

DIFFUSION OF CHEMICALLY REACTIVE SPECIES OF A MAXWELL FLUID DUE TO AN UNSTEADY STRETCHING SHEET WITH SLIP EFFECT

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The influence of a first-order slip boundary condition on the diffusion of chemically reactive species in an unsteady MHD boundary layer flow of a non-Newtonian Maxwell fluid over a vertical permeable linearly stretching or shrinking sheet is considered. The whole analysis is performed taking the first-order chemical reaction and linearly varying wall concentration. Choosing appropriate similarity variables, the governing equations of the problem are transformed into a set of nonlinear coupled self-similar equations, which are then solved numerically by the shooting technique. A comparison with previously published results is performed for certain cases and the results are found to be in excellent agreement. The flow features and mass transfer characteristics for different values of the governing parameters are graphically presented and discussed in detail.

Key words: Maxwell fluid; chemical reaction; unsteady flow; diffusion; slip.

1. Introduction

Chemical reactions are often classified as either heterogeneous or homogeneous processes. This relies on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the chemical reaction is heterogeneous, if it takes place at associate interface and homogenized, if it takes place in solution. In generality states of chemical reactions, the reaction rate based on the concentration of the species itself. A reaction is so called a first order, if the rate of reaction is directly proportional to concentration itself. Some representative fields of interest during which combined heat and mass transfer beside chemical process play an important role in chemical reaction industries like food process and polymer production. Additionally, the no-slip boundary condition in fluid mechanics cases that the fluid velocity coincides the velocity of the solid body. This boundary condition is well passable at the macroscopic level. There exists, of course, an outsized body of literature on the interpretation and rotation of bodies in Stokes flow with no-slip boundary conditions, e.g., Refs. [1–11]. Altogether antecedently mentioned works, the flow obeys the conventional no-slip boundary condition that is thought because the central tenets of the Navier–Stokes theory. But, no-slip assumption is not consistent with all physical characteristic i.e., some workable flow situations seems wherever it becomes necessary to interchange the no-slip boundary condition by partial

slip boundary condition. It is a helpful model for few flows through pipes during which chemical reactions occurred at the walls, for flows with laminar film condensation, surely two-phase flows and for flows in porous slider bearings [12]. A slip boundary condition is usually used in hydrodynamic simulations at solid boundaries. Kistler [13] observed that a slip boundary condition may be a means of circumventing the “kinematic paradox” of a moving contact line at a no-slip boundary.

Non-Newtonian fluid flows generated by a stretching sheet have been widely analyzed for the importance in many producing processes like extrusion of melted polymers through a slit die for the manufacture of plastic sheets, process of food stuffs, and wire and fiber coating. On the other hand, convective heat transfer plays an important role throughout the handling and process of non-Newtonian fluid flows. The flow characteristics of non-Newtonian fluids are quite completely different as compared to Newtonian fluids. So, as to get a transparent plan of non-Newtonian fluids and their varied applications, it's necessary to study their flow behavior. As a result of the complexness of those fluids, there's not a one constituent equation that exhibits all properties of such non-Newtonian fluids. Among these, the overwhelming majority of non-Newtonian fluid models are involved with the simple models viz. the power law and grade two or three [14–22]. These simple fluid models have some drawbacks that they are unable to produce results not having accordance with fluid flows in reality. The power-law model is employed modeling fluids with shear-dependent viscosity. But it cannot predict the effects of physical property. On the other side, though the fluids of grade two or three can calculate the effects of elasticity, the viscosity in these patterns is not shear dependent. Moreover, they are unable to surmise the effects of stress relaxation. Maxwell model, a category of rate kind fluids, can surmise the stress relaxation and therefore, have become more popular (Sadeghy et al. [23], Abel et al. [24]). This model excludes the complicating effects of shear-dependent viscosity from any boundary layer analysis and enables one to focus solely on the effects of a fluid's elasticity on the characteristics of its boundary layer. Hayat et al. [25] constructed an analytic solution for unsteady MHD flow in a rotating Maxwell fluid through a porous medium. Swati and Krishnendu [26] presented the problem of unsteady flow of a Maxwell fluid over a stretching surface in the presence of chemical reaction.

The purpose of the present investigation is to study the unsteady boundary layer slip flow and mass transfer characteristics of a non-Newtonian Maxwell fluid along a stretching vertical sheet taking into account the effect of a magnetic field and a chemical reaction. Similarity transformation is employed, and the reduced ordinary differential equations are solved numerically. The results of this parametric study are shown graphically and the physical aspects of the problem are highlighted and discussed.

