From the Guest Editors

MATHEMATICAL MODELS FOR THERMAL SCIENCE

by

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Thermal science is a moving boundary of innovation and appears anywhere in engineering and life, all cute frontiers of all advanced technologies will definitely relate to thermodynamics. Take for example, the absolute zero temperature derived by the fractal E-infinity theory reads [1]:

$$T = 1 - 40\varphi^4 = -273.16\ ^\circ\text{C} \tag{1}$$

where φ is the golden meaning, $\varphi = (1+\sqrt{5})/2.$ This theoretical prediction is very much closed to the practical observation, −273.15, on the Celsius scale. The golden meaning is widely appeared in nature, for example, wool [2], polar bear's hair [3], and cocoon [4], all have a fractional dimension of φ, the temperature distribution through a cocoon wall reads [4]:

$$T = a + \frac{bx^\alpha}{\Gamma(1+\alpha)} \tag{2}$$

where a and b are constants and α is the value of fractal dimensions. For the inner wall, $x = 0$. It is obvious that:

$$\frac{\mathrm{d}T}{\mathrm{d}x}(x=0) = \lim_{x\to 0}\frac{\alpha b x^{\alpha-1}}{\Gamma(1+\alpha)} = \begin{cases} 0, & \alpha > 1 \\ b, & \alpha = 1 \\ \infty, & \alpha < 1 \end{cases} \tag{3}$$

When $\alpha = \varphi$, the temperature at inner wall will not change much regardless of the environmental changes. When $\alpha = 0.99999999$, $\alpha = 1.0000000000$, $\alpha = 1.0000000001$, respectively, the temperature change at inner wall will differentiate remarkably. In mathematics $\alpha = 0.\dot{9} = 1,$ but for practical applications $\alpha = 0.\dot{9} < 1.$ God is a geometrician.

Another example is the spinning process in textile engineering, which is actually a thermal process. Sudden and fast solvent evaporation in electrospinning, bubble electrospinning [5, 6] and bubbfil spinning [7, 8] plays a great important for controlling fiber morphology. To control the evaporation process, we have to use Bernoulli equation from fluid mechanics:

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$$\frac{1}{2}u^2 + \frac{p}{\rho} = B \tag{4}$$

where u is the jet velocity, p – the pressure, ρ – the density, and B – the Bernoulli constant. A higher velocity of jets in the spinning process, according to eq. (4), means lower flow pressure, this is extremely useful for solvent evaporation (see the paper by Liu, *et al.,* p. 1821, and Zhao, *et al.*, p. 1827 in this issue). In bubble electrospinning, the initial speed of jet is almost 300 m/s. Phase separation due to solvent evaporation occurs when the polymer solution is ejected into a solvent-free air environment, according to the state equation:

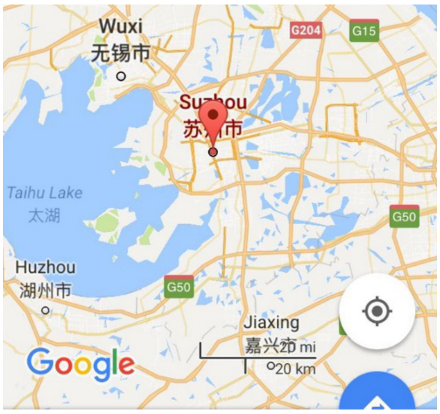
$$\frac{p}{\rho} = \mathrm{R}T \tag{5}$$

the temperature of the jet decreases greatly for a jet moving a high velocity until to freezing point of solvent, when the solvent is removed from the jet, a porous nanofiber can be obtained. Without the knowledge of modern thermal science, it is impossible to optimize its spinning process.

In this issue, mathematical models for various thermal problems are established, for example, a fluid-mechanic model for fabrication of nanoporous fibers by electrospinning is proposed by Fan, *et al.* (p. 1621 in this issue), a predictive model of thermal conductivity of plain woven fabrics is suggested by Wu *et al*. Mathematical models in this issue include differential equations, differential-difference equations, and fractional differential equations, which are solved either analytically or numerically. It is obvious that fractional calculus [9-15] is an effective approach to model various thermal problems in porous media, for example, hydrogen diffusion occurs in a scale within 1 nanometer, at such a small scale all the diffusion process must be described using a fractional model [19]. Tsunami prevention is first proposed in this issue by Wang and his students [19], which can also be modelled by a fractional differential equation, and tsunami's wave morphology can be effectively controlled by the fractional order. In this issue, some powerful analytical methods for fractional calculus are effectively used, mainly the homotopy perturbation method, the variational iteration method, the exp-function method, Adomian method, sub-equation method, and their modifications [16]. It should be emphasized that the fractional complex transformation [17, 18] proposed by He and Li is especially effective to approximately deal with fractional calculus. The solution of fractional calculus should be unsmooth everywhere on the studied scale, and on a larger scale, the solution appears approximately smooth. To elucidate this solution property, we consider a map on Suzhou city, China. On the scale of 20 km, we can see discontinuous map due to various small lakes distributed on the land, and these lakes disappear except Taihu Lake when the scale is as large as 200 km, and the land become continuous, see fig. 1. The smaller islands on Taihu Lake also disappear on a larger scale. On an even larger scale, Taihu Lake will also disappear on the map. The scale is everything, when we study water flow in a tube, a macroscopic scale is needed, where classic fluid mechanics works. When we study solvent diffusion in a solution, a molecule scale has to be adopted, and the solution becomes discontinuous, any laws derived from the continuum assumption are forbidden, and their fractional partners have to be considered.

The fractional complex transformation [17, 18]:

$$X = \frac{x^{\alpha}}{\Gamma(\alpha + 1)} \tag{6}$$



(a)

(b)

Figure 1. Discontinuous and continuous maps at different scales, (a) scale of 20 km, (b) scale of 200 km

On the scale of x, the solution is generally discontinuous, and on the scale of X, it becomes approximately smooth. So the fractional complex transformation is to convert a fractal space to its continuous partner.

Numerical methods in this issue include the reproducing kernel method, barycentric interpolation collocation method, gray-encoded evolution algorithm, CFD, finite element method, and other. The numerical results are widely used as imagined experiments to save time and effort. The thermal problems in this issue covers from textile engineering to ocean engineering, architectural engineering including Mongolian yurt, and nanotechnology.

This issue is especially helpful for researchers in fluid mechanics, thermal science, mathematics, chemistry, material science, textile engineering, and nanotechnology, the given examples can be easily followed for other practical applications.

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