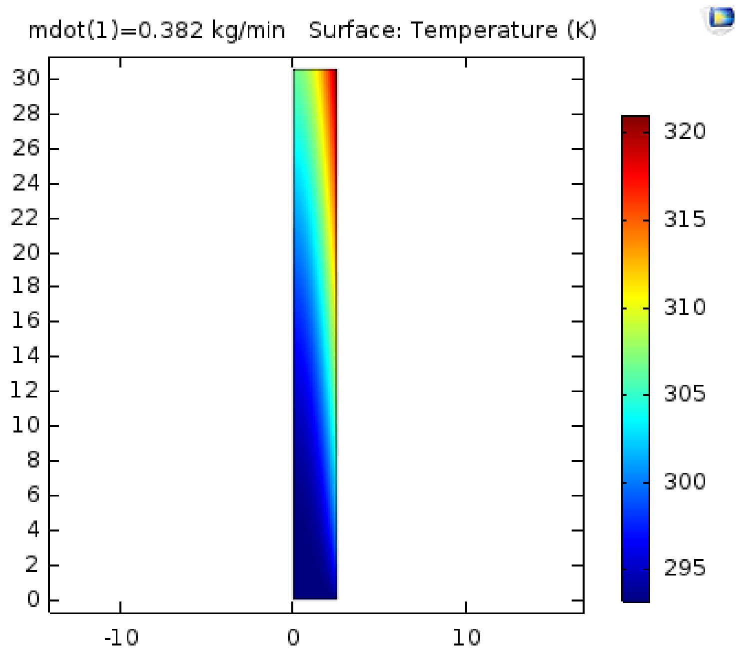

Figure 8: Temperature distribution within the domain for 0.382 kg/min.

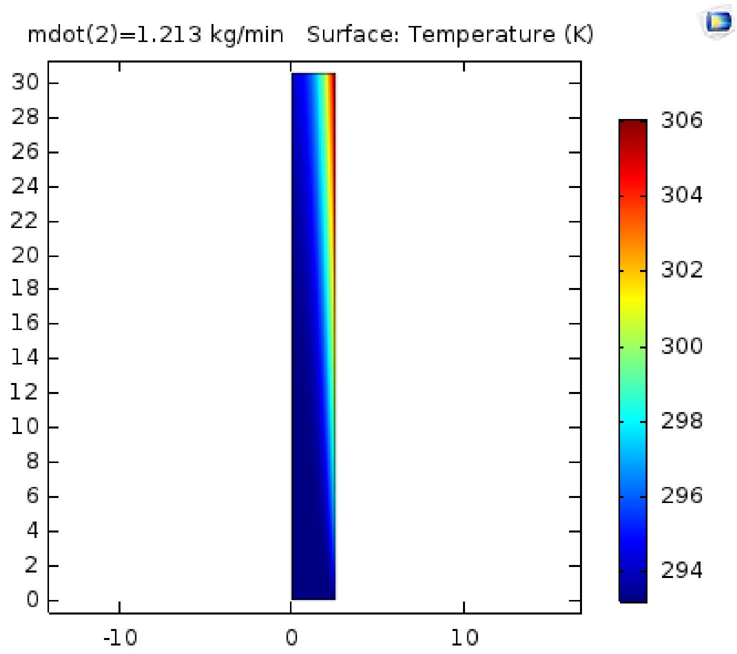

Figure 9: Temperature distribution within the domain for 1.213 kg/min.

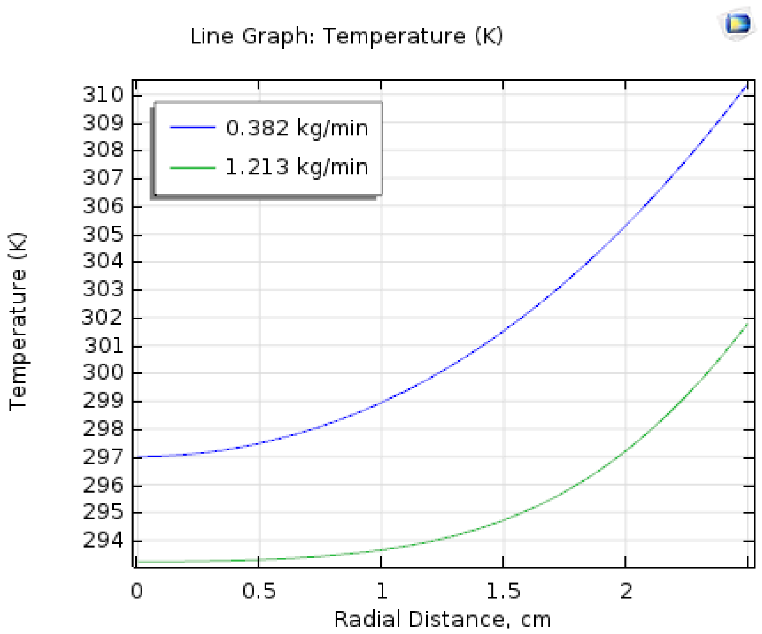

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Figure 10: Radial temperature distribution at the middle of the pipe