2. Governing Equations

A laminar boundary-layer two-dimensional flow of an incompressible, conducting non-Newtonian Maxwell fluid undergoing a first-order chemical reaction over an unsteady stretching sheet with slip effect has been analyzed. Two equal and opposite forces are applied along the x -axis so that the wall is stretched keeping the origin fixed. A magnetic field of strength $B = B_0\sqrt{(1 - \gamma t)^{-1}}$ is applied in the y direction, i.e., normal to the flow direction. The external electric field is assumed to be zero and the magnetic Reynolds number is assumed to be small. Hence, the induced magnetic field is small compared with the externally applied magnetic field.

The unsteady fluid and heat flows start at $t = 0$. The sheet emerges out of a slit at origin ($x = 0, y = 0$) and moves with non-uniform velocity $U(x, t) = bx(1 - \gamma t)^{-1}$, where $b > 0; \gamma \geq 0$ are constants with dimension (time)⁻¹, b is the stretching rate. The reaction of the species be the first order homogeneous chemical reaction of rate k which varies with time. The equations for steady two-dimensional flow and the reactive concentration can be written in usual notation as

$$u_x + v_y = 0 \quad (1)$$

$$u_t + uu_x + vv_y + \beta(u^2u_{xx} + v^2u_{yy} + 2uvu_{xy}) = \nu u_{yy} - (\sigma B^2 / \rho)u \pm g\beta^*(C - C_\infty) \quad (2)$$

$$C_t + uC_x + vC_y = DC_{yy} - k(C - C_\infty) \quad (3)$$

where, (u, v) are the velocity components in (x, y) directions, respectively, ρ is the density of the fluid, σ is the electrical conductivity, ν is the kinematic viscosity, g is the acceleration due to gravity, k is the time-dependent reaction rate of the solute $k = k_0(1 - \gamma t)$, $k > 0$ stands for destructive reaction whereas; $k < 0$ stands for constructive reaction, k_0 is a constant, $\beta = \beta_0(1 - \gamma t)$ is the relaxation time of the period, β_0 is a constant, C is the concentration of the species of the fluid, D is the diffusion coefficient of the diffusing species in the fluid, β^* the volumetric coefficient of concentration expansion, Furthermore, the last term on the right-hand side of Eq. (2) represents the influence of the diffusion of chemically reactive species on the flow field, with “+” sign corresponding to the flow assisting region and “-” sign the flow opposing region.

The relevant boundary conditions of the governing equations are

$$y = 0 : \quad u = U(x, t) + Lu_y, \quad v = 0, \quad C = C_w(x, t) \quad (4a)$$

$$y \rightarrow \infty : \quad u \rightarrow 0, \quad C \rightarrow C_\infty \quad (4b)$$

where, L is the slip length, b is a constant known as the stretching rate, the concentration at the surface of the sheet is similarly assumed to vary both along the sheet and with time, in accordance with $C_w = C_\infty + bx(1 - \gamma t)^{-2}$ where C_∞ is the constant free stream concentration. The wall concentration $C_w(x, t)$ represents a situation in which the sheet concentration increases (reduces) if b is positive (negative) in proportion to x and such that the amount of concentration increase (reduction) along the sheet increases with time. The expressions for $U(x, t)$, $C_w(x, t), k(t), \tilde{\beta}(t)$ are valid for time $t < \gamma^{-1}$.

Most of the actual exact solutions in hydrodynamic are similarity solutions which shorten the number of independent variables by one or more. Similarity solutions are often asymptotic solutions to a given problem and may have utility in this field of limiting solutions. Similarity solutions may be utilized to obtain the physical insight into these details of complex fluid flows and these solutions display most of the characteristic as well as the effect of the physical and thermal parameters of the existing problem. In order to get a similarity solution of the problem we define the following transformations

$$\eta = \sqrt{b / (\nu(1 - \gamma t))} y, \quad \psi = \sqrt{\nu b / (1 - \gamma t)} x f(\eta), \quad \phi(\eta) = (C - C_\infty) / b x (1 - \gamma t)^{-2} \quad (5)$$

where the stream function ψ satisfies the continuity equation and defines in the usually way as $u = \psi_y$, $v = -\psi_x$, η is the dimensionless similarity variable.

