

# A new model to measure the performance of the fins based on exergy analysis

Seyyed Masoud SEYYEDI \*, Tahereh RAJAEI-ZADEH, Mehdi HASHEMI-TILEHNOEE

Department of Mechanical Engineering, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran

\* Email: [s.masoud\\_seyyedi@yahoo.com](mailto:s.masoud_seyyedi@yahoo.com), Tel: +98 915 1089309, Fax: +98 173 4225166

**Abstract:** *Currently, two basic measures performance models are conventionally defined to evaluate the performance of the extended surfaces or the fins. First is the fin efficiency that is defined as the ratio of actual heat transferred by a fin to heat that would be transferred if the entire fin were at base temperature. Second is the fin effectiveness that is defined as the ratio of heat flux from the wall with the fin to heat flux from the wall without the fin. In the present work, a new criterion is proposed to measure the performance of the fins. The new criterion is defined as the ratio of exergy of convective heat transferred by the fin to the irreversibility of the fin. The new criterion named fin ecological coefficient of performance ( $ECOP_f$ ) based on the second law of thermodynamics whereas the fin efficiency and fin effectiveness carried out by the first law of thermodynamics. A code has been developed using these models to compare the performances of a typical fin with respect to the fin parameters and cooling fluid. According to the results, it can be concluded that the  $ECOP_f$  model is a rational criterion rather than two other models. In addition, since  $ECOP_f$  model considers the irreversibility of the control surface so it is a better measuring performance model in new fin design.*

**Keywords:** Extended surface, Fin efficiency, Fin effectiveness,  $ECOP$ , Exergy analysis

## 1. Introduction

One of the most important problems in the field of heat transfer is good design of extended surfaces. Extended surfaces that are called fins are used for promoting heat transfer from a hot surface to the surrounding. In addition, the fins are widely used to increase the heat transfer rate between a solid and cooling fluid. The major variables that influence the heat and mass transfer through a fin include, fin geometry, fin material and operating conditions [1]. Commonly fin geometries are classified to annular fins, pin fins, longitudinal or straight fins [2]. Fin profiles are classified as rectangular, triangular or trapezoidal, convex parabolic, and concave parabolic, too [3]. Fins optimization is a process to obtain the optimum

dimensions of a fin for a required heat transfer rate, or calculating the maximum heat transfer rate where the dimensions of the fin are known [4]. Many researchers have been analyzed the effect of radiation, curvature, thermal conductivity, and heat transfer coefficient [5-7]. The minimization of entropy generation to design of fins has been applied by Poulikakos and Bejan [8]. By entropy generation analysis, they have derived the optimum dimensions for different fin geometry. Three types of fins been studied by Laor and Kalman. They considered the effect of the tip condition and fins with convective tips then compared the result with insulated tip fins [9]. Bahadur and Bar-Cohen [10] analytically derived the correlation for cylindrical spines of orthotropic material. In other work, a dimensionless closed form solution for cylindrical spines with orthotropic material is derived by Zubair et al. [11]. Mustafa et al. [12] derived a closed form analytical solution to study the thermal performance of orthotropic material annular fins with a contact resistance, too. Medrano et al. [13] compared the performance of five different commercial heat exchangers experimentally. They concluded that compact heat exchangers equipped with the fins are the most promising heat exchangers for future applications. As a renewable resource, solar stills are simple devices used to produce distilled water from saline water and effluents. For example, basin solar stills with rectangular fins in a basin have been used to produce the distilled water from effluents where the performance of the system was enhanced [14,15]. However, the effects of fin configuration on single basin solar stills had been theoretically and experimentally investigated [16]. In 2016, hollow circular and square fins have been integrated into the basin plate and its performance has been investigated by Rajaseenivasan and Srithar [17]. Li and Byon [18] parametrically studied the orientation effect of a radial heat sink with rectangular fins. Almendros-Ibanez et al. studied the prescribed temperature at the fin tip with constant cross sectional area [19]. Sadollah et al. [20] suggested a novel approach to numerically study the longitudinal fins. Kundu and Lee developed an analytical method to study the annular step fin that made of porous material under a moving condition [21]. Lee et al. [22] proposed a correlation for estimating the Nusselt number of natural convection flows from cylinders equipped with triangular fins. In 2016, hyperbolic annular fins studied under dehumidifying operating conditions and its thermal performance optimized analytically [1]. According to the above literatures, performances of all the fins have been evaluated using the definition of the fin efficiency or the fin effectiveness. In this study, a new model is proposed for evaluating the performance of the fins based on second law thermodynamics. Then this model verify for a typical fin that surrounded by fluid.

## 2. Fin Analysis

The simplest fin geometry has a constant cross sectional area. Since the cross-sectional area does not vary with the axial coordinate, so the temperature profile differential equation becomes easy to integrate. The solution of this basic heat transfer problem can be found in references [23-30]. Figure 1 shows the longitudinal fins having rectangular, trapezoidal, triangular, and concave parabolic profiles, with length  $L$  and the pin fins with diameter  $D$ .

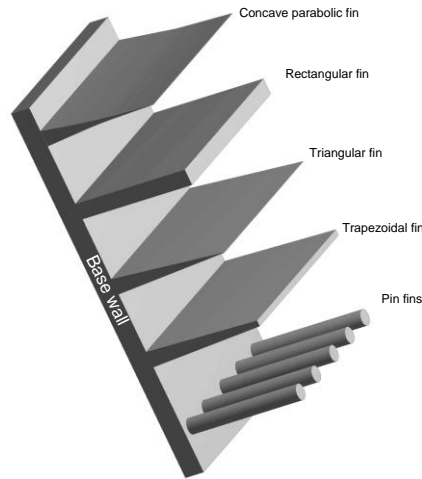


Figure 1. Schematic view of longitudinal fin having different profiles

The differential equation for a fin with the constant cross sectional area is as follows [23]:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \quad (1)$$

where

$$\theta = T - T_\infty \quad \text{and} \quad m^2 = hP/kA \quad (2)$$

$A$  is the cross sectional of the fin and  $P$  is the perimeter of the fin. One boundary condition is:

$$\theta = \theta_B = T_B - T_\infty \quad \text{at} \quad x = 0 \quad (3)$$

The other boundary condition depends on the physical situation. Several cases may be considered.

**Case1:** The fin is very long and the temperature at the end of the fin is considered equal to the temperature of surrounding fluid.