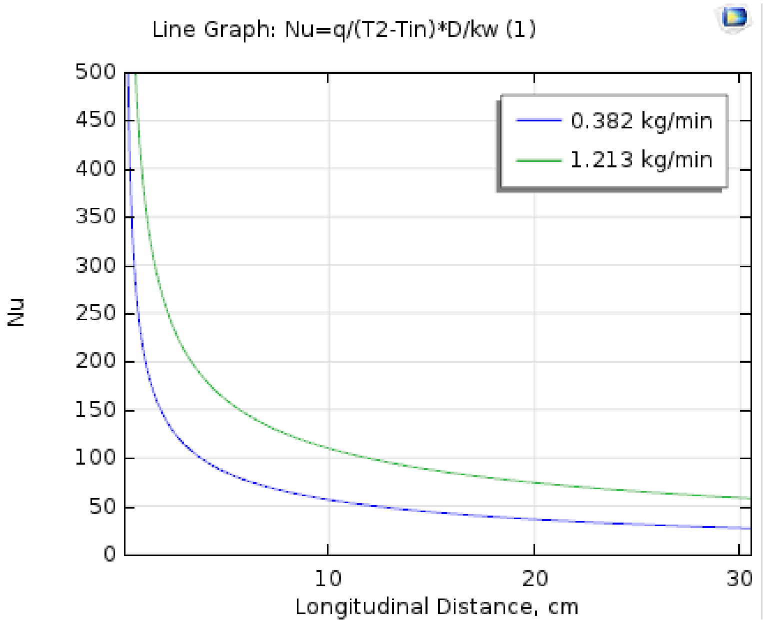

Figure 11: Nusselt number variation along the pipe.

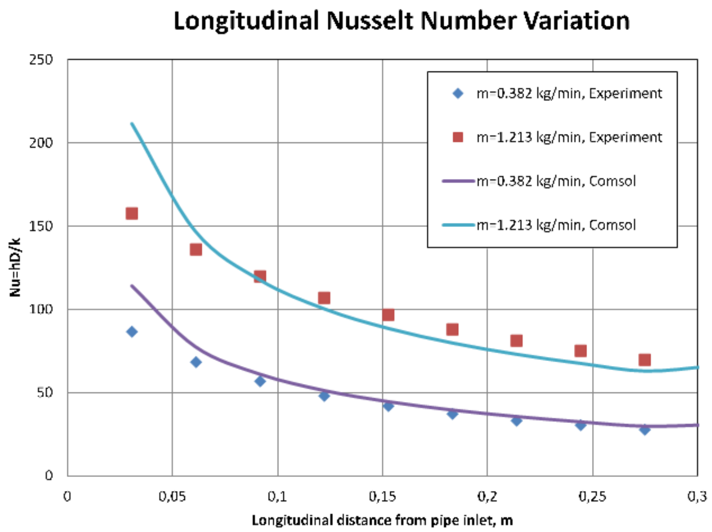

Figure 12: Comparison Nusselt number variations.

Conclusion

Heat transfer in flow through a porous medium of steel balls where the fluid phase is water, is investigated numerically in this study using the commercial software Comsol©. The results are then compared to those obtained in a series of past experiments, also conducted by the author of this study, validating the numerical model with a great success. It has thus been shown that Comsol© is a powerful computational instrument in dealing with fluid flow and heat transfer problems in porous media, realizing the aim of the study. It can be concluded that with careful modeling including a flawless geometry with a fine enough mesh grid size, although similar results can obtained in less computational time using coarser mesh sizes if geometry is as simple as that of the current study, finer mesh sized should be given preference for more complex geometries, optimizing between the precision of results and computational time together with a set of carefully and correctly chosen parameters and boundary/initial conditions, the software is very capable to produce reliable solutions, as shown in this work, thus making it unnecessary to prepare expensive experimental setups and spending extensive time to conduct experiments.