With the change of variables (5), Eq. (1) is identically satisfied and Eqs. (2)–(4) are transformed to

$$f''' + ff'' - f'^2 - Mn f' - A(f' + 2^{-1}\eta f'') - \beta(f^2 f''' - 2ff'f'') + \lambda\phi = 0 \quad (6)$$

$$Sc^{-1}\phi'' + f\phi' - f'\phi - A(2^{-1}\eta\phi' + 2\phi) - K\phi = 0 \quad (7)$$

The transformed boundary conditions are turn into

$$f(0) = 0, \quad f'(0) = 1 + \delta f''(0), \quad \phi(0) = 1, \quad f'(\infty) \rightarrow 0, \quad \phi(\infty) \rightarrow 0 \quad (8)$$

In the above equations, the prime denotes ordinary differentiation with respect to the similarity variable η , where, $\lambda = g\beta^* / b$ is the diffusion parameter,. Further, ($\lambda > 0$) corresponding to flow assisting region, whereas ($\lambda < 0$) corresponding to flow opposing region. $Sc = \nu / D$, $K = k_0 / b$ are the Schmidt number and reaction rate parameter, respectively, $\delta = L\sqrt{b / \nu(1 - \gamma t)}$ is the first-order slip flow parameter, $A = \gamma / b$ is the unsteadiness parameter, $Mn = \sigma B_0^2 / \rho b$ is the magnetic field parameter and $\beta = \beta_0 b$ is the Maxwell parameter (elastic parameter).

Expressions for the skin friction coefficient C_f and the local Sherwood number Sh are

$$C_f = \tau_w / \rho U^2, \quad Sh = x q_m / D(C_w - C_\infty) \quad (9)$$

where τ_w is the wall shear stress, q_m is the mass flux, respectively, which define as

$$\tau_w = \mu(1 + \beta)u_y, \quad q_m = -DC_y, \quad y = 0$$

Applying transformation (5), then the dimensionless forms of Eq. (9) takes the form

$$\sqrt{Re} C_f = (1 + \beta)f''(0), \quad Sh = -\sqrt{Re}\phi'(0) \quad (10)$$

where $Re = Ux / \nu$ is the local Reynolds number depend on the stretching velocity U .

3. Numerical Solution and Discussion

The nonlinear coupled self-similar equations (6) and (7) along with the boundary conditions (8) from a boundary value problem (BVP) and is solved by the shooting method with the fourth-order Runge-Kutta technique after converting into an initial value problem (IVP). First we have to choose a suitable finite value of $\eta \rightarrow \infty$, say η_∞ . We set following first-order systems

$$f' = p, \quad p' = g, \quad g' = (Mnp + p^2 - fg - \lambda\phi + A(p + 2^{-1}\eta g) - 2\beta fpg) / (1 - \beta f^2) \quad (11)$$

$$\text{and } \phi' = J, \quad J' = Sc(A(2^{-1}\eta J + 2\phi) - fJ + (p + K)\phi) \quad (12)$$

with the boundary conditions

$$f(0) = 0, \quad p(0) = 1 + \delta g(0), \quad \phi(0) = 1$$

To solve Eqs. (11) and (12) as an IVP, we must need values for $g(0)$ i.e. $f''(0)$ and $J(0)$ i.e. $\phi'(0)$ but no such values are given in the problem. The initial guess values for $f''(0)$ and $\phi'(0)$ is chosen and the solution procedure is carried out. We compare the calculated values of $f'(\eta)$ and $\phi(\eta)$ at $\eta_\infty (= 15)$ with the given boundary conditions $f'(\eta_\infty) = 0$ and $\phi(\eta_\infty) = 0$ and adjust values $f''(0)$ and $\phi'(0)$ using the Secant method to find better approximation for the solution. The step size is taken as $h = 0.01$. The process is repeated until we get the results corrected up to the desired accuracy of 10^{-5} level. In addition, to validate the method applied in this work and to judge the accuracy of the present study, comparisons with the available results of Anderson [27] corresponding to the skin-friction coefficient $f''(0)$ for different values of slip parameter for a Newtonian fluid is made (Table 1) and Sharidan et al. [28] corresponding to the skin-friction coefficient $f''(0)$ for unsteady flow of a Newtonian fluid is made (Table 2) and the results are to be found in excellent agreement.

In order to get a clear insight of the patterns of velocity and temperature fields for a non-Newtonian Maxwell fluid, an extensive numerical computation is accomplished for several values of the parameters that describe the flow characteristics, and the results are reported graphically.