**Case2:** The fin is of finite length and loses heat by convection from its end.

**Case 3:** The end of the fin is insulated so that  $dT/dx = 0$  at  $x = L$ .

The general solution for Eq. (1) can be written as follows:

$$\theta = C_1 e^{-mx} + C_2 e^{mx} \quad (4)$$

In this work, case 3 is considered. For case 3 boundary conditions are:

$$\begin{cases} \theta = \theta_B & \text{at } x = 0 \\ \frac{d\theta}{dx} = 0 & \text{at } x = L \end{cases} \quad (5)$$

Therefore the solution is:

$$\frac{\theta}{\theta_B} = \frac{e^{-mx}}{1+e^{-2mL}} + \frac{e^{mx}}{1+e^{2mL}} = \frac{\cosh[m(L-x)]}{\cosh mL} \quad (6)$$

All of the heat lost by the fin must be conducted into the base at  $x = 0$ . Therefore, the heat loss from the fin can be obtained by:

$$q_B = -kA \left. \frac{dT}{dx} \right|_{x=0} \quad (7)$$

In case 3 it is

$$q_B = -kA\theta_B m \left( \frac{1}{1+e^{-2mL}} - \frac{1}{1+e^{+2mL}} \right) = \sqrt{hPkA} \theta_B \tanh mL \quad (8)$$

## 2.1 Fin efficiency

Two basic measures of fin performance are particularly useful in a fin design. The first is called the efficiency ( $\eta_f$ ) that is defined as follows [24]:

$$\eta_f = \frac{\text{actual heat transferred by a fin}}{\text{heat that would be transferred if the entire fin were at base temperature}} \quad (9)$$

The fin efficiency is:

$$\eta_f = \frac{\sqrt{hPkA} \theta_B \tanh mL}{hPL\theta_B} = \frac{\tanh mL}{mL} \quad (10)$$

## 2.2 Fin effectiveness

A second measure of fin performance is called the effectiveness ( $\varepsilon_f$ ) that is defined as follows [30]:

$$\varepsilon_f = \frac{\text{heat flux from the wall with the fin}}{\text{heat flux from the wall without the fin}} \quad (11)$$

This can be easily computed from the efficiency:

$$\varepsilon_f = \eta_f \frac{\text{surface area of the fin}}{\text{cross sectional area of the fin}} \quad (12)$$

The fin effectiveness for case 3 is:

$$\varepsilon_f = \frac{q \text{ with fin}}{q \text{ without fin}} = \frac{\eta_f A_f h \theta_0}{h A_b \theta_0} = \eta_f \frac{A_f}{A_b} = \eta_f \frac{PL}{A} = \frac{\tanh mL}{\sqrt{hA/kP}} \quad (13)$$

It should be mentioned that for a proper fin, the fin efficiency is less than one, whereas fin effectiveness must be greater than one.

### 3. Entropy generation

The Gouy-Stodola relation for a control volume is as follows [31]:

$$\dot{I} = T_0 \dot{S}_{gen} \quad (14)$$

which means that the irreversibility rate of a process is the product of the entropy generation rate for all systems participating in the process and the temperature of the environment.

Extended of Eq. (14) is:

$$\dot{I} = T_0 \left[ \sum_{OUT} \dot{m}_e s_e - \sum_{IN} \dot{m}_i s_i - \sum_r \frac{\dot{Q}_r}{T_r} \right] \quad (15)$$

The fin entropy generation rate is [8]:

$$\dot{S}_{gen} = \frac{q_B \theta_B}{T_\infty^2 (1 + \theta_B / T_\infty)} + \frac{F_D U_\infty}{T_\infty} \quad (16)$$

In this expression  $\theta_B$  is the temperature difference between the base of the fin and free stream; i.e.  $\theta_B = T_B - T_\infty$ . For more details, see Refs. [8, 32].

### 4. Exergy analysis

Exergy is one of the important concepts of the second law of thermodynamics, which is the maximum useful work that we can obtain from flow of matter or energy [33]. For a heat transfer rate  $\dot{Q}$  and a temperature at the control surface where the heat transfer is taking place  $T_r$  the maximum rate of conversion of thermal energy to work is [31]:

$$\dot{W}_{max} = \dot{E}^Q = \dot{Q}_r \tau \quad (17)$$

where:

$$\tau = 1 - \frac{T_0}{T_r} \quad (18)$$

$\tau$  is called dimensionless exegeric temperature. If the heat flux  $\dot{Q}_A$  (heat transfer rate per area) distribution at the various temperatures,  $T$ , is known, the associated thermal exergy flux can be determined from [31]:

$$\dot{E}^Q = \int_A \left( \frac{T - T_0}{T} \right) \dot{Q}_A dA \quad (19)$$

where  $A$  is the heat transfer area.

### 5. Fin ecological coefficient of performance

$$ECOP_f = \frac{\text{exergy rate corresponding to convective heat transfer of fin}}{\text{irreversibility rate of fin}} \quad (20)$$

$$ECOP_f = \frac{\dot{E}Q_{conv}}{i} = \frac{\int_0^L hP(T-T_\infty)\left(1-\frac{T_0}{T}\right)dx}{T_0\dot{S}_{gen}} \quad (21)$$

In case 3 the ECOP can be obtained as follows:

$$ECOP_f = \frac{\int_0^L hP(T-T_\infty)dx - \int_0^L hPT_0dx + \int_0^L hPT_\infty\frac{T_0}{T}dx}{T_0 \left[ \frac{q_B\theta_B}{T_\infty^2\left(1+\frac{\theta_B}{T_\infty}\right)} + \frac{F_D U_\infty}{T_\infty} \right]} \quad (22)$$

$$ECOP_f = \frac{\sqrt{hPkA}\theta_B \tanh mL + hPT_0L + hPT_\infty T_0 \frac{c}{b} \left[ \text{Arctan}\left(\frac{1+\frac{1}{2a}}{b}\right) - \text{Arctan}\left(\frac{e^{mL}+\frac{1}{2a}}{b}\right) \right]}{T_0 \left[ \frac{q_B\theta_B}{T_\infty^2\left(1+\frac{\theta_B}{T_\infty}\right)} + \frac{F_D U_\infty}{T_\infty} \right]} \quad (23)$$

where

$$a = \frac{\theta_B}{T_\infty} \frac{1}{2 \cosh mL} \quad (24)$$

$$b^2 = 1 - \frac{1}{4a^2} \quad (25)$$

$$c = \frac{-1}{mT_\infty a} \quad (26)$$

The numerator and denominator of ECOP are divided by  $G_0$  as a dimensionless analysis, so

$$G_0 = T_0 \frac{q_B^2 U_\infty}{kvT_0T_\infty} \quad (27)$$

For a pin fin with boundary condition in case 3, the final result for ECOP is:

$$ECOP_f = \frac{G_1 + G_2 + G_3}{G_4 + G_5} \quad (28)$$

where

$$G_1 = \frac{4}{\pi} \times \frac{1}{mL \tanh mL} \times \frac{\Gamma^2}{\beta Re_L} \quad (29)$$

