A NEW ANALYTICAL METHOD FOR DEFINING THE PUMP'S POWER OPTIMUM OF A WATER-TO-WATER HEAT PUMP HEATING SYSTEM USING COP

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Abstract- This paper analyzes the energy efficiency of the heat pump and the complete heat pump heating system. Essentially, the maximum of the coefficient of performance - COP_{max} of the heat pump and the heat pump heating system are investigated and determined by applying a new analytical optimization procedure. The analyzed physical system consists of the water-to-water heat pump, circulation and well pump. In the analytical optimization procedure the "first derivative equal to zero" mathematical method is applied. The objective function is the coefficient of performance of the heat pump, and the heat pump heating system. By using the analytical optimization procedure and the objective function, as the result, the local and the total energy optimum conditions with respect to the mass flow rate of hot and cold water i.e. the power of circulation or well pump are defined.

Keywords: objective function, heat pump, heat pump heating system, coefficient of performance, local optimum, global optimum.

1. Introduction

From the aspect of the coefficient of performance, COP, of the heating system with specified heating capacity of the water-to-water heat pump, it is important to select the adequate capacity of the pumps. In fact, there is an optimal capacity of the well and circulation pump that provide the maximum coefficient of performance, COP of the heating system.

In a similar manner, Granryd, E. [1] physically described and defined the task of energy optimization of the heat pumping systems, as well as in this study, however, the objective function, optimization variables and process, he performed on completely different way. He defined the COP of the system through the ideal Carnot process, i.e. through the air temperature of the heat source and sink, while in our study the COP is defined through the heat flux in the condenser and the required power of the compressor and the pumps. He optimized the airflow velocity through the evaporator and the condenser while the author of this paper optimized the mass flow rate of water in the hot and cold water loop and the power of well and circulation pump. In both studies the mathematical optimization procedure "first derivative equal to zero" is applied. In paper [1] the analytical energy optimization, numerical simulation and analysis are performed in detail for the cooling system, however, optimization of the heating system with a heat pump is only mentioned. In this study the main target was just an analytical energy optimization of the heating system with a heat pump. The objective functions, optimization methods, as well as the structure of the heat pump system in this paper is different relating to other papers, it can be seen based on the topics of the following papers. Sayyaadi, H. et al. [2] dealt with thermodynamic and thermo-economic optimization of a vertical ground source heat pump system. Esenand, H., Turgut, E. [3] focused on the optimization of vertical ground coupled heat pump system. In several papers the economical optimization of the heat pump systems is examined [4], [5], [6]. Some authors optimized the constructive details of the system [7], [8], [9]. In addition to the mentioned cases, optimization also be developed for different structures and for various applications of heat pumps [10], [11], [12]. The authors, and articles that deal with the topic of optimization, heat pumps systems and renewable energy sources are Kalmar, F. [13], [14], Kajtár, L. and Kassai, M. [15], [16].

In the enclosed study analytically derived the basic relations of the real, energy optimum conditions of the heating system with water-to-water heat pump. The main goals were: 1. define exactly the objective function and based on that, apply an analytical optimization procedure to determine the energy optimum conditions of the considered system. 2. fill in the gap in the world scientific literature related to the analytical energy optimization, (it can be seen from the review of literature, the basic analytical energy optimization conditions were not performed and published).

The considered physical system, i. e. the heating system with water-to-water heat pump, consists of the following components: heat pump, circulation and well pump. The heat pump is divided into the following components: evaporator, condenser and compressor. It should be mentioned, that the procedure is applicable for both air-to-air, air-to-water heating-cooling heat pump system. Energy optimization is performed for the established regime of the heating system, because in the time domain observed 98% of cases are taking place in this mode.

Nyers, J. et al. [17], [18], [19], [20] created a steady-state mathematical model with the aim to implementing the analytical energy optimization of the heat pump and the heat pump heating system. The model contains the overall coefficient of performance, COP, for the heat pump and heating system with a heat pump. The overall coefficient of performance, COP, of the heating system is the ratio between the obtained heat flux for heating and electric power used to drive the electric motor of the compressor, circulation and well pumps. Part of analytical optimization is the modification of the basic steady-state model for the evaporator and the condenser according to the requirements of the optimization process, [17].

