

ENTROPY ANALYSIS IN MOVING WAVY SURFACE BOUNDARY LAYER

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Abstract: *It is a well-established fact that significant gain in the heat transfer rate can be obtained by altering that flat surface texture of the working body. The most convenient alteration, in view of mathematical handling, is the wavy one. Existing studies reveal that the convective heat transfer phenomenon is affected significantly due to the presence of a solid wavy surface. How does the phenomena of entropy generation is effected due to a wavy surface is the question investigated in this manuscript. The expressions for irreversibility distribution rate, Bejan number and volumetric entropy generation number have been evaluated by Keller-Box method. The effect of important parameters of interest, such as wavy amplitude, Prandtl number and group parameter on irreversibility distribution rate, Bejan number and entropy generation number, have been discussed in detail. The study reveals that entropy generation number decreases and irreversibility rate increases by increasing the amplitude of the wavy surface.*

Keywords: *Entropy generation, viscous fluid, irreversibility, Bejan number, wavy surface, Keller Box method*

1. Introduction

Efficiency of the thermal engineering systems can always be upgraded by minimizing the heat losses. So far, no mechanical device has been invented which can completely convert the supplied energy into useful work. The heat losses are always tried to be minimized by utilizing various techniques. The most effective design of a thermal system can be accomplished by reducing entropy generation in the systems. Entropy generation is the key issue in several engineering equipment for example cooling of nuclear reactors and electronic devices, geophysical fluid dynamics, energy storage systems and heat exchangers etc. Thermodynamic irreversibility is linked with entropy generation and it exists in all types of heat transfer phenomenon. Characteristics of convective heat transfer, that is, viscous dissipation and finite temperature gradient are the main causes of irreversibility and are accountable for entropy generation. Entropy generation minimization (EGM) is the method credited to Bejan [1] by which thermodynamic optimization of a real system can be achieved by controlling both the causes of irreversibility. Since the primary objective of EGM analysis of flow and heat transfer phenomenon is to investigate the situations for the full use of the energy resources by reducing energy losses and improving the thermal systems. To do this the second law of thermodynamics is used because it examines the reversibility in the system through entropy generation rate.

The analysis of entropy generation in convective heat transfer phenomena is associated to some physical parameters, like entropy generation number, Bejan number and irreversibility rate, as determined by Bejan [2] and subsequent investigators (e.g., Paoletti *et al.* [3] and Benedetti and Sciubba [4]). Using this approach, the causes of entropy production in four different configurations of convective heat transfer process were investigated by Bejan [5]. After this groundbreaking work of Bejan a lot of investigations have been made on entropy analysis in convective heat transfer during the last few years. The investigation of mixed convection flow with entropy generation was studied by Abu-Hijleh and Heilen [6]. They found that increase in the buoyancy parameter and the Reynolds number also increases the entropy generation rate. Tasnim *et al.* [7] analyzed entropy generation with hydromagnetic effect in a porous channel. Forced convection inside a channel was studied by Mahmud and Fraser [8]. They derived analytic expression for the entropy generation number and the Bejan number. Carrington and Sun [9] used control volume method approach to calculate entropy generation in some flow situations of internal and external flows. Selamet and Arpaci [10] discussed entropy generation in boundary layer flows and determined the same two major causes of entropy production. Munawar *et al.* [11] discussed the causes of entropy production in flow over an oscillatory stretching cylinder and concluded that the entropy rate increases with amplitude of oscillations. Recently, Munawar *et al.* [12] investigated the second law analysis in a peristaltic flow of a variable viscosity fluid and reveal that entropy production is high in the contracted region and reduces in the wider part of channel. Some more interesting studies on the topic could be of interest for readers [13-20].