Table 1. The skin-friction coefficient $-f''(0)$ for different values of δ for Newtonian fluid ($\beta = 0$)

Present	Anderson [27]	δ
1.000000	1.0000	0.0
0.872077	0.8721	0.1
0.776402	0.7764	0.2
0.591199	0.5912	0.5
0.430201	0.4302	1.0
0.283983	0.2840	2.0
0.144804	0.1448	5.0
0.081223	0.0812	10.0
0.043782	0.0438	20.0
0.018605	0.0186	50.0
0.009513	0.0095	100.0

Table 2. The skin-friction coefficient $-f''(0)$ for different values of A for Newtonian fluid ($\beta = 0$)

Present	Sharidan et al. [28]	A
1.261519	1.261512	0.8
1.378061	1.378052	1.2

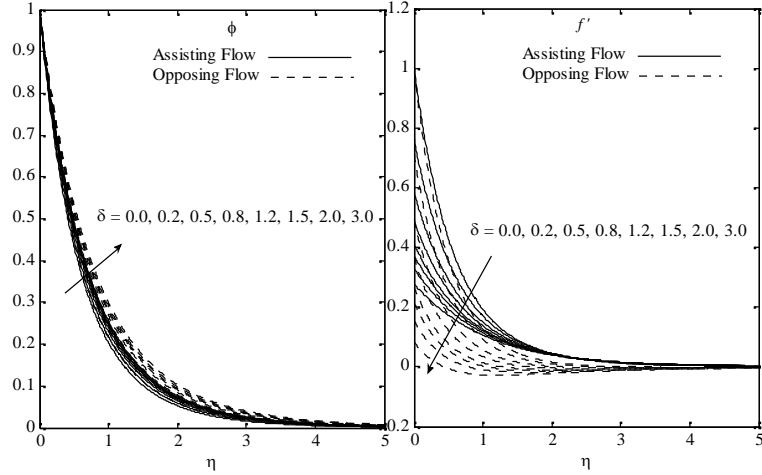


Fig. 1. Concentration (ϕ) and velocity (f') profiles for various values of slip parameter when $A = 0.7$, $Mn = 2.0$, $K = 0.6$ $\beta = 0.3$, and $Sc = 0.9$

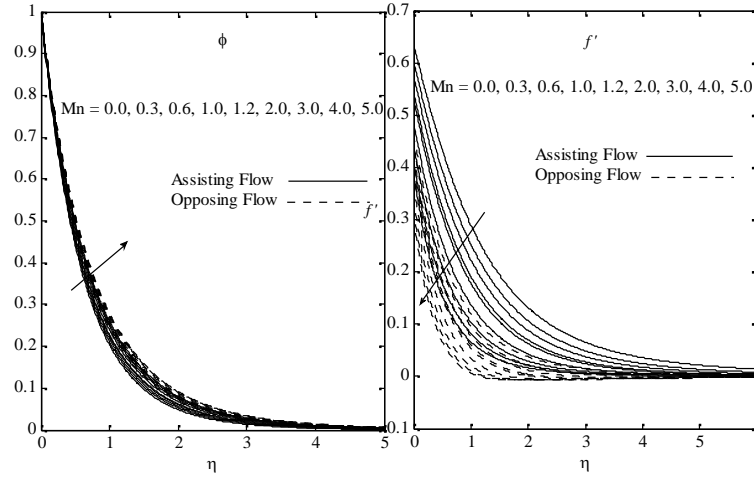


Fig. 2. Concentration (ϕ) and velocity (f') profiles for various values of magnetic field parameter when $A = 0.7$, $\delta = 0.8$, $K = 0.6$ $\beta = 0.3$, and $Sc = 0.9$

Fig. 1 illustrates the behavior of the x -component of the translational velocity and concentration distributions for different values of the slip parameter considering the two cases, namely assisting and opposing region flow. The slip parameter δ measures the amount of slip at the surface. It can be seen that, the velocity distribution decreases with the increasing values of the slip parameter. Consequently, with the increase of the slip parameter, the thickness of boundary layer increases. It can further be seen that as $\delta \rightarrow \infty$, the fluid velocity at the surface will coincide with the free stream velocity of the fluid, because if we promote the slip parameter δ to a value tending to infinity then the boundary layer structure will disappear. Moreover, the concentration profiles enhances with increasing the slip parameter.

Fig. 2 depicts the influence of the magnetic field parameter Mn on the fluid velocity and concentration profiles, considering the two cases namely assisting and opposing region flow. Application of a magnetic field normal to an electrically-conducting fluid has the tendency to provide a drag-like force referred to as the Lorentz force which acts in the reverse direction to

that of the flow, causing a flow retardation influence. This causes the fluid velocity to reduce. However, this reduction in the flow speed is taken by corresponding enhancement in the fluid thermal state standard. These conducts are clearly shown in the decrease in the fluid velocity and the enhancement in the concentration profiles. Furthermore, the magnetic parameter tends to reduce the velocity gradient at the wall and increase concentration gradient (see Figs. 7 to 12). The velocity gradient, as well as, the concentration gradient tends to decrease or increase quickly at first, then progressively level off as the magnetic field parameter increases.