$$G_2 = \frac{-4}{\pi} \left(\frac{T_0}{T_\infty}\right) \times \frac{1}{\tanh^2 mL} \times \frac{\Gamma}{\beta^2 Re_D} \quad (30)$$

$$G_3 = \frac{-8}{\pi} \left(\frac{T_0}{T_\infty}\right) \times \frac{1}{mL \tanh^2 mL} \times \frac{\Gamma^2}{\beta^2 Re_L} \times \frac{J}{\sqrt{\left(\frac{\beta}{\cosh mL}\right)^2 - 1}} \quad (31)$$

$$\text{where } J = \text{Arctan}\left(\frac{1+\frac{1}{2a}}{b}\right) - \text{Arctan}\left(\frac{e^{mL}+\frac{1}{2a}}{b}\right) \quad (32)$$

$$G_4 = \frac{4}{\pi} \left(\frac{T_0}{T_\infty}\right) \times \frac{1}{(1+\beta) \times mL \tanh mL} \times \frac{\Gamma^2}{Re_L} \quad (33)$$

$$G_5 = \frac{1}{2} BC_D Re_D Re_L \text{ and } B = \frac{\rho kv^3 T_0}{q_B^2} \quad (34)$$

where

$$\Gamma = \frac{L}{D} = \frac{Re_L}{Re_D} \quad (35)$$

$$\beta = \frac{\theta_B}{T_\infty} \quad (36)$$

$$mL = 2 Nu^{1/2} (\lambda/k)^{1/2} \Gamma \quad (37)$$

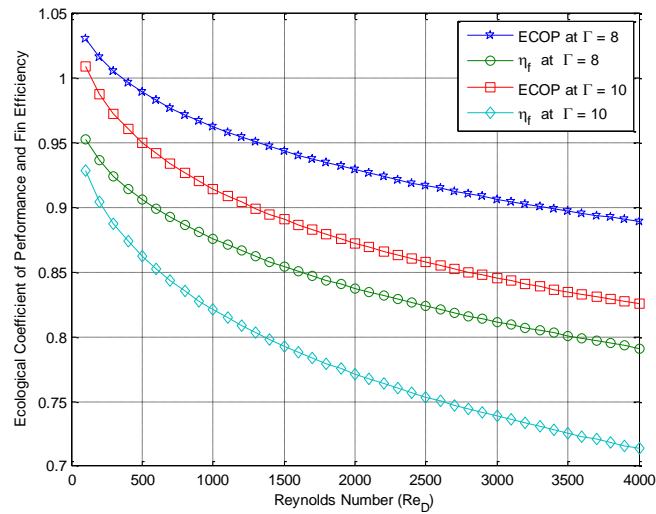
In this analysis, the Nusselt number and the drag coefficient have been evaluated from the results developed for a single cylinder in cross flow by giving equations [8, 32, 34].

$$Nu = 0.683 Re_D^{0.466} Pr^{\frac{1}{3}} \quad \text{and} \quad C_D = 5.484 Re_D^{-0.246} \quad \text{for} \quad 40 < Re_D < 4 \times 10^3 \quad (38)$$

The details of proven for ECOP is given in [Appendix A](#).

## 6. Results and discussion

In order to compare the fin efficiency and the ECOP of the fin, a code was developed in MATLAB and these parameters were plotted with respect to Reynolds number. [Figure 2](#) shows these results for a pin fin. In this figure the fin efficiency and ECOP of the fin are plotted with respect to Reynolds number at two values of slenderness ratio.



[Figure 2](#). The fin efficiency and ecological coefficient of performance with respect to Reynolds number

The figure shows both the fin efficiency and ECOP decrease with increasing of Reynolds number in a fixed slenderness ratio. For each Reynolds number, the value of fin ECOP is more than fin efficiency. In addition, in a fixed Reynolds number by increasing the slenderness ratio ECOP<sub>f</sub> decreases. This means that by increasing the fin's L/D the fin

irreversibility increased, too. In the calculations, the values of Table 1 have been considered as constants.

Table 1. The values of constants [24].

Parameter (unit)	$\theta_B/T_\infty$	$U_\infty$	$T_0$	$T_\infty$	$\nu$	$\rho$	$\lambda$	M	Pr	$\Gamma$
	(-)	(m/s)	(K)	(K)	(m <sup>2</sup> /s)	(kg/m <sup>3</sup> )	(W/m <sup>2</sup> K)	(-)	(-)	(-)
Value	0.1	0.2	298.15	300.15	15.7E-6	1.17	0.026	100	0.707	8 and 10

Figure 3 shows the fin effectiveness and ECOP of the fin with respect to Reynolds number for two different values of slenderness ratio.

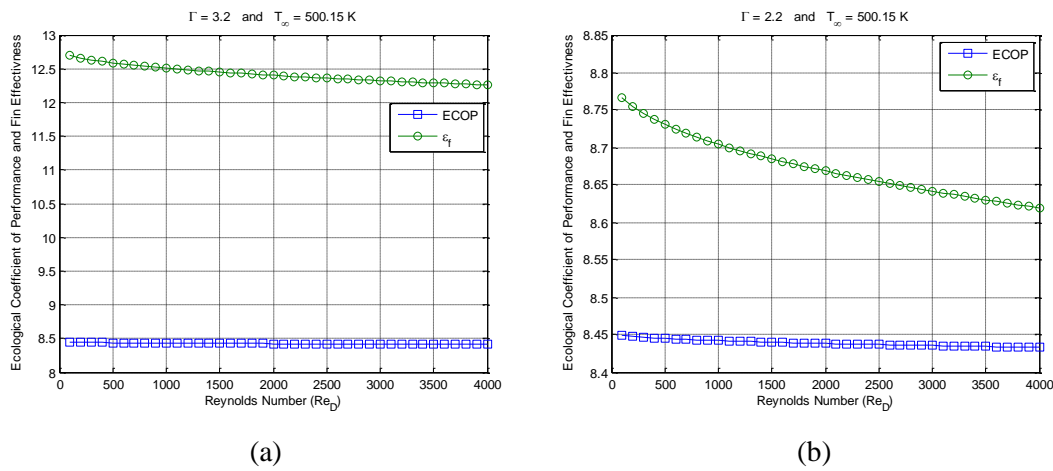


Figure 3. The fin effectiveness and ecological of performance with respect to Reynolds number at a)  $\Gamma = 2.2$  and  $T_\infty = 500.15$  K b)  $\Gamma = 3.2$  and  $T_\infty = 500.15$  K

As it is seen, the difference between the fin effectiveness and ECOP of the fin increases as slenderness ratio increases from 2.2 to 3.2. In addition, increasing Reynolds number had no effect on fin effectiveness. In these figures, the absolute temperature of free stream has been considered equal to 500.15 K. Air properties at 500.15 K read from the table of air properties as listed in Table 2.