Despite of the above mentioned studies a more significant and frequently occurring flow configuration is the convective heat transfer over irregular surface which is seen in several practical applications. To enhance the convection phenomena surfaces are intentionally roughened. Since this roughening element disrupts the flow, consequently, the convection strengthens and hence heat transfer rate increases. In this regard, Yao [21] investigated the natural convective heat transfer phenomena from vertical sinusoidal wavy surfaces in a viscous fluid. Rees and Pop [22] analyzed natural convective flow along vertical wavy surface in porous media. Hossain and Rees [23] examined heat and mass transfer in a natural convection flow along a vertical wavy surface. Rees and Pop [24] analyzed free convective and heat transfer along wavy horizontal plate. Hossain and Pop [25] discussed the influence of MHD on boundary layer flow on a moving wavy surface. Narayana *et al.* [26] investigated effects of double diffusive on a horizontal wavy plate in porous medium. We found two investigations; Chen *et al.* [27] and Chen *et al.* [28] who examined entropy production in two different flow situations over a wavy plate under the influence of thermal radiation effects. They concluded that the increase in thermal radiation results in increase of entropy. This can be explained as the fluid absorbs thermal radiation heat flux which increases heat transfer of the flow field. Therefore, the entropy generation due to heat transfer is more prominent for wavy surface. Readers are referred to some other interesting investigations on heat transfer through wavy surface [28-34]. A literature survey immediately reveals that the entropy analysis of convective heat transfer phenomena is limited to self-similar flows and no attention has been given to the non-similar flows. Owing to this fact, our focus here is to examine entropy generation in a non-similar boundary-layer flow due to a uniformly moving wavy plate. The impact of surface texture upon the

entropy generation number and irreversibility rate have been investigated in detail and discussed in section 5 with the help of graphs and tables.

2. Mathematical description

We consider steady incompressible viscous flow due to a horizontal wavy sheet moving uniformly in x -direction. The surface shape is assumed to be smooth differentiable described by the function

$$\bar{y} = \bar{S}(\bar{x}) = \bar{\alpha} \sin\left(\frac{\pi \bar{x}}{l}\right). \quad (1)$$

The schematic of the plate geometry and the associated coordinate system is shown in Fig. 1.

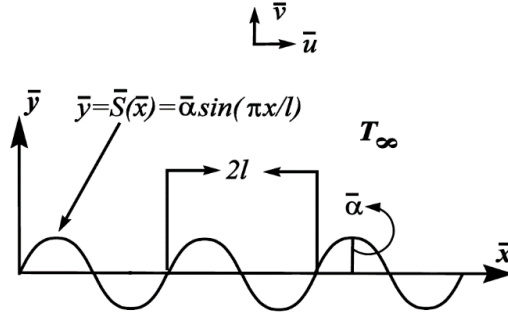


Figure 1: Schematic diagram of the flow configuration.

Essentially the flow is two dimensional and non-similar in nature. The problem has already been modelled in detail by Hossain and Pop [25] in which the authors have considered an electrically conducting fluid along with uniform magnetic field. The governing non-similar equations in the absence of magnetic field immediately follow as:

$$f''' + \frac{1}{2} ff'' - \xi \frac{\sigma_\xi}{\sigma} [ff'' - f'^2] = \xi \left[f' \frac{\partial f'}{\partial \xi} - f'' \frac{\partial f}{\partial \xi} \right], \quad (2)$$

$$\frac{1}{Pr} \theta'' + \frac{1}{2} f \theta' + \xi \frac{\sigma_\xi}{\sigma} f \theta' = \xi \left[f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right], \quad (3)$$

where the following set of dimensionless variables has been utilized:

$$\left. \begin{aligned} x &= \frac{\bar{x}}{l}, \quad y = \frac{\bar{y}}{l}, \quad u = \frac{\bar{u}}{U}, \quad v = \frac{\sqrt{\text{Re}}}{U} (\bar{v} - S_\xi \bar{u}), \quad p = \frac{\bar{p}}{\rho U^2}, \quad S = \frac{\bar{S}(\bar{x})}{l}, \\ \eta &= \frac{(y - S) \sqrt{\text{Re}}}{\sigma \sqrt{\xi}}, \quad \xi = x, \quad \sigma = \sqrt{1 + S_\xi^2}, \quad \alpha = \frac{\bar{\alpha}}{l}, \quad \text{Re} = \frac{Ul}{\nu}, \quad \text{Pr} = \frac{\mu C_p}{k}, \\ u &= \frac{\partial \psi}{\partial x}, \quad v = -\frac{\partial \psi}{\partial y}, \quad \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \psi(\xi, \eta) = \sigma \sqrt{\xi} f(\xi, \eta), \end{aligned} \right\} \quad (4)$$

where Pr is the Prandtl number, Re the Reynolds number and the prime denotes differentiation with respect to η . The parameter σ and σ_ξ represent the wavy contribution in Eqs. (2) and (3). The appropriate boundary conditions of the system (2) and (3) read as Hossain and Pop [25]:

$$\left. \begin{aligned} f(\xi, 0) = 0, \quad f'(\xi, 0) = \frac{1}{\sigma}, \quad \theta(\xi, 0) = 1, \\ f'(\xi, \infty) = 0, \quad \theta(\xi, \infty) = 0. \end{aligned} \right\} \quad (5)$$