Nomenclature

A	cross-sectional area of the test chamber [m^2]
A_p	peripheral area of the test chamber [m^2]
d	ball diameter [m]
D	inner diameter of test chamber [m]
F	inertial coefficient [-]
h	heat transfer coefficient [$Wm^{-2}K^{-1}$]
K	permeability [m^2]
L	length of the porous medium [m]
ΔP	pressure difference [Pa]
$\Delta P/L$	pressure gradient [$Pa.m^{-1}$]
k	conduction coefficient [$Wm^{-1}K^{-1}$]
k_d	dispersion conduction coefficient [$Wm^{-1}K^{-1}$]
k_e	effective conduction coefficient [$=k_m+k_d$, $Wm^{-1}K^{-1}$]
Pe_d	Particle diameter based Peclet number [$=PrRe_d$, -]
Pr	Prandtl number [-]
Re	Reynolds number [$=\rho UD/\mu$, -]
Q	joule heating obtained from ribbon heaters [W]
q	heat flux at the wall [Wm^{-2}]
r	radial distance from the centerline the test chamber
r_o	radius of the test chamber [m]
U	amplitude of mean fluid velocity [ms^{-1}]
$\mathbf{v}$	velocity vector [ms^{-1}]

Greek Symbols

α	heat diffusion coefficient
ε	porosity [-]
μ	dynamic viscosity [Pa.s]
ρ	fluid density [kgm^{-3}]

Subscripts:

f	fluid
m	medium
s	solid
w	wall

References

[1] Ergun, S., Fluid flow through packed columns, *Chemical Engineering Progress, Vol. 48, No.2* (1952) pp. 89-94.

[2] Özdemir, M., 1996. Forced convective heat transfer in porous medium of wire screen meshes, Ph.D. Thesis, Istanbul Technical University, Turkey, 1996.

[3] Özdemir, M. and Özgüç, A. F., Forced convective heat transfer in porous medium of wire screen meshes, *Heat and Mass Transfer, 33* (1997) 129-136 © Springer-Verlag .

[4] Shivakumara, I. S.,. Boundary and inertia effects on convection in porous media with throughflow, *Acta Mechanica, 137* (1998), pp.151-165.

[5] Chen, G. and Hadim, H.A., 1999. Numerical study of three-dimensional non-Darcy forced convection in a square porous duct, *International Journal of Numerical Methods for Heat & Fluid Flow, Vol. 9 No. 2* (1999), pp. 151-169.

[6] Pulsifer, J. E. and Raffray, A. R., *19th Symposium* on structured porous media for high flux applications, Fusion Engineering (2002).

[7] Çelik, N. and Kurtbaş, İ.. Alüminyum köpük malzemelerin ısı değiştiricisi performansına etkisi, Fırat Üniversitesi, *Mühendislik Fakültesi, Makine Mühendisliği Bölümü* (2007).

[8] Nield, D.A. and Kuznetsov, A.V., 2005. Heat transfer in bidisperse porous media, *International Journal of Heat and Mass Transfer, Vol. 49, Issues 17-18* (2006), pp. 3068-3074.

[9] Jiang, P., Xu, R. and Gong W., 2006. Particle-to-fluid heat transfer coefficients in miniporous media", *Chemical Engineering Science, 61* (2006) 7213 – 7222.

[10] Hooman, K., Gurgenci, H. and Merrikh, A.A., 2006. Heat transfer and entropy generation optimization of forced convection in porous-saturated ducts of rectangular cross-section, *International Journal of Heat and Mass Transfer, 50* (2007) 2051.

[11] Yang, C. and Nakayama, A., A synthesis of tortuosity and dispersion in effective thermal conductivity of porous media, *International Journal of Heat and Mass Transfer, 53* (2010) 3222–3230

[12] Pamuk, M.T., 2012. Heat transfer in oscillating flow through porous media consisting of steel balls, Ph.D. Thesis, Istanbul Technical University, Turkey, 2012.

[13] Dukhan, N., Bağcı, Ö., Ozdemir, Thermal development in open-cell metal foam: An experiment with constant wall heat flux, *International Journal of Heat and Mass Transfer, 85 (2015)*, pp. 852–859.

[14] Bağcı, Ö., Dukhan, N., Ozdemir, M. And Kavurmacıoğlu, A.L., Experimental heat transfer due to oscillating water flow in open-cell metal foam, *International Journal of Thermal Sciences, Vol. 101* (2016), pp. 48-58.

[15] Sayehvand, H., Yari S.M., Basiri P.A., Numerical study of forced convection heat transfer over three cylinders in staggered arrangement immersed in porous media, *Thermal Science, DOI: 10.2298/TSCI150808249S, PUBLISHED ONLINE: 2016-11-06.*