Table 2. Air properties [24]

Parameter (unit)	$T_\infty$ (K)	$\nu$ (m <sup>2</sup> /s)	$\rho$ (kg/m <sup>3</sup> )	$\lambda$ (W/m <sup>2</sup> K)	Pr (-)
Value	500.15	37.9E-6	0.704	0.04	0.68

Figure 4 represents ECOP of the fin with respect to Reynolds number for three different values of property group (M) and three different values of dimensionless temperature ( $\beta$ ) at a fixed slenderness ratio ( $\Gamma = 10$ ).



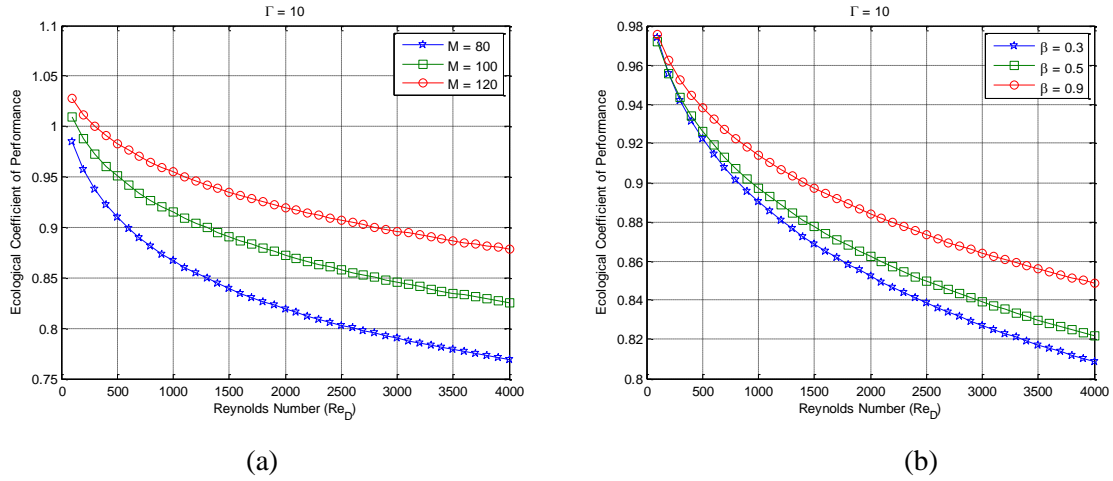


Fig. 4. Fin ecological coefficient of performance with respect to Reynolds number at a) different values of  $M$  and  $\Gamma = 10$  b) different values of  $\beta$  and  $\Gamma = 10$

The figure shows that the ECOP of the fin decreases with increasing of Reynolds number for each property group. Fin ECOP increases, or fin irreversibility decreases, with increasing of property group for each value of the Reynolds number. In addition, the figure shows that the ECOP of the fin decreases with increasing of Reynolds number for each dimensionless temperature. Also, ECOP of fin increases or fin irreversibility decreases with increasing of dimensionless temperature for each value of the Reynolds number. Fig. 5 shows a comparison between the values of ECOP in two values of the Reynolds number and three values of velocity of free stream.

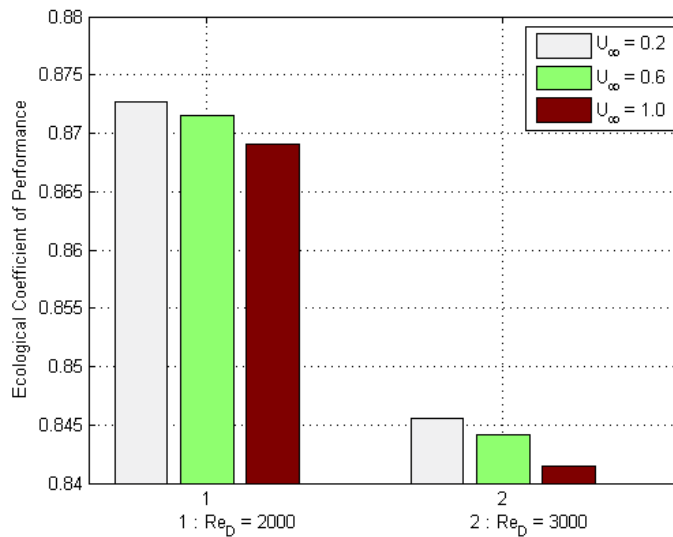


Fig. 5. Fin ecological coefficient of performance at two different values of Reynolds number and three different values of velocity of free stream

The figure shows that the values of  $ECOP_f$  related to  $Re_D = 2000$  are more than that of the  $Re_D = 3000$ . For example, the value of  $ECOP_f$  almost 3.2% decreases as the Reynolds number increases from 2000 to 3000 for  $U_\infty = 0.6 \text{ m/s}$ . Finally the velocity of free stream has no special effects on fin irreversibility.

## 7. Conclusion

In this work, a new criterion for evaluation of performance of the fins is introduced that is called  $ECOP_f$ . The other criteria are fin efficiency and fin effectiveness that based on the first law of thermodynamics whereas the  $ECOP_f$  is based on the second law of thermodynamics. The fin efficiency is less than one, whereas the fin effectiveness must be more than one.  $ECOP$  can be less or more than one. Increasing the value of fin  $ECOP$  or decreasing the fin irreversibility can be the goal of the designer. Here, the  $ECOP_f$  has been calculated and plotted with respect to Reynolds number for a pin fin. For future works, it can be evaluated for other geometries of the fins under various boundary conditions.