The local skin friction coefficient and the local Nusselt number are given by

$$C_{fx} = \frac{\tau_w}{\rho U^2}, \quad Nu_x = \frac{\bar{x} q_w}{k(T_w - T_\infty)}, \quad (6)$$

where τ_w is the wall shear stress and q_w is the wall heat flux which are given as

$$\tau_w = \mu(\nabla \bar{u} \cdot \hat{n})_{y=0}, \quad q_w = -k(\nabla T \cdot \hat{n})_{y=0}, \quad (7)$$

in which \hat{n} is the unit normal to the wavy surface. After using Eqs. (4) and (7), the skin friction coefficient and the local Nusselt number, respectively, take the form

$$C_f = C_{fx} (Re_x)^{1/2} = \frac{1}{\sigma} f''(\xi, 0), \quad Nu = Nu_x (Re_x)^{-1/2} = -\theta'(\xi, 0). \quad (8)$$

3. Entropy Analysis

For two-dimensional viscous incompressible fluid which follows the Fourier law of heat conduction and is in local thermodynamic equilibrium. The volumetric rate of entropy generation in Cartesian coordinate system is defined as follow [1-2], [5] and [36]:

$$S_G = \overbrace{\frac{k}{T_\infty^2} \left[\left(\frac{\partial T}{\partial x} \right)^2 + \left(\frac{\partial T}{\partial y} \right)^2 \right]}^{\text{Entropy generated by heat transfer, } S_{G,H}} + \overbrace{\frac{\mu}{T_\infty} \left[2 \left(\frac{\partial \bar{u}}{\partial x} \right)^2 + 2 \left(\frac{\partial \bar{v}}{\partial y} \right)^2 + \left(\frac{\partial \bar{v}}{\partial x} + \frac{\partial \bar{u}}{\partial y} \right)^2 \right]}^{\text{Entropy generated by fluid friction, } S_{G,F}}. \quad (9)$$

The above expression shows that the viscosity (μ) and the thermal conductivity (k) create irreversibility. Entropy rate is positive and finite only if velocity gradient or temperature exists in the medium. The entropy generation number is the ratio of volumetric entropy generation rate to the characteristic entropy generation rate. Using set of dimensional variables listed in Eq. (4) the total entropy generation number takes the form

$$\begin{aligned} N_S &= \frac{S_G}{S_{G_0}} = \frac{1}{\xi} \theta'^2 + \frac{\omega(1 + S_\xi^2)}{\xi} f''^2, \\ &= N_H + N_F. \end{aligned} \quad (10)$$

where $S_{G_0} = k(\Omega/l)^2$, $\Omega = \Delta T/T_\infty$, T_∞ , $\omega = Br/\Omega$ and $Br = \mu U_w^2/k\Delta T$ are the characteristic entropy, the dimensionless temperature difference, the reference temperature of the fluid, group parameter and

Brinkman number, respectively. The first term represents the heat transfer irreversibility denoted by N_H and the second term represents fluid friction irreversibility and is denoted by N_F . The ratio of heat transfer irreversibility to the total irreversibility is known as the Bejan number which is given by

$$\text{Be} = \frac{1}{1 + \Phi}, \quad (11)$$

where $\Phi = N_F/N_H$ is the irreversibility distribution ratio. The Bejan number helps in characterizing the major role of irreversibilities with in the range $[0, 1]$. If the value of the Bejan number is near 0 the entropy is dominated by fluid friction effects. However, if $\text{Be} \approx 1$ then the irreversibility due to heat transfer is dominant. Similarly, effect of irreversibility due to both factors are equal if $\text{Be} = 1/2$.