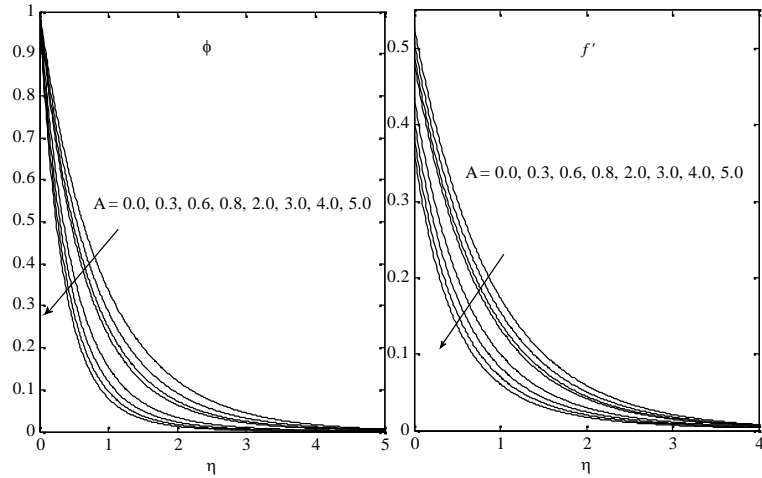


Fig. 3. Concentration (ϕ) and velocity (f') profiles for various values of unsteadiness parameter when $Mn = 2.0$, $\delta = 0.8$, $K = 0.6$ $\beta = 0.3$, and $Sc = 0.9$

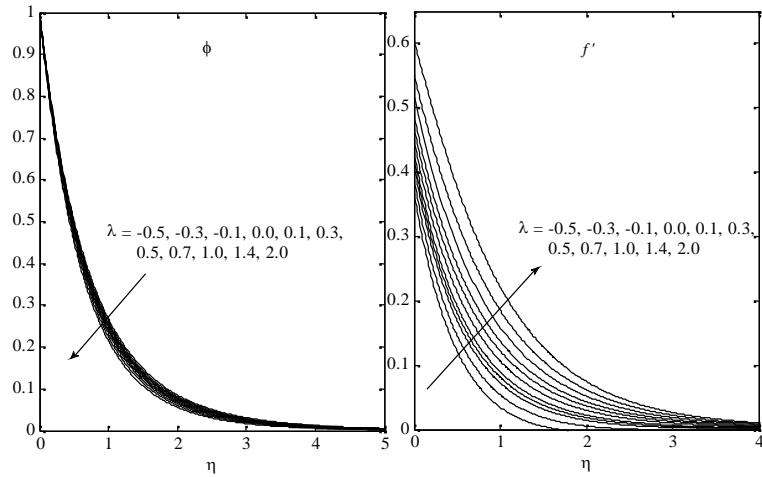


Fig. 4. Concentration (ϕ) and velocity (f') profiles for various values of diffusion parameter when $Mn = 2.0$, $A = 0.7$, $\delta = 0.8$, $K = 0.6$ $\beta = 0.3$, and $Sc = 0.9$

Fig. 3 exhibits the velocity and the concentration distributions for different values of the unsteadiness parameter A . It is noticed that the velocity along the sheet reduces at first with the increase in the unsteadiness parameter A , and this implies an accompanying decreases of the thickness of the momentum boundary layer near the surface. The steady case is obtained when

$A = 0$. Furthermore, it is observed that the concentration at a specific point is found to decrease significantly with increasing unsteadiness parameter.

The velocity and concentration profiles are depicted for several values of the diffusion parameter λ in Fig. 4. With increasing values of λ , the velocity at a point increases, but the converse effect is noticed for the concentration profiles i.e. $\phi(\eta)$ at a fixed point of η decreases with λ . This effect of diffusion of reactive species on the flow field is very significant from a physical and practical point of view.

From Fig. 5, one can observe that both of velocity profile $f'(\eta)$ and concentration decrease with an increase in the elastic parameter β . Therefore, the boundary layer thickness decreases for the same values of β . Physically, it is clearly that an increase in the elastic parameter increases the resistance of the fluid motion.