In the analytical mathematical procedure of energy optimization, the objective function is the overall coefficient of performance, COP, of the heating system. The analytical optimization procedure is based on a method of "first derivative equal to zero". The overall coefficient of performance, COP, of the heating system with respect to the power of circulation and well pumps is partially differentiated. Individually, the obtained equations represent conditions of the local energy optimum with respect to the pumps' power. Together, these two equations, the conditions of local energy optimum, represent the condition of the global energy optimum. The common additional condition at local and global optimization is the compressor power constant.

At local optimization, besides the compressor power, the power of circulation pump or the power of well pump alternately is constant. However, when locally optimized with respect to the power of circulation pump, then, the power of the well pump is the constant and vice versa.

The global and local conditions of energy optimum with respect to the power of the circulation and well pump provide only trivial solutions; the optimal power of pumps equals to zero. The problem is, that there is no mathematical relation between the heat fluxes in the condenser and the evaporator as well as among the power of the circulation, the well pump and the compressor power.

The triviality problem is solved by introducing new variables: the mass flow rate of hot and cold water. Namely, the heat flux in the condenser and the power of the circulation pump depends on the mass flow rate of hot water, while the heat flux in the evaporator and the power of well pump depends on the mass flow rate of cold water.

Applying the mass flow rates of hot and cold water, we created new local and global energy optimum conditions. Equations of the energy optimum conditions are the basis of relations for analytic optimization of the heating system. This part of optimization process will be shown in the following article.

The new approach and the optimization procedure create a new opportunity to determine the optimal power of circulation and well pump. Optimal powers provide the maximum of the overall coefficient of performance, COP, of heat pump heating system.

The novelty in the paper: l. the condition equations of the local and global energy optimum of the heating system with respect to the power of circulation and well pump. 2. The condition equations of the local and global energy optimum of the heating system with respect to the mass flow rate of hot and cold water. 3. The complete proposed analytical energy optimization procedure. The method and procedure of energy optimization have general importance, because it is applicable not only for water-water, but also for air-to-air, air-water heat pump heating systems.

Th**is** paper is an introduction to a complete study of the analytical optimization procedure of the energy optimum of heat pump heating systems. The final theme will be the total energy optimization of the heating system according to three energy components: power of the compressor, circulation and well pump.

2. The physical model of the water-to-water heat pump heating system

2.1. Structure and functioning

From the aspect of the overall coefficient of performance, COP, of the water-to-water heat pump heating system, the system's structure consists of three loops.

1. Cold water loop. In heat pump's evaporator the ground water transports the heat from the layers of ground. The well pump provides energy for the water circulation in the evaporator and the loop using electrical power.

2. Hot water loop. From the heat pump's condenser the hot water transports the heat to heating bodies. The circulation pump ensures the power for the water circulation.

3. The refrigeration loop. The refrigerant receives heat from the cold well water by evaporation. The compressor sucks the refrigerant vapor and, by applying electrical power, compresses it. During compression the pressure and temperature of superheated vapor rise.

The condition of the heating system operation: all three circulation loops must be in operation.

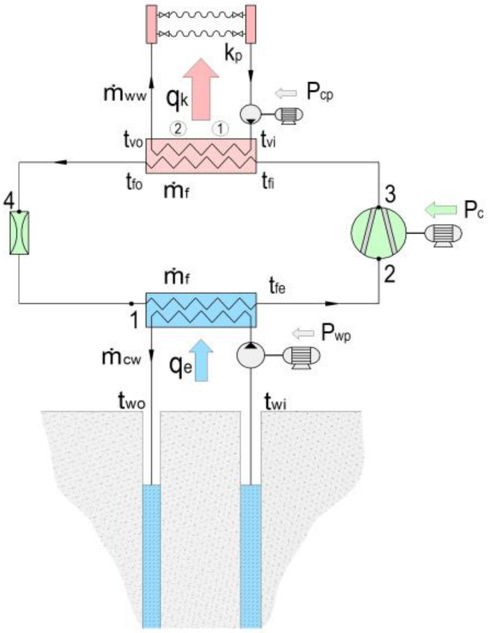

Figure1. Functional scheme of the heat pump heating system with components and symbols.