## Appendix A: Derivation of $ECOP$ for a pin fin

In case 3, we have (see Eqs. 1-6):

$$\frac{\theta}{\theta_B} = \frac{e^{-mx}}{1+e^{-2mL}} + \frac{e^{mx}}{1+e^{2mL}} = \frac{\cosh[m(L-x)]}{\cosh mL} \quad (\text{A.1})$$

Ecological coefficient of performance is defined as follows:

$$ECOP_f = \frac{\text{exergy rate corresponding to convective heat transfer of fin}}{\text{irreversibility rate of fin}} \quad (\text{A.2})$$

$$ECOP_f = \frac{\dot{E}Q_{\text{conv}}}{i} = \frac{\int_0^L hP(T-T_\infty)\left(1-\frac{T_0}{T}\right)dx}{T_0\dot{S}_{\text{gen}}} = \frac{\int_0^L hP(T-T_\infty)dx - \int_0^L hPT_0dx + \int_0^L hPT_\infty\frac{T_0}{T}dx}{T_0\left[\frac{q_B\theta_B}{T_\infty^2(1+\theta_B/T_\infty)} + \frac{F_D U_\infty}{T_\infty}\right]} \quad (\text{A.3})$$

$$r_1 = \int_0^L hP(T-T_\infty)dx = \int_0^L hP\theta dx = \int_0^L hP\theta_B\left[\frac{\cosh[m(L-x)]}{\cosh mL}\right]dx$$

$$\Rightarrow r_1 = hP\theta_B\left[\frac{-m\sinh[m(L-x)]}{\cosh mL}\right]_0^L = hP\theta_B\left(\frac{m\sinh mL}{\cosh mL}\right) = \sqrt{hPkA}\theta_B \tanh mL \quad (\text{A.4})$$

$$r_2 = \int_0^L hPT_0dx = hPT_0L \quad (\text{A.5})$$

$$r_3 = \int_0^L hPT_\infty\frac{T_0}{T}dx = hPT_\infty T_0 \int_0^L \frac{dx}{T} = y hPT_\infty T_0 \text{ where } y = \int_0^L \frac{dx}{T} \quad (\text{A.6})$$

$$\frac{\theta}{\theta_B} = \frac{T-T_\infty}{T_B-T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)} = F(x) \quad (\text{A.7})$$

$$\frac{\frac{T}{T_\infty}-1}{\frac{\theta_B}{T_\infty}} = F(x) \Rightarrow T = T_\infty \left[ 1 + \frac{\theta_B}{T_\infty} F(x) \right] \quad (\text{A.8})$$

$$y = \frac{1}{T_\infty} \int_0^L \frac{dx}{1 + \frac{\theta_B}{T_\infty} \frac{e^{m(L-x)} + e^{-m(L-x)}}{2 \cosh mL}} \quad (\text{A.9})$$

$$\text{where } u = e^{m(L-x)} \Rightarrow du = -me^{m(L-x)} dx \Rightarrow du = -mudx \Rightarrow dx = \frac{-du}{mu}$$

$$\Rightarrow y = \frac{1}{T_\infty} \int_0^L \frac{\frac{-du}{mu}}{1 + \frac{\theta_B}{T_\infty} \frac{1}{2 \cosh mL} \left(u + \frac{1}{u}\right)} = -\frac{1}{mT_\infty} \int_0^L \frac{du}{u+a(u^2+1)} \text{ where } a = \frac{\theta_B}{T_\infty} \frac{1}{2 \cosh mL}$$

$$\Rightarrow y = -\frac{1}{mT_\infty a} \int_0^L \frac{du}{u^2 + \frac{1}{a}u + 1} = -\frac{1}{mT_\infty a} \int_0^L \frac{du}{\left(u + \frac{1}{2a}\right)^2 + \left(1 - \frac{1}{4a^2}\right)}$$

$$\text{where } v = u + \frac{1}{2a} \Rightarrow dv = du \text{ and } b^2 = 1 - \frac{1}{4a^2}$$

$$\Rightarrow y = \frac{-1}{mT_\infty a} \int_0^L \frac{dv}{v^2 + b^2} = \frac{c}{b} \text{Arctan} \left( \frac{v}{b} \right) \Big|_0^L \text{ where } c = \frac{-1}{mT_\infty a}$$

$$\Rightarrow y = \frac{c}{b} \text{Arctan} \left( \frac{e^{m(L-x)} + \frac{1}{2a}}{b} \right) \Big|_0^L$$

$$\Rightarrow y = \frac{c}{b} \left[ \text{Arctan} \left( \frac{1 + \frac{1}{2a}}{b} \right) - \text{Arctan} \left( \frac{e^{mL} + \frac{1}{2a}}{b} \right) \right] \quad (\text{A.10})$$

$$\Rightarrow r_3 = hPT_\infty T_0 \frac{c}{b} \left[ \text{Arctan} \left( \frac{1 + \frac{1}{2a}}{b} \right) - \text{Arctan} \left( \frac{e^{mL} + \frac{1}{2a}}{b} \right) \right] \quad (\text{A.11})$$

$$\text{ECOP}_f = \frac{\sqrt{hPkA}\theta_B \tanh mL - hPT_0 L + hPT_\infty T_0 \frac{c}{b} \left[ \text{Arctan} \left( \frac{1 + \frac{1}{2a}}{b} \right) - \text{Arctan} \left( \frac{e^{mL} + \frac{1}{2a}}{b} \right) \right]}{T_0 \left[ \frac{q_B \theta_B}{T_\infty^2 (1 + \theta_B/T_\infty)} + \frac{F_D U_\infty}{T_\infty} \right]} \quad (\text{A.12})$$

The numerator and denominator of ECOP are divided by  $G_0$  as a dimensionless analysis, so

$$G_0 = T_0 \frac{q_B^2 U_\infty}{kvT_0 T_\infty} \quad (\text{A.13})$$

The term  $mL$  can be simplified as follows:

$$mL = \left( \frac{hP}{kA} \right)^{1/2} L = \left( \frac{h\pi D}{k\pi D^2/4} \right)^{1/2} L = \left( \frac{4h}{kD} \right)^{1/2} L = \left( \frac{4Nu\lambda}{kD^2} \right)^{1/2} L = 2Nu^{1/2} \left( \frac{\lambda}{k} \right)^{1/2} \left( \frac{L}{D} \right)$$

$$\Rightarrow mL = 2Nu^{1/2} (\lambda/k)^{1/2} \Gamma \quad (\text{A.14})$$

where Reynolds number, slenderness ratio and Nusselt Number are as follows:

$$Re_D = \frac{\rho U_\infty D}{\mu} = \frac{U_\infty D}{\nu} \quad (A.15)$$

$$Re_L = \frac{\rho U_\infty L}{\mu} = \frac{U_\infty L}{\nu} \quad (A.16)$$

$$\Gamma = \frac{L}{D} = \frac{Re_L}{Re_D} \quad (A.17)$$

$$Nu = \frac{hD}{\lambda} \quad (A.18)$$

For a pin fin with boundary condition in case 3, the final result for ECOP is:

$$ECOP_f = \frac{G_1 + G_2 + G_3}{G_4 + G_5} \quad (A.19)$$

where

$$G_1 = \frac{r_1}{G_0} = \frac{\sqrt{hPkA}\theta_B \tanh mL}{T_0 \frac{q_B^2 U_\infty}{kvT_0 T_\infty}} = \frac{\sqrt{h\pi D k \frac{\pi D^2}{4}} \theta_B \tanh mL}{T_0 \frac{q_B^2 U_\infty}{kvT_0 T_\infty}} \quad (A.20)$$