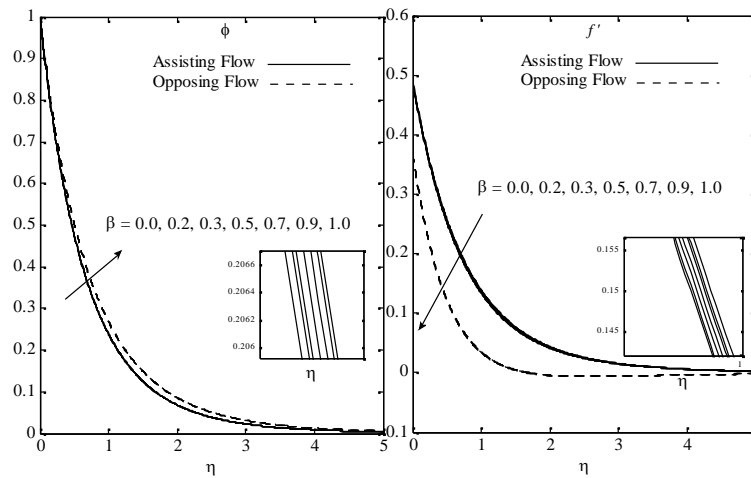


Fig. 5. Concentration (ϕ) and velocity (f') profiles for various values of Maxwell parameter when $Mn = 2.0$, $\delta = 0.8$, $K = 0.6$, $A = 0.7$, and $Sc = 0.9$

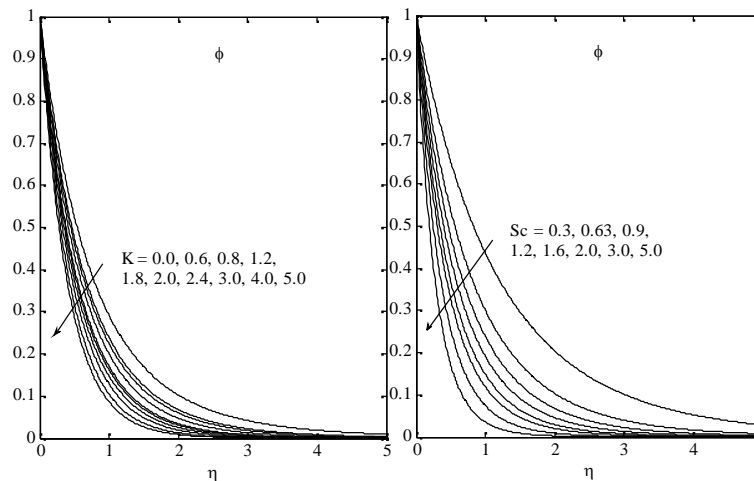


Fig. 6. Concentration (ϕ) profiles for various values of chemical reaction parameter and Schmidt number when $Mn = 2.0$, $\delta = 0.8$, $\beta = 0.3$, $\lambda = 0.7$ and $A = 0.7$,

The effects of the Schmidt number and the chemical reaction are displayed in Fig. 6 on the species concentration profiles. It is shown that, an increase in the values of Sc causes the concentration and its boundary-layer thickness to reduce significantly. This decrease in the solute concentration causes a reduction in the solutal buoyancy effects resulting in less induced flow along the sheet surface. The same effect of the chemical reaction parameter i.e. the chemical reaction parameter opposes the flow and the diffusion of the concentration distribution.

Variations of the skin-friction coefficient $f''(0)$ and the mass transfer coefficient $-\phi'(0)$ for various values of the elastic parameter β (Maxwell parameter) are displayed in Figs. 7 and 8. As it is noted that, the magnitude values of $f''(0)$ increases with increasing values of the Maxwell parameter, but the opposite effect occurs with mass transfer, i.e. the magnitude values of $-\phi'(0)$ decrease with increasing values of the Maxwell parameter.

Fig. 9 displays the influence of the unsteadiness parameter A on the velocity gradient at the wall f''_w . The magnitude of $f''(0)$ related to the skin-friction coefficient decreases with increasing values of the unsteadiness parameter A . A drop in the skin-friction coefficient as analyzed in this problem has an important application that in free coating operations and elastic properties of the coating formulations may be useful for the whole process. This implies that less force may be required to drag a moving sheet at a given withdrawal velocity, or equivalently higher withdrawal speeds can be carried out for a given driving force resulting in, increase in the rate of production. The behavior of the rate of mass transfer (from the sheet to the fluid) decreases with increasing values of A as shown in Fig. 10. As the unsteadiness parameter A increases, less mass is transferred from the sheet to the fluid; therefore, the concentration $\phi(\eta)$ reduces (Fig. 3) since the fluid flow is caused exclusively by the stretching surface.