3. The mathematical model

3.1. Introduction

The heating system, i.e. the system for the transport of heat is energy-optimized, so it is important to define and analyze the energy efficiency of the components separately and the energy efficiency of the whole system. It is suitable to do it through the coefficient of performance, COP, separately, for the heat pump and for the entire heat pump heating system.

3.2. COP of the heat pump

The COP of the individual heat pump in heating regime is the ratio between heat pump heating capacity and the compressor power.

$$\eta_{hp} = \frac{q_k}{P_c} \tag{1}$$

The heat pump's heating capacity is equal to the heat, which the superheated vapor transfers in the condenser to the hot water. Heat transferred in the condenser is the sum of the heat pump cooling capacity and the compressor power.

$$q_k = q_e + P_c \tag{2}$$

After substitution:

$$\eta_{hp} = \frac{q_e + P_c}{P_c} = \frac{q_e}{P_c} + 1 \tag{3}$$

It can be seen that the heat pump COP depends only on the heat pump cooling capacity and compressor power. The power consumption of the circulation and well pump or fans has not been taken into account.

4. The coefficient of performance of the heat pump heating system

For the operation of the whole heat pump heating system, it is necessary to enable the simultaneous circulation of the cold and hot water. When the system's COP is defined, besides the compressor power, the circulation and well pump power should be taken into account as well.

The overall COP of the heating system is the total heating capacity of the system divided by the sum of all consumers power demand.

$$\eta = \frac{q_t}{\sum_1^3 P_i} \tag{4}$$

The heat transferred in the condenser is the sum of the cooling capacity and the compressor power.

$$q_k = q_e + P_c \tag{5}$$

The total heating capacity of the system is heat transferred in the condenser plus the power of the circulation pump.

$$q_t = q_k + P_{cp} = q_e + P_c + P_{cp} \tag{6}$$

The consumed power contains the power of the compressor, circulation pump and well pump.

$$\sum_1^3 P_i = P_c + P_{wp} + P_{cp} \tag{7}$$

After substitution, the expression of the overall coefficient of performance of the heat pump heating system is obtained in two forms.

1. from the point of condenser:

$$\eta = \frac{q_k + P_{cp}}{P_c + P_{wp} + P_{cp}} \tag{8}$$

2. from the point of evaporator:

$$\eta = \frac{q_e + P_c + P_{cp}}{P_c + P_{wp} + P_{cp}} \tag{9}$$

5. The analytical optimization procedure

5.1. Introduction

The goal of the energy optimization is that the overall coefficient of performance of the heat pump heating system achieves the maximum value. For a certain heating system with defined heating power of the water-to-water heat pump, the maximum value of the overall COP is achieved by optimizing the power of circulation and well pump.

Applying the mathematical model and the analytical or numerical optimization procedure, the optimum value of the mass flow rate of hot and cold water can be determined. Then, on the basis of the optimum value of the mass flow rates, the optimum power of circulation and well pump are defined. The obtained optimum values are the local optimums.

From the aspect of mathematics, there is only one global energy optimum, i. e. global maximum overall COP of the heating system. The two local optimum conditions together present the condition of the global energy optimum. One of the local optimum is, when optimization procedure determines the optimum mass flow rate of hot water. Furthermore, on the basis of the mass flow rate, the optimum power of circulation pump is determined. The other local optimum is when the optimization procedure determines the optimum mass flow rate of well water. Again, on the basis of the mass flow rate, the optimum power of well pumps is determined.

The applied analytical method of optimization is "the first differential equal to zero."

6. Direct optimization procedure

In direct procedure, the optimization is done directly through the power of circulation and well pump. It is the first step in the analytical optimization. The goal of optimization is the objective function, i.e. the overall COP of the heating system reaches the maximum value.