4. Numerical solution

The governing non-similar system of partial differential Eqs. (2) and (3) are solved by Keller-Box scheme (for details see. [37-40]) together with implicit finite difference method along with boundary conditions (5). According to this procedure, Eqs. (2) and (3) are reduced to a system of first order ODE. Using the central difference formula we approximate the functions and their derivatives in finite difference form to get non-linear difference equations. After this we use Newton's method to linearize non-linear difference equations and solve with the help of block tri-diagonal algorithm.

Table 1: Comparison of present results with already published data when $\text{Pr} = 0.7$ and $\alpha = 0$

	Present	Mehmood <i>et al.</i> [32]	Rees and Pop [24]	Hossain and Pop [25]
$C_{fx} (\text{Re}_x)^{1/2}$	-0.44375	-0.44375	-0.4438	-0.4439
$\text{Nu}_x (\text{Re}_x)^{-1/2}$	-0.34924	-0.34924	-0.3492	-0.3509

Table 2: Comparison of present results with already published data when $\text{Pr} = 7$, $\zeta = 0.5$ and $\alpha = 0.2$

	Present	Mehmood <i>et al.</i> [32]
$-C_{fx} (\text{Re}_x)^{1/2}$	0.87458	0.87458
$-\text{Nu}_x (\text{Re}_x)^{-1/2}$	0.92201	0.92201

In order to investigate the accuracy and validity of the computed results, a comparison of present results with available results in literature are shown in Tables 1 and 2. We clearly see from the table that our results of the skin friction coefficient and the local Nusselt number are very close to the existing results reported in Rees and Pop [24], Hossain and Pop [25] and Mehmood *et al.* [32]. This provides confidence in the present numerical results. We use the same procedure to solve Eqs. (2), (3) and (5) in order to make the further analysis.

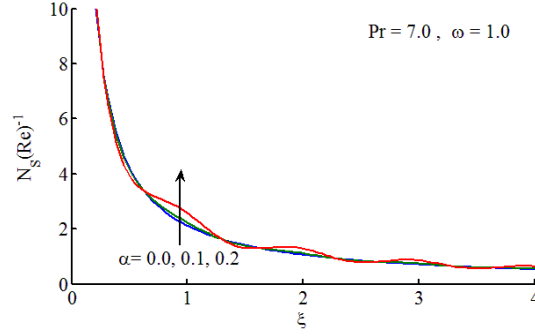


Fig. 2: N_S against ζ for different α

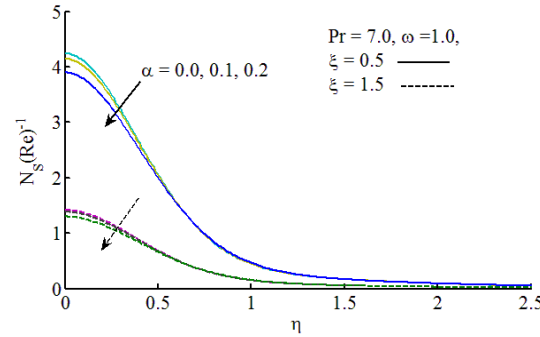


Fig. 3: N_S against η for different α

5. Results and discussion

The numerical results are used to examine the entropy generation phenomenon in the flow with the help of graphs shown in Figs. 2-18. The entropy number is plotted against the variable ζ for different values of amplitude of surface oscillations. It is noticed that the entropy generation is high for small ζ and reduces gradually in the downstream direction. This is because of the velocity and temperature gradients at the wall are less at downstream than at upstream. Moreover, it is also observed that the magnitude of oscillation in entropy number increases as the parameter α increases. The reason of this fact is somewhat trivial; large values α of enhance the velocity fluctuations due to which the fluctuations as the velocity and temperature gradients also increase. Consequently, the amplitude of entropy fluctuations also increases. Fig. 3 depicts the behavior of entropy generation across the boundary layers. It is seen from the figure that entropy number is higher near the wall and vanishes in the freestream flow region. The reason behind this fact is again the large velocity gradients in the near wall region. To discriminate the entropy generation rates corresponding to the viscous and thermal dissipation the Bejan number is plotted in Figs. 4 and 5 against variables ζ and η , respectively. Fig. 4 demonstrates that the size of fluctuations in Be increases as one moves in downstream direction. In this graph an interesting role of α is highlighted; the fluctuations in the Bejan number are of very small amplitude at the upstream locations which significantly develop at the downstream locations. Fig. 5 shows that near wall irreversibility is dominated by heat transfer effects and far away from the wall is the region of fluid friction dominancy. As one moves away

from the wall the dominance of entropy due to heat transfer becomes weaker and weaker and the two rates, namely the heat transfer entropy rate and the viscous entropy rate balance each other at $\eta = 0.9$ (roughly).

