EXPERIMENTAL RESEARCH OF PRESSURE DROP IN PACKED BEDS OF MONOSIZED SPHERES A NOVEL CORRELATION FOR PRESSURE DROP CALCULATION

by

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Flow through packed beds of spheres is a complex phenomenon and it has been extensively studied. Although, there is many different correlations there is still no reliable universal equation for prediction of pressure drop. The paper presents the results of experimental research of pressure drop in packed bed of monosized spheres of three different diameters, 8, 11 and 13 mm set within cylindrical vessel of diameter d_k *= 74 mm, and two different heights of packed bed* h_s *= 300 and 400 mm. It has been proposed modification of widely used Ergun's equation in the form of* $f_p = [150+1.3\cdot(Re_p/(1-\varepsilon))]\cdot(1-\varepsilon)^2/(\varepsilon^3\cdot Re_p)$ *and new correlation* $f_p = 1/[(27.4-25700\cdot d_h)/Re_p+0.545+6.85\cdot d_h]$ *for pressure drop calculation in simple and convenient form for hand and computer calculations. For total number of 362 experimental runs the correlation ratio of the modified Ergun's relation was CR = 99.3%, and standard deviation SD = 12.2%, while novel relation has CR = 93.7% and SD = 5.4%.*

Key words: *packed bed, monosized spheres, friction factor, pressure drop, bed porosity, laminar and turbulent flow.*

Introduction

Packed bed columns and reactors have wide application in process industries. Packed bed is typically used to improve contact between two phases during mass and/or heat transfer. It is usually used as a catalyst carrier in chemical reactors, as a packing in separation processes – absorption, stripping/distillation, as filter filler and as heat storage in regenerative heat exchangers. Recently, packed beds are used in porous ceramic burners for combustion of low calorific gaseous fuels [1, 6].

Porosity, specific surface of packed bed and mean pressure drop across it are the most significant for operating performance of apparatus with packed beds. The variables affecting pressure drop through packed bed can be classified into two groups: (1) variables related to the fluid – viscosity, density, velocity and (2) variables related to the bed – size, shape and orientation of particles, bed porosity, particle surface roughness and bed geometric aspect ratio (d_k / d_p).

One of key parameters to be assessed during the design is pressure drop through the packed bed. Flow through packed beds of spheres, as a very complex phenomenon, has been extensively studied, but there is still no reliable universal equation for prediction of the pressure drop [2]. Only few studies investigated packed bed pressure drop at elevated

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temperatures [3]. There is large number of correlations for calculation of pressure drop for fluid flow through the packed bed. For laminar water flow through the bed of sand Darcy observed that pressure drop through the bed is proportional to the superficial fluid velocity (w_k). On the other hand, the pressure drop for laminar fluid flow through a randomly packed bed of monosized spheres with diameter d_p can be calculated according to Carman-Kozeny equation:

$$\frac{\Delta p_k}{h_s} = 180 \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} \cdot \frac{\mu_f \cdot w_k}{d_p^2} \tag{1}$$

Still, one of the most popular and widely used is Ergun's equation [4]:

$$\frac{\Delta p_k}{h_s} = f_p \cdot \frac{\rho_f \cdot w_k^2}{d_p}, \quad \frac{\Delta p_k}{h_s} = 150 \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3} \cdot \frac{\mu_f \cdot w_k}{d_p^2} + 1.75 \cdot \frac{1-\varepsilon}{\varepsilon^3} \cdot \frac{\rho_f \cdot w_k^2}{d_p} \tag{2}$$

Two parts of equation describe viscous (laminar) and inertial (turbulent) pressure losses.

Comprehensive review of widely used correlations coverred in relevant literature, sistematized using a uniform notation for mutual comparison is presented in [5]. Table 1 shows correlations for particle friction factors tested in this paper.

Table 1. Particle friction factors

Author(s)	Relation	Range of applicability
Ergun [4]	$f_p = \left[150 + 1.75 \cdot \left(\frac{\mathrm{Re}_p}{1-\varepsilon}\right)\right] \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3 \cdot \mathrm{Re}_p}$ (3)	$0.2 < \mathrm{Re}_1 < 700$
Brauer [10]	$f_p = \left[160 + 3.1 \cdot \left(\frac{\mathrm{Re}_p}{1-\varepsilon}\right)^{0.9}\right] \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3 \cdot \mathrm{Re}_p}$ (4)	$2 < \mathrm{Re}_m < 20{,}000$

Experimental setup

Experimental study has been performed in Laboratory for process engineering, energy efficiency and environmental protection of the Faculty of Mechanical Engineering, University of Belgrade as a first part of research work on PhD Thesis on working parameters of combustion the low calorific gaseous fuels and waste industrial gases in porous ceramic burner.