6.1. Objective function

In the energy optimization procedure the objective function is the overall COP of heat pump heating system.

$$\eta = \| \frac{q_e + P_c + P_{cp}}{P_c + P_{wp} + P_{cp}} \| \; max \tag{10}$$

$$\eta = \| \frac{q_k + P_{cp}}{P_c + P_{wp} + P_{cp}} \| \; max \tag{11}$$

In the applied analytical optimization procedure, the condition of extreme, the first differential with respect to the independent variable equals to zero. Extreme is maximum if the second differential is less than zero. In our case, the independent variables are the power of the circulation and well pump. The mathematical condition of the local optimum, i.e. extreme, the first differential of the overall heating system COP with respect to the power of circulation pump and well pump equals to zero.

$$\frac{\partial \eta}{\partial P_{cp}} = 0 \quad or \quad \frac{\partial \eta}{\partial P_{wp}} = 0 \tag{12}$$

The mathematical condition of the maximum is that the second derivative is less than zero. The second derivative of the COP, with respect to the power of circulation pump, well pumps or combined is:

$$\frac{\partial^2 \eta}{\partial {P_{cp}}^2} < 0 \quad or \quad \frac{\partial^2 \eta}{\partial {P_{wp}}^2} < 0 \quad or \quad \frac{\partial^2 \eta}{\partial P_{cp} \partial P_{wp}} < 0 \tag{13}$$

Remark: In the given case, in the optimization procedure, it is not necessary to determine the second derivative, because, on the basis of the simulations and measurements, we know that the extreme is always maximum.

The first derivative with respect to the power of the circulation pump, after substituting the expression of COP, is:

$$\frac{\partial \eta}{\partial P_{cp}} = \frac{\partial}{\partial P_{cp}} \left(\frac{q_k + P_{cp}}{P_c + P_{wp} + P_{cp}} \right) = 0 \tag{14}$$

The first derivative with respect to the power of well pump is:

$$\frac{\partial \eta}{\partial P_{wp}} = \frac{\partial}{\partial P_{wp}} \left(\frac{q_e + P_c + P_{cp}}{P_c + P_{wp} + P_{cp}} \right) = 0 \tag{15}$$

Remark: From the aspect of the circulation pump, the partial derivative of the first form is more convenient. From the aspect of the well pump, the second form of the COP derivative is more suitable.

6.2. Local energy optimum conditions with respect to the pump's power

The first derivative with respect to the circulation pump power is the first local energy optimum condition, i.e. the first local maximum. After partial derivation:

$$\left(\frac{\partial q_k}{\partial P_{cp}}+\frac{\partial P_{cp}}{\partial P_{cp}}\right)\cdot\left(P_c+P_{wp}+P_{cp}\right)-\left(q_k+P_{cp}\right)\cdot\left(\frac{\partial P_c}{\partial P_{cp}}+\frac{\partial P_{wp}}{\partial P_{cp}}+\frac{\partial P_{cp}}{\partial P_{cp}}\right)=0 \quad (16)$$

The first derivative, with respect to the well pump power, is the second local energy optimum condition, i.e. the second local maximum. After partial derivation:

$$\left(\frac{\partial q_e}{\partial P_{wp}}+\frac{\partial P_c}{\partial P_{wp}}+\frac{\partial P_{cp}}{\partial P_{wp}}\right)\left(P_c+P_{wp}+P_{cp}\right)-\left(q_e+P_c+P_{cp}\right)\left(\frac{\partial P_c}{\partial P_{wp}}+\frac{\partial P_{wp}}{\partial P_{wp}}+\frac{\partial P_{cp}}{\partial P_{wp}}\right)=0 \quad (17)$$

6.2.1. Substitutions

The circulation pump and the well pump are the autonomous, independent components of the heating system and there is no direct connection between them. In traditional control systems, in on-off control strategy, the pumps are started up at full power. In modern pumps there is frequent power, speed control. If the control algorithm is adaptive, then there is the possibility of harmonizing the pump's power, so as to achieve optimum performance.

Since, between the circulation and well pump there is no direct interaction, thus, mathematically, the mutual partial derivatives are equal to zero. The own derivative is equal to 1.

$$\frac{\partial P_{wp}}{\partial P_{cp}}=0 \quad and \quad \frac{\partial P_{cp}}{\partial P_{wp}}=0 \quad (18)$$

$$\frac{\partial P_{cp}}{\partial P_{cp}}=1 \quad and \quad \frac{\partial P_{wp}}{\partial P_{wp}}=1 \quad (19)$$

After substituting the partial derivatives (18) and (19), the local energy optimum conditions obtain a simpler form.