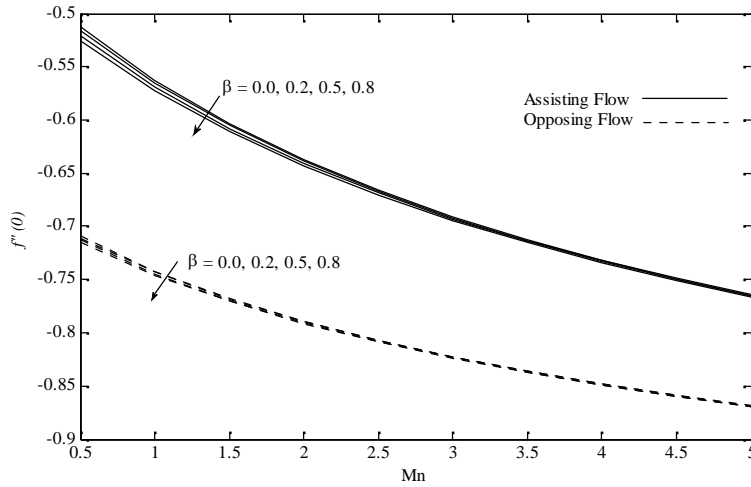


Fig. 7. Effect of Maxwell parameter on skin-friction coefficient $f''(0)$

Fig. 11 demonstrates the effect of the first-order chemical reaction rate parameter K on the variation of mass transfer coefficient $-\phi'(0)$. It is clearly shown that as the value of K increases, the magnitude values of $-\phi'(0)$ increase. Fig. 12 illustrates the effect of the slip

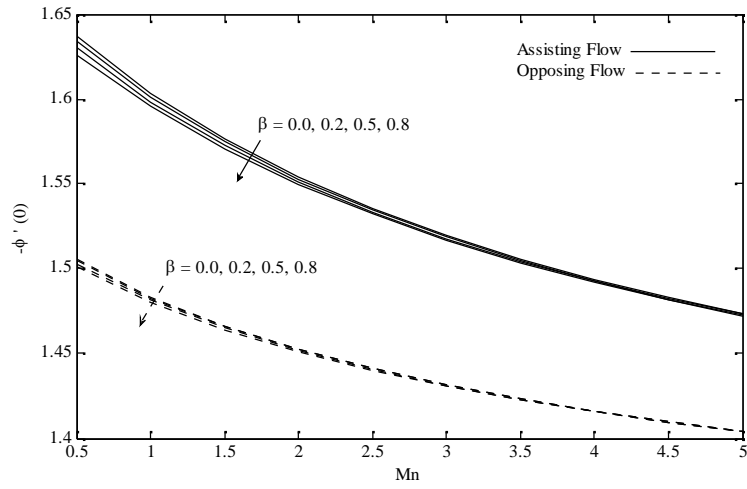


Fig. 8. Effect of Maxwell parameter on mass transfer rate $-\phi'(0)$

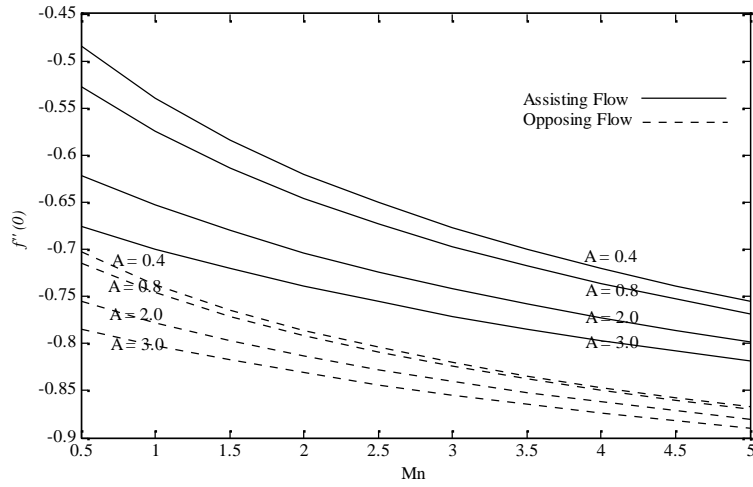


Fig. 9. Effect of unsteadiness parameter on skin-friction coefficient $f''(0)$

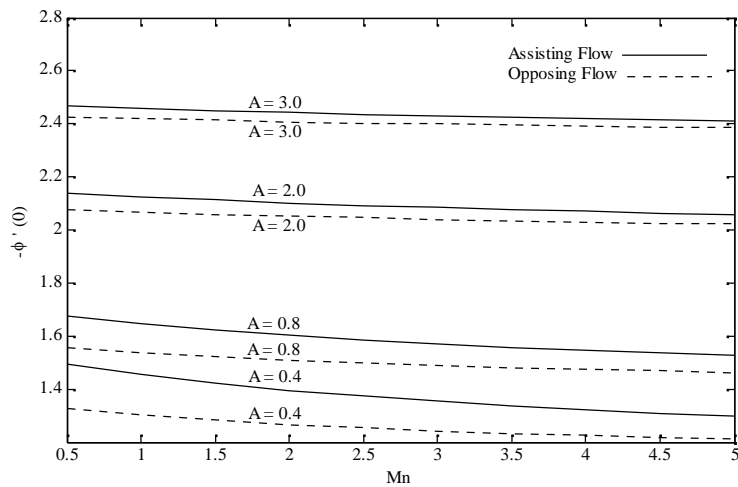


Fig. 10. Effect of unsteadiness parameter on mass transfer rate $-\phi'(0)$

parameter on the velocity gradient. As it is illustrated in the figure, the magnitude of the velocity gradient decreases with the increase of the slip flow parameter.