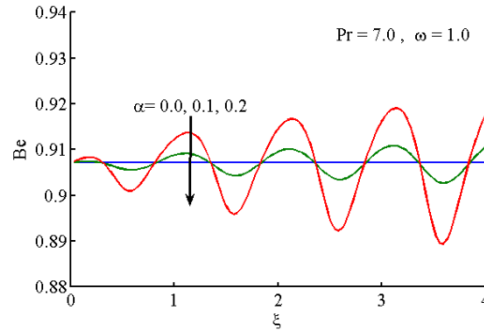


Fig. 4: Graph of the Bejan number for different value of α

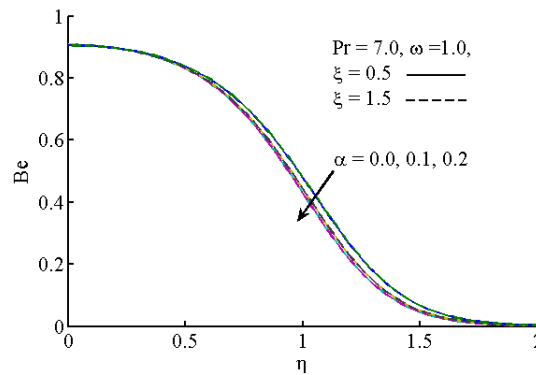


Fig. 5: Graph of the Bejan number for different α

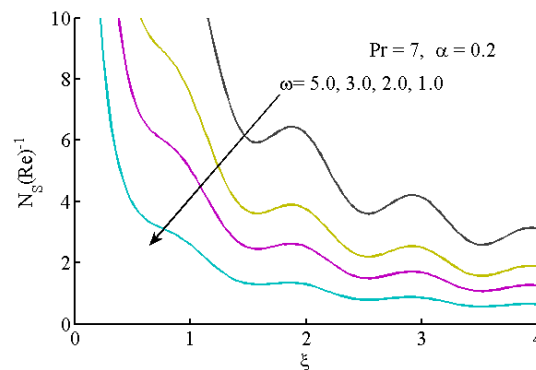


Fig. 6: Effects on N_s for different ω when $\alpha = 0.2$

Figs. 6-9 unveil the behavior of entropy generation phenomenon for different values of group parameter ω . It is revealed from Figs. 6 and 7 that entropy number rises as ω increases. Such type of result is quit expected since an increase in ω gives rise to the viscous dissipation effects which causes the production of entropy. In addition, increasing amplitude of oscillation in entropy number profiles is noticed as ω increases. This is because of the strong frictional effect of the fluid particles which produces

more fluctuations in the velocity gradient. Interesting effects of α on the Bejan number are observed in Figs. 8 and 9. Upon increasing the values of ω the Bejan number decreases significantly. For sufficiently large values of ω the dominant role of the heat transfer entropy can be reversed (Figs. 8 and 9.). This is because of the fact that the increasing values of ω give rise to the ongoing convection phenomena due to which the entropy rate due to viscous resistance increases. Consequently the Bejan number reduces upon increasing the values of ω . Growing amplitude of the Bejan number can also be confirmed from Fig. 8 at the downstream locations.