The local energy optimum condition with respect to the circulation pump power is:

$$\left(\frac{\partial q_k}{\partial P_{cp}}+1\right)\cdot\left(P_c+P_{wp}+P_{cp}\right)-\left(q_k+P_{cp}\right)\cdot\left(\frac{\partial P_c}{\partial P_{cp}}+1\right)=0 \quad (20)$$

$$P_c = constant, \qquad P_{wp} = constant,$$

$$P_{cp} = optimum$$

Local energy optimum condition with respect to the well pump power is:

$$\left(\frac{\partial q_e}{\partial P_{wp}}+\frac{\partial P_c}{\partial P_{wp}}\right)\cdot\left(P_c+P_{wp}+P_{cp}\right)P_c-\left(q_e+P_c+P_{cp}\right)\cdot\left(\frac{\partial P_c}{\partial P_{wp}}+1\right)=0 \quad (21)$$

$$P_c = constant, \qquad P_{cp} = constant,$$

$$P_{wp} = optimum$$

6.3. Global energy optimum condition with respect to the power of pumps

In principle: The global energy optimum condition of the heating system is obtained if the equations of the local energy optimum condition (20) and (21) are solved according to the power of circulation and well pump.

But, it is not possible to achieve the direct analytical or numerical solution of equations (20) and (21), because there is no mathematical relation between heat flux and pumps, and between the powers themselves. Consequently, it is not possible to determine the partial derivatives. However, introducing the mass flow rates as new independent variables, we can obtain the solution of equations of local energy optimum condition.

7. Indirect optimization procedure

7.1. Introduction

Energy optimum conditions obtained in direct procedure have to be further developed. Instead of the power of circulation and well pump, it is necessary to introduce new independent variables, i.e. the mass flow rate of the hot and cold water. The replacement is justifiable because:

1. The heat flux in the condenser and the power of circulating pump depend on the mass flow rate of hot water.

$$q_k = q_k(\dot{m}_{ww}) \quad and \quad P_{cp} = P_{cp}(\dot{m}_{ww}) \tag{22}$$

2. The heat flow in the evaporator and the power of well pump depend on the mass flow rate of cold water.

$$q_e = q_e(\dot{m}_{cw}) \quad and \quad P_{wp} = P_{wp}(\dot{m}_{cw}) \tag{23}$$

3. The power of the compressor depends on the mass flow rate of refrigerant and the refrigerant mass flow rate depends on the mass flow rate of cold and hot water.

$$P_c = P_c\left(\dot{m}_f(\dot{m}_{ww}, \dot{m}_{cw})\right) \tag{24}$$

7.2. Partial differentials with respect to water mass flow rates

The transition to the new independent variables requires some mathematical operations. Instead of the power of pumps, a new variable of the mass flow rates of water appears. Mathematically, the transformations are as follows:

- the heat flux in the condenser, instead of the power of circulation pump, is differentiated with respect to the mass flow rate of hot water:

$$\frac{\partial q_k}{\partial P_{cp}} = \frac{\partial q_k}{\partial \dot{m}_{ww}} \cdot \frac{\partial \dot{m}_{ww}}{\partial P_{cp}} = \frac{\partial q_k}{\partial \dot{m}_{ww}} \cdot \frac{1}{\frac{\partial P_{cp}}{\partial \dot{m}_{ww}}} \tag{25}$$

- the heat flux in the evaporator, instead of the power of well pump, is differentiated with respect to the mass flow rate of cold water:

$$\frac{\partial q_e}{\partial P_{wp}} = \frac{\partial q_e}{\partial \dot{m}_{cw}} \cdot \frac{\partial \dot{m}_{cw}}{\partial P_{wp}} = \frac{\partial q_e}{\partial \dot{m}_{cw}} \cdot \frac{1}{\frac{\partial P_{wp}}{\partial \dot{m}_{cw}}} \tag{26}$$

- the power of compressor, instead of the power of circulation pump, is differentiated with respect to the mass flow rate of hot water:

$$\frac{\partial P_c}{\partial P_{cp}} = \frac{\partial P_c}{\partial \dot{m}_f} \cdot \frac{\partial \dot{m}_f}{\partial \dot{m}_{ww}} \cdot \frac{\partial \dot{m}_{ww}}{\partial P_{cp}} = \frac{\partial P_c}{\partial \dot{m}_f} \cdot \frac{\partial \dot{m}_f}{\partial \dot{m}_{ww}} \cdot \frac{1}{\frac{\partial P_{cp}}{\partial \dot{m}_{ww}}} \tag{27}$$