$$q_B = -kA \left. \frac{dT}{dx} \right|_{x=0} \quad (A.21)$$

For case 3 it is

$$q_B = -kA\theta_B m \left( \frac{-1}{1+e^{-2mL}} + \frac{1}{1+e^{+2mL}} \right) = \frac{\pi}{4} kD^2 \theta_B m \tanh mL \quad (A.22)$$

$$\Rightarrow G_1 = \frac{\sqrt{hk \frac{\pi^2 D^3}{4}} \theta_B \tanh mL kvT_\infty}{U_\infty \theta_B^2 \frac{\pi^2}{16} k^2 D^4 m^2 \tanh^2 mL} = \frac{16\sqrt{hkD} vT_\infty L^2}{2\pi U_\infty \theta_B kD^3 (mL)^2 \tanh mL}$$

$$\Rightarrow G_1 = \frac{16}{2\pi (mL)^2 \tanh mL} \frac{\sqrt{hkD} vT_\infty L^3}{U_\infty kD^3 \theta_B L} = \frac{16}{2\pi (mL)^2 \tanh mL} \frac{\sqrt{(Nu \times \lambda) \times k}}{k Re_L} \left(\frac{L}{D}\right)^3 \left(\frac{T_\infty}{\theta_B}\right)$$

$$\Rightarrow G_1 = \frac{16}{2\pi} \left(\frac{T_\infty}{\theta_B}\right) \frac{1}{(mL)^2 \tanh mL} \times \sqrt{Nu \times (\lambda/k)} \times Re_L^{-1} \left(\frac{L}{D}\right)^3$$

$$\Rightarrow G_1 = \frac{4}{\pi} \left(\frac{T_\infty}{\theta_B}\right) \frac{1}{mL \tanh mL} \times Re_L^{-1} \left(\frac{L}{D}\right)^2$$

$$\Rightarrow G_1 = \frac{4}{\pi} \times \frac{1}{mL \tanh mL} \times \frac{\Gamma^2}{\beta Re_L} \quad (A.23)$$

$$G_2 = \frac{-r_2}{G_0} = \frac{-hPT_0 L}{T_0 \frac{q_B^2 U_\infty}{kvT_0 T_\infty}} = \frac{-hPT_0 L kvT_\infty}{q_B^2 U_\infty} \quad (A.24)$$

$$\Rightarrow G_2 = \frac{-h\pi D k L v T_0 T_\infty}{\theta_B^2 \pi^2 k^2 D^4 m^2 \tanh^2 mL U_\infty} = \frac{-16 D h v T_0 T_\infty L^3}{\pi \theta_B^2 k D^4 U_\infty (mL)^2 \tanh^2 mL}$$

$$\Rightarrow G_2 = \frac{-16}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \left(\frac{T_0}{T_\infty}\right) \frac{1}{(mL)^2 \tanh^2 mL} \times Nu \times (\lambda/k) \times Re_D^{-1} \left(\frac{L}{D}\right)^3$$

$$\Rightarrow G_2 = \frac{-4}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \left(\frac{T_0}{T_\infty}\right) \frac{1}{\tanh^2 mL} \times Re_D^{-1} \frac{L}{D}$$

$$\Rightarrow G_2 = \frac{-4}{\pi} \left(\frac{T_0}{T_\infty}\right) \times \frac{1}{\tanh^2 mL} \times \frac{\Gamma}{\beta^2 Re_D} \quad (A.25)$$

$$G_3 = \frac{r_3}{G_0} = \frac{h P T_\infty T_0 \frac{c}{b} \left[ \text{Arctan}\left(\frac{1+\frac{1}{2a}}{b}\right) - \text{Arctan}\left(\frac{e^{mL} + \frac{1}{2a}}{b}\right) \right]}{T_0 \frac{q_B^2 U_\infty}{k v T_0 T_\infty}} = \frac{k v h P T_0 T_\infty^2 \frac{c}{b} \times J}{q_B^2 U_\infty} \quad (A.26)$$

$$\text{where } J = \text{Arctan}\left(\frac{1+\frac{1}{2a}}{b}\right) - \text{Arctan}\left(\frac{e^{mL} + \frac{1}{2a}}{b}\right)$$

$$\Rightarrow G_3 = \frac{k v h \pi D T_0 T_\infty^2 \frac{c}{b} \times J}{\theta_B^2 \pi^2 k^2 D^4 m^2 \tanh^2 mL U_\infty} = \frac{16}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \frac{T_0}{(mL)^2 \tanh^2 mL} \times \frac{L^2 v h D \frac{c}{b} \times J}{k D^4 U_\infty}$$

$$\text{where } a = \frac{\theta_B}{T_\infty} \frac{1}{2 \cosh mL} \text{ and } b^2 = 1 - \frac{1}{4a^2} \text{ and } c = \frac{-1}{m T_\infty a}$$

$$\Rightarrow G_3 = \frac{16}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \frac{T_0}{(mL)^2 \tanh^2 mL} \times \frac{L^3 v (Nu \lambda) \times \left(\frac{-1}{m T_\infty a}\right) \times \frac{J}{b}}{k D^4 U_\infty L}$$

$$\Rightarrow G_3 = \frac{16}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \left(\frac{T_0}{T_\infty}\right) \frac{1}{(mL)^2 \tanh^2 mL} \times \frac{L^4 (Nu \lambda/k) \times \left(\frac{-1}{a}\right) \times \frac{J}{b}}{D^4 mL (U_\infty L / v)}$$

$$\Rightarrow G_3 = \frac{16}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \left(\frac{T_0}{T_\infty}\right) \frac{1}{(mL)^2 \tanh^2 mL} \times Nu \times (\lambda/k) \times Re_L^{-1} \left(\frac{L}{D}\right)^4 \frac{(-J)}{a \times b \times mL}$$

$$\text{where } a \times b = a \times \sqrt{1 - \frac{1}{4a^2}} = a \times \frac{\sqrt{4a^2 - 1}}{2a} = \frac{\sqrt{4a^2 - 1}}{2} = \frac{\sqrt{4\left(\frac{\theta_B}{T_\infty} \frac{1}{2 \cosh mL}\right)^2 - 1}}{2}$$

$$\Rightarrow G_3 = \frac{-16}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \left(\frac{T_0}{T_\infty}\right) \frac{1}{(mL)^2 \tanh^2 mL} \times Nu \times (\lambda/k) \times Re_L^{-1} \left(\frac{L}{D}\right)^4 \times \frac{2J}{mL \times \sqrt{4\left(\frac{\theta_B}{T_\infty} \frac{1}{2 \cosh mL}\right)^2 - 1}}$$