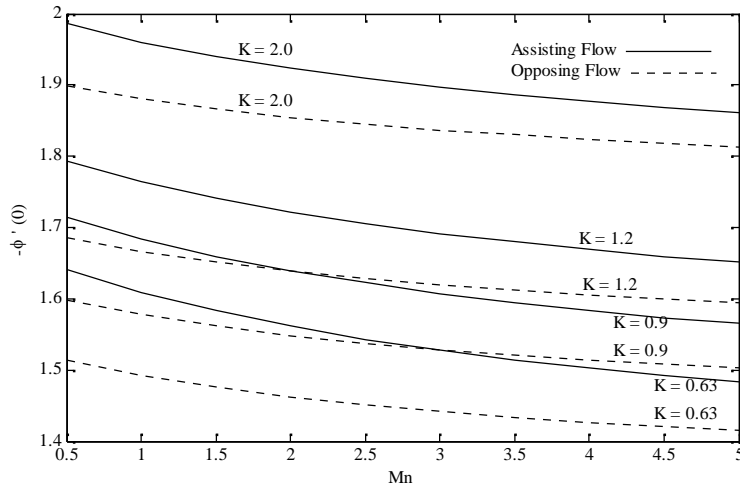


Fig. 11. Effect of chemical reaction parameter on mass transfer rate $-\phi'(0)$

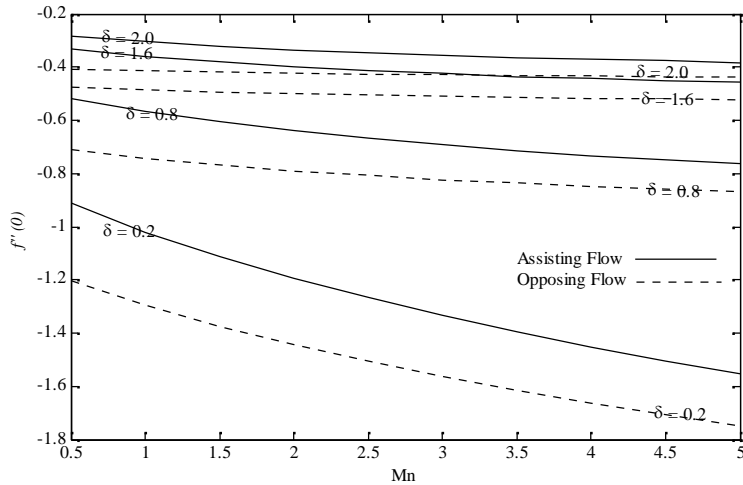


Fig. 12. Effect of slip parameter on skin-friction coefficient $f''(0)$

Conclusions

Our objective is to analyze the slip effects on diffusion of chemically reactive species in an unsteady MHD boundary layer flow of a Maxwell fluid due to a vertical stretching sheet with in the presence of a magnetic field effect. The whole analysis is performed taking the first-order chemical reaction and a linearly varying wall concentration. The transformed self-similar equations are solved numerically using the shooting method. Results for the velocity and

concentration distributions as well as the velocity gradient $f''(0)$ and the concentration gradient $-\phi'(0)$ are given for representative governing parameters. As a summary, it can be concluded that

- The fluid velocity decreases initially due to an increase in the unsteadiness parameter; and the concentration also decreases significantly in this case.
- The effect of increasing values of the slip and magnetic field parameters is to suppress the velocity field, whereas the concentration is enhanced with increasing slip and magnetic field parameters.
- Both of $f''(0)$ and $-\phi'(0)$ increase with the increase in the slip flow parameter, whereas the magnetic field and the unsteadiness parameters have the opposite effect.
- Increasing the chemical reaction parameter tends to decrease the concentration distribution and the chemical reaction parameter can be used to increase the rate of mass transfer.
- As the elastic parameter increases, the magnitude of $f''(0)$ increases while the reverse influence occurs with the mass transfer rate $-\phi'(0)$.

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