- the power of compressor, instead of the power of well pump, is differentiated with respect to the mass flow rate of cold water:

$$\frac{\partial P_c}{\partial P_{wp}} = \frac{\partial P_c}{\partial \dot{m}_f} \cdot \frac{\partial \dot{m}_f}{\partial \dot{m}_{cw}} \cdot \frac{\partial \dot{m}_{cw}}{\partial P_{wp}} = \frac{\partial P_c}{\partial \dot{m}_f} \cdot \frac{\partial \dot{m}_f}{\partial \dot{m}_{cw}} \cdot \frac{1}{\frac{\partial P_{wp}}{\partial \dot{m}_{cw}}} \tag{28}$$

7.3. Local energy optimum conditions of the heating system with respect to water mass flow rates

After substituting the derivatives of the new independent variables the local energy optimum condition with respect to the mass flow rate of hot water is:

$$\left(\frac{\partial q_k}{\partial \dot{m}_{ww}} \cdot \frac{1}{\frac{\partial P_{cp}}{\partial \dot{m}_{ww}}} + 1\right)\left(P_c + P_{wp} + P_{cp}\right) - \left(q_k + P_{cp}\right)\left(\frac{\partial P_c}{\partial \dot{m}_f} \cdot \frac{\partial \dot{m}_f}{\partial \dot{m}_{ww}} \cdot \frac{1}{\frac{\partial P_{cp}}{\partial \dot{m}_{ww}}} + 1\right) = 0 \qquad (29)$$

$$P_c = \ constant, \ \ P_{wp} = constant,$$

$$\dot{m}_{ww} = \boldsymbol{optimum}$$

The local energy optimum condition with respect to the mass flow rate of cold water is:

$$\left(\frac{\partial q_e}{\partial \dot{m}_{cw}} \cdot \frac{1}{\frac{\partial P_{wp}}{\partial \dot{m}_{cw}}} + \frac{\partial P_c}{\partial P_{wp}}\right)\left(P_c + \ P_{wp} + \ P_{cp}\right) \qquad (30)$$

$$- \left(q_e \ + \ P_c + \ P_{cp}\right)\left(\frac{\partial P_c}{\partial \dot{m}_f} \cdot \frac{\partial \dot{m}_f}{\partial \dot{m}_{cw}} \cdot \frac{1}{\frac{\partial P_{wp}}{\partial \dot{m}_{cw}}} + 1\right) = 0$$

$$P_c = \ constant, \ \ P_{cp} = \ constant,$$

$$\dot{m}_{cw} = optimum$$

7.4. Global energy optimum condition of the heating system with respect to water mass flow rates

The global energy optimum condition of the heating system obtained by direct or indirect procedure is essentially the same. The difference is that, instead of power of pumps, the mass flow rate of the hot and cold water appear as independent variables.

Mathematically, the global energy optimum is obtained, if the two equations of local energy optimum condition (28), (29) are solved according to mass flow rates. The solution gives the optimum of mass flow rate of hot and cold water at the same time. The two local energy optimums represent the global optimum and provide the maximum value of COP of the heating system. Determining the optimal value of the mass flow rate of hot and cold water can determine the optimal value of the power of circulation and well pump, while the power of the compressor is assumed constant.

$$P_c = \ constant$$

$$\dot{m}_{ww} = optimum, \ \ P_{cp} = optimum, \ \ \ \dot{m}_{cw} = optimum, \ \ P_{wp} = optimum$$

8. The energy optimum of the heat pump

8.1. Introduction

It is possible to find the energy optimum of heat pump mathematically, but practically, it has no any particular importance. This is explained by the fact that the mass flow rate of hot and cold water is assumed constant, the type and the surface area of the evaporator is a known value.

8.2. Direct optimization procedure

The energy optimum of the heat pump in direct optimization procedure is obtained by differentiating the heat pump's COP with respect to the power of the compressor.