$$\Rightarrow G_3 = \frac{-32}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \left(\frac{T_0}{T_\infty}\right) \frac{1}{(mL)^3 \tanh^2 mL} \times Nu \times (\lambda/k) \times Re_L^{-1} \left(\frac{L}{D}\right)^4 \times \frac{J}{\sqrt{\left(\frac{\theta_B}{T_\infty} \frac{1}{\cosh mL}\right)^2 - 1}}$$

$$\Rightarrow G_3 = \frac{-8}{\pi} \left(\frac{T_\infty}{\theta_B}\right)^2 \left(\frac{T_0}{T_\infty}\right) \frac{1}{mL \tanh^2 mL} \times Re_L^{-1} \left(\frac{L}{D}\right)^2 \times \frac{J}{\sqrt{\left(\frac{\theta_B}{T_\infty} \frac{1}{\cosh mL}\right)^2 - 1}}$$

$$\Rightarrow G_3 = \frac{-8}{\pi} \left(\frac{T_0}{T_\infty}\right) \times \frac{1}{mL \tanh^2 mL} \times \frac{\Gamma^2}{\beta^2 Re_L} \times \frac{J}{\sqrt{\left(\frac{\beta}{\cosh mL}\right)^2 - 1}} \quad (\text{A.27})$$

$$G_4 = \frac{T_0 \times \frac{q_B \theta_B}{T_\infty^2 (1 + \theta_B/T_\infty)}}{T_0 \frac{q_B^2 U_\infty}{kv T_0 T_\infty}} = \frac{T_0 \theta_B kv}{q_B T_\infty U_\infty (1 + \theta_B/T_\infty)} = \frac{T_0 \theta_B kv}{T_\infty U_\infty (1 + \theta_B/T_\infty) \times \frac{\pi}{4} k D^2 \theta_B m \tanh mL} \quad (\text{A.28})$$

$$\Rightarrow G_4 = \frac{4}{\pi} \frac{T_0}{T_\infty} \frac{kvL}{U_\infty (1 + \theta_B/T_\infty) \times k D^2 mL \tanh mL} = \frac{4}{\pi} \frac{T_0}{T_\infty} \frac{L^2}{(1 + \theta_B/T_\infty) \times D^2 mL \tanh mL} \frac{v}{U_\infty L}$$

$$\Rightarrow G_4 = \frac{4}{\pi} \frac{T_0}{T_\infty} \frac{1}{(1 + \theta_B/T_\infty) \times mL \tanh mL} \times Re_L^{-1} \left(\frac{L}{D}\right)^2$$

$$\Rightarrow G_4 = \frac{4}{\pi} \left(\frac{T_0}{T_\infty}\right) \times \frac{1}{(1 + \beta) \times mL \tanh mL} \times \frac{\Gamma^2}{Re_L} \quad (\text{A.29})$$

$$G_5 = \frac{T_0 \times \frac{F_D U_\infty}{T_\infty}}{T_0 \frac{q_B^2 U_\infty}{kv T_0 T_\infty}} = \frac{T_0 kv F_D}{q_B^2} = \frac{T_0 kv}{q_B^2} \times \left(\frac{1}{2} C_D \rho U_\infty^2 DL\right) = \frac{1}{2} \times \frac{\rho kv^3 T_0}{q_B^2} \times \frac{U_\infty L}{v} \times \frac{U_\infty D}{v} \times C_D \quad (\text{A.30})$$

$$\Rightarrow G_5 = \frac{1}{2} B C_D Re_D Re_L \quad \text{where} \quad B = \frac{\rho kv^3 T_0}{q_B^2} \quad (\text{A.31})$$

where dimensionless temperature ( $\beta$ ) is defined as follows:

$$\beta = \frac{\theta_B}{T_\infty} \quad (\text{A.32})$$

Also, for simplicity property group is defined as follows [8];

$$M = \frac{(k/\lambda)^{1/2}}{Pr^{1/6}} \quad (\text{A.33})$$

In this analysis, the Nusselt number and the drag coefficient have been evaluated from the results developed for a single cylinder in cross flow [8, 32, 34]:

$$Nu = 0.683 Re_D^{0.466} Pr^{\frac{1}{3}} \quad \text{and} \quad C_D = 5.484 Re_D^{-0.246} \quad \text{for} \quad 40 < Re_D < 4 \times 10^3 \quad (\text{A.34})$$

Eq. (A.14) is a general form that used for pin fins. Using Eqs. (A.17), (A.33), and (A.34), the Eq. (A.14) can be rewritten as follows:

$$mL = 2 Nu^{\frac{1}{2}} (\lambda/k)^{\frac{1}{2}} \left(\frac{L}{D}\right) = 2 \times \left(0.683 Re_D^{0.466} Pr^{\frac{1}{3}}\right)^{\frac{1}{2}} (\lambda/k)^{\frac{1}{2}} \left(\frac{L}{D}\right) = 1.6529 Re_D^{0.233} Pr^{\frac{1}{6}} (\lambda/k)^{\frac{1}{2}} \left(\frac{L}{D}\right)$$

$$\Rightarrow mL = 1.6529 Re_D^{0.233} \times \frac{\Gamma}{M} \quad (\text{A.35})$$