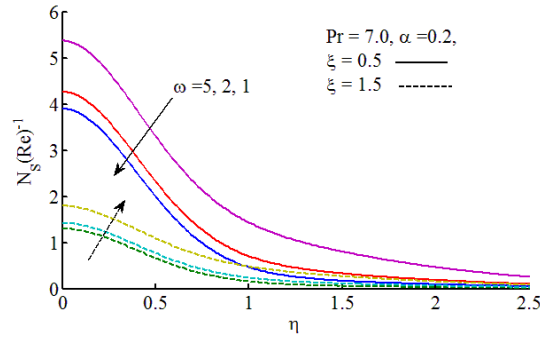


Fig. 7: Effect of varying ω on N_S against η

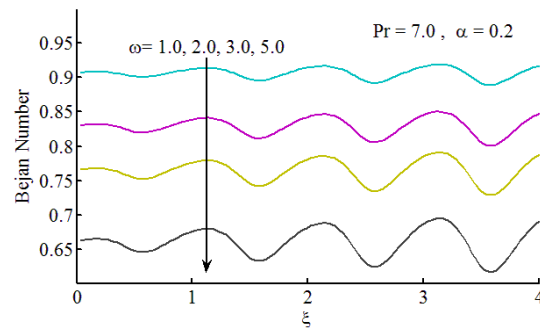


Fig. 8: Variation of the Bejan number against different ω

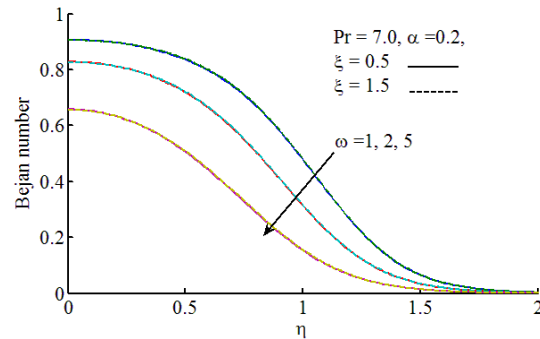


Fig. 9: Plot of Bejan number for different ω

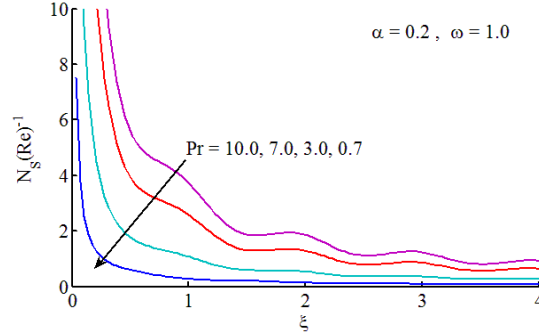


Fig. 10: N_s against ξ for different values of Pr

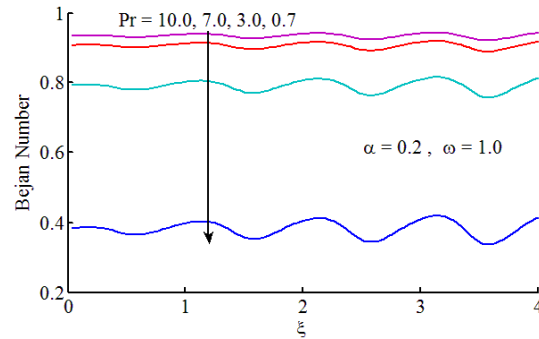


Fig. 11: Effects of Pr on the Bejan number

Fig. 10 shows the effect of Prandtl number on the total entropy generation number. It is noticed that the entropy production rises in the flow as the Prandtl number increases. This shows that the entropy production in the fluids having large Prandtl numbers is high. Fig. 11 depicts that the heat transfer irreversibility dominates in the fluids having large Pr values and fluid friction irreversibility dominates for those having small Pr such as the air.

6. Concluding Remarks

In this study, the second law of thermodynamics is investigated in a convective heat transfer phenomena over a continuous horizontally moving wavy surface. A numerical scheme is applied to calculate accurate velocity and temperature profiles. The numerical results are used to calculate the expression for the entropy generation. This study reveals that, the entropy generation is high in upstream region and low in the downstream region. In the upstream region, the contribution of heat transfer and fluid friction irreversibility is distinguishable due to less fluctuation. However in the downstream region the fluctuation in the Bejan number is high due to which dominance of heat transfer and fluid friction irreversibility varies. Interesting observations regarding the Bejan number has been noted. The Bejan number fluctuates in ξ where the amplitude of fluctuations increases at the downstream locations. Due to this behavior it seems possible to have a situation when the Bejan number fluctuates in such a manner that the dominance of heat transfer entropy and the viscous friction entropy interchanges at different ξ locations. The group parameters ω is noted to depreciate the dominance of heat transfer entropy. The

observations regarding the effects of Pr on the Bejan number are in accordance with the already observed facts available in literature.

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