The first differential is equal to zero.

$$\frac{d\eta}{dP_c} = \frac{dq_e}{dP_c} = \frac{dq_e}{dP_c} \cdot P_c - q_e \cdot \frac{dP_c}{dP_c} = 0 \tag{31}$$

The obtained differential equation separates the variables, and it can be solved as follows:

$$\frac{dq_e}{q_e} = \frac{dP_c}{P_c} \tag{32}$$

The solution of the differential equation is:

$$lnq_e = lnP_c$$
$$q_e = P_c \tag{33}$$

The solution of differential equation is trivial and is not useful in practice. However, the process continues and a new variable is introduced, i.e. the mass flow rate of the refrigerant, instead of the compressor power.

8.3. Indirect optimization procedure

The heat flux in the evaporator and the compressor power depends on the mass flow rate of the refrigerant.

$$q_e = q_e(\dot{m}_f) \tag{34}$$
$$P_c = P_c(\dot{m}_f) \tag{35}$$

The transition of the compressor power to the mass flow rate of refrigerant is:

$$\frac{dq_e}{dP_c} = \frac{dq_e}{d\dot{m}_f} \cdot \frac{d\dot{m}_f}{dP_c} = \frac{dq_e}{d\dot{m}_f} \cdot \frac{1}{\frac{dP_c}{d\dot{m}_f}} \tag{36}$$

$$\frac{dP_c}{dP_c} = 1 \tag{37}$$

The equation of energy optimum condition of the heat pump is:

$$\frac{dq_e}{d\dot{m}_f} \cdot \frac{1}{\frac{dP_c}{d\dot{m}_f}} \cdot P_c - q_e \cdot 1 = 0 \tag{38}$$

Solving the equation of the energy optimum condition (38) according to the mass flow rate of refrigerant, the optimum mass flow rate of the refrigerant and on the basis of that the optimum compressor power is obtained.

$$\dot{m}_f = optimum \rightarrow P_c = optimum$$

$$\dot{m}_{ww} =, \quad \dot{m}_{cw} =, \quad F_e = , \quad and \quad evaporator\ type = \ assumed\ constant$$

The optimum mass flow rate of the refrigerant and the optimum compressor power is valid under the condition that the mass flow rate of hot and cold water as well as the size and type of the evaporator surface are known, i.e. ussumed constants.

9. Summary

In this study, the aim was to set up a physical and mathematical basis for analytical energy optimization of heat pump and heat pump heating system. In other words, define the maximum COP according to the optimum mass flow rate of the hot and cold water. As a final goal, on the basis of the optimum mass flow rates, to define the optimum power of circulation and well pump.

Energy efficiency of the considered heat pump and heat pump heating system is analyzed through the coefficient of performance. In the process of energy optimization of the heating system, the objective function is taken as the coefficient of performance of the complete system. In the mathematical optimization procedure for obtaining the maximum coefficient of performance the principle of "first derivative equal to zero" is applied.

Based on the mentioned principle, in the first step, the local optimum is determined with respect to the power of circulation and well pump. In other words, the partial derivation of the objective function was carried out with respect to the well and circulation pump power. The obtained first derivatives equaled to zero. The equations of the COP derivative are implicit and contain partial derivatives of the circulation and well pump power. The obtained equations are the analytical local energy optimum conditions. Together, the two local optimum conditions provide the global energy optimum condition.

By direct procedure, it is not possible to obtain the partial derivatives for defining the conditions of the local and the global energy optimum. The procedure should be continued with the indirect determination of the partial derivatives that means, instead of the pump's power, the mass flow rate of cold and hot water should be introduced as new variables. This is possible, because the cooling capacity, compressor power and power of circulation and well pump depend on the mass flow rates. The mentioned relations have a mathematical description, that is, there are mathematical relations. In the paper the basic equations of the local and the global energy optimum conditions are defined by direct and indirect procedure.

The following studies will defined he mathematical relations of cooling capacity, compressor power and the power of circulation and well pump, depending on the mass flow rates.

Nomenclature

$\dot{m}$ = mass flow rate, kg s^{-1}

P = power, W

q = heat flux, W

Greek symbols

η = coefficient of performance - COP,

Subscripts and Superscripts

ww = hot water,

cw = cold water,

f = refrigerant,

e = evaporator,

k = condenser,

cp = circulation pump,

wp = well pump,

hp = heat pump,

c = compressor,

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