## References

- [1] Pashah, S., Moinuddin, A., Zubair, S.M., Thermal performance and optimization of hyperbolic annular fins under dehumidifying operating conditions – analytical and numerical solutions, *International Journal of Refrigeration* 65 (2016), pp. 42-54.
- [2] Kraus, A.D., Aziz, A., Welty, J., *Extended Surface Heat Transfer*, John Wiley and Sons, New York, 2001.
- [3] Yovanovich, M.M., Heat balance method for spines, longitudinal and radial fins with contact conductance and end cooling. 37th AIAA Thermophysics Conference Portland, Oregon: AIAA- (2004), pp. 2004–2569.
- [4] Ullmann, A., Kalman, H., Efficiency and optimized dimensions of annular fins of different cross-section shapes. *International Journal of Heat and Mass Transfer* 32 (1989), pp. 1105-1110.
- [5] Maday, C. J., The minimum weight one dimensional straight fin, *ASME Journal of Engineering for Industry* 96 (1974), pp.161-165.
- [6] Guceri, S., Maday, C. J., A least weight circular cooling fin, *ASME Journal of Engineering for Industry* 97 (1975), pp. 1190-1193.
- [7] Razelos, P., Imre, K., The optimum dimensions of circular fins with variable thermal parameters, *ASME Journal of Heat Transfer* 102 (1980), pp. 420-425.
- [8] Poulidakos, D., Bejan, A., Fin geometry for minimization entropy generation in forced convection, *Journal of Heat Transfer* 104 (1982), pp. 616-623.
- [9] Laor, K. , Kalman, H., The effect of tip convection on the performance and optimum dimensions of cooling fins, *Int. Commun. Heat Mass Transfer* 19 (1992), pp. 569-584
- [10] Bahadur R., Bar-Cohen, A., Orthotropic thermal conductivity effect on cylindrical pin fin heat transfer. *International Journal of Heat and Mass Transfer* 50 (2007), pp. 1155-1162.
- [11] Zubair, S.M., Arif, A.F.M., Sharqawy, M.H., Thermal Analysis and Optimization of Orthotropic Pin Fins: A Closed-Form Analytical Solution. *Journal of Heat Transfer* 132 (2010), pp. 031301-1–031301-8.
- [12] Mustafa, M., Zubair, S.M., Arif, A., Thermal analysis of orthotropic annular fins with contact resistance: A closed-form analytical solution. *Applied Thermal Engineering* 31 (2011), pp. 937–945.
- [13] Medrano, M., Yilmaz, M.O., Nogués, M., Martorell, I., Roca, J., Cabeza, L.F., Experimental evaluation of commercial heat exchangers for use as PCM thermal storage systems, *Applied Energy* 86 (2009), pp. 2047-2055.
- [14] Velmurugan, V., Deenadayalan, C.K., Vinod, H., Srithar, K., Desalination of effluent using fin type solar still, *Energy* 33 (2008), pp. 1719–1727.
- [15] Velmurugan, V., Gopalakrishnan, M., Raghu, R., Srithar, K., Single basin solar still with fin for enhancing productivity, *Energy Convers. Manag.* 49 (2008), pp. 2602–2608.
- [16] El-Sebaili, A.A., Ramadan, M.R.I., Aboul-Enein, S., El-Naggar, M., Effect of fin configuration parameters on single basin solar still performance, *Desalination* 365 (2015), pp.15–24.

- [17] Rajaseenivasan, T., Srithar, K., Performance investigation on solar still with circular and square fins in basin with CO<sub>2</sub> mitigation and economic analysis, *Desalination* 380 (2016), pp. 66–74.
- [18] Li, B., Byon, C., Experimental and numerical study on the heat sink with radial fins and a concentric ring subject to natural convection *Applied Thermal Engineering* 90 (2015), pp. 345-351.
- [19] Almendros-Ibanez, J.A., Belmonte, J.F., Molina, A.E., Fins with a prescribed temperature at the tip: Efficiency and effectiveness expressions *Applied Thermal Engineering* 91 (2015), pp. 447-455.
- [20] Sadollah A., Choi Y., Yoo, D.G., Kim, J.H., Metaheuristic algorithms for approximate solution to ordinary differential equations of longitudinal fins having various profiles, *Applied Soft Computing* 33 (2015), pp. 360–379.
- [21] Kundu, B., Lee, K.S., A proper analytical analysis of annular step porous fins for determining maximum heat transfer, *Energy Conversion and Management* 110 (2016), pp. 469–480.
- [22] Lee, M., Kim, H.J., Kim, D.K., Nusselt number correlation for natural convection from vertical cylinders with triangular fins, *Applied Thermal Engineering* 93 (2016), pp. 1238–1247.
- [23] Bejan, A., *Heat Transfer*, John Wiley and Sons, 1993.
- [24] Holman, J.P., *Heat Transfer*, 10<sup>th</sup> edition, McGraw Hill, 2010.
- [25] Nellis, G., Klein, S., *Heat Transfer*, Cambridge University Press, New York, 2008.
- [26] Bejan, A., Kraus, A.D., *Heat Transfer Handbook*, John Wiley & Sons Inc., New York, 2003, pp.161-260.
- [27] Han, J.C., *Analytical Heat Transfer*, CRC Press, New York, 2012.
- [28] Incropera, F.P., Dewitt, D.P., Bergman, T.L., Lavine, A.S., *Principles of Heat and Mass Transfer, International Student Version*, 7<sup>th</sup> edition, John Wiley and Sons, 2012.
- [29] Kreith F., R. Manglik, M. Bohn, *Principles of Heat Transfer* 7<sup>th</sup> edition, Cengage Learning, 2011.
- [30] Lienhard IV, J.H., Lienhard V, J.H, *A heat transfer textbook*, Phlogiston Press Cambridge Massachusetts, USA, 2000.
- [31] Kotas, T.J., *The Exergy Method of Thermal Plant Analysis*, Department of Mechanical Engineering, Queen Mary College, University of London, 1985.
- [32] A. Bejan, *Entropy generation through Heat and fluid flow*, John Wiley and Sons, 1982.
- [33] Seyyedi, S.M., Ajam, H., Farahat, S., A new criterion for the allocation of residues cost in exergoeconomic analysis of energy systems, *Energy* 35 (2010),8, pp. 3474-3482. [34] B. Gebhart, *Heat Transfer*, McGraw Hill, New York, 1971, pp.212-214.



## Nomenclature

$A_b$	Area for the case without fin, ( $m^2$ )	$\dot{S}_{gen}$	Entropy generation rate, (W/K)
$A_f$	Area for the case with fin, ( $m^2$ )	$s$	Specific entropy, (J/kg.K)
$A$	Area, ( $m^2$ )	$T_\infty$	Absolute temperature of free stream, (K)
$C_D$	Drag coefficient, (-)	$T_0$	Absolute temperature of environment, (K)
$D$	Pin diameter, (m)	$U_\infty$	Velocity of free stream, (m/s)
ECOP	Ecological coefficient of performance	<b>Greek symbols</b>	
$F_D$	Net drag force, (N)	$\Gamma$	Slenderness ratio, (-)
$h$	Convective heat transfer coefficient, (W/ $m^2$ .K)	$\varepsilon_f$	Fin effectiveness, (-)
$\dot{I}$	Irreversibility rate, (J)	$\eta_f$	Fin Efficiency, (-)
$k$	Thermal conductivity of pin material, (W/m.K)	$\theta$	Dimensionless temperature, (-)
$L$	Fin Length, (m)	$\theta_B$	Base stream temperature difference, (k)
$\dot{m}$	Mass flow rate, (Kg/s)	$\lambda$	Thermal conductivity of fluid, (W/m. k)
$Nu$	Nusselt number, (-)	$\rho$	Density of fluid, ( $kg/m^3$ )
$P$	Perimeter, (m)	$\nu$	Kinematic Viscosity, ( $m^2/s$ )
$Pr$	Prandtl number, (-)		
$Re$	Reynolds number, (-)		
$q_B$	Base heat flux, (W/ $m^2$ )		
$\dot{Q}_{conv}$	Convective heat transfer rate, (W)		

Paper submitted: 17. December 2016.

Paper accepted: 10. October 2017.

Paper revised: 1. November 2017.