Experimental setup for research of pressure drop in porous layer of Al_2O_3 spheres (tabular alumina, Almatis Iwakuni, Japan) is presented in Figure 1. We have used atmospheric air as a working fluid. There were three dimensions of Al_2O_3 spheres: d_p = {8, 11, 13} mm. The main elements of the experimental setup are:

1 Glass column with internal diameter d_k = 74 mm,

2, 3 Porous partition wall with filter cloth

4 Anemometer

6 Porous bed of Al_2O_3 spheres (h_s is layer height),

7, 8 Valves

9 Atmospheric pipeline
10 Blower
11 Thermometer (t_v).

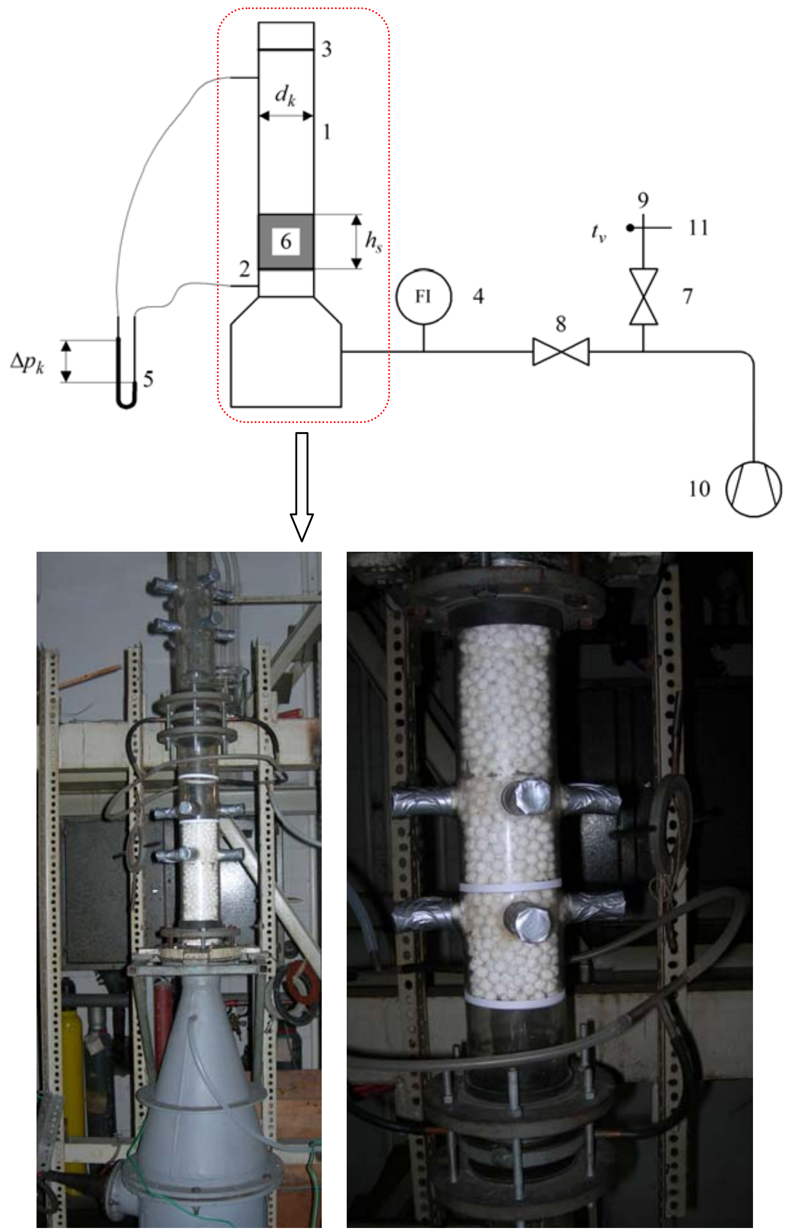

Figure 1. Experimental setup

Glass column (1) was filled with porous layer (6) with heights of h_s = 300 mm and h_s = 400 mm. Air flow rate was provided by a blower (10), and valves (7) and (8) were used for flow rate regulation.

Air flow was measured using (4) as well as its temperature and pressure for correction.

First measurements were done with empty glass column in order to establish correlation

$$\Delta p_k = a \cdot w_k^b \tag{5}$$

thus, taking into account all of the friction losses and minor pressure losses due to contractions, enlargements, swirl flows [11], etc.

After measuring pressure drop on the column filled with Al_2O_3 spheres, pressure drop due to porous layer was calculated using

$$\Delta p = \Delta p_{uk} - \Delta p_k \tag{6}$$

where:

Δp,	Pa,	pressure drop through the layer of Al_2O_3 spheres,
Δp_{uk},	Pa,	total pressure drop,
Δp_k,	Pa,	pressure drop through empty glass column.

Results and discussion

Statistical analysis of the results of measurements provided us the following equation for pressure drop of an empty column

$$\Delta p_k = 252 \cdot w_k^{1.25} \tag{7}$$

with the following statistical parameters: CR = 99.8% and SD = 3.5% (Figure 2).

There were 362 working regimes gathered as original measurements on experimental setup and the range of working conditions were: t_v = 17.9-28.4°C, w_k = 0.47-3.83 m/s. Porosity of packed bed of monosized spheres was in the range 0.42 - 0.45. Re_p was in the range 218 - 3188. Raw results are presented in Figure 3.

As stated before Ergun [4] was the first one who made the analysis of gathered laminar and turbulent flow of fluid through the porous layer. His model was analogously implemented in many cases of two phase flow, like flow of fluid through the packed distillation or absorption columns, adsorption columns with granular bed of activated carbon or other adsorbents, two phase flow in froth in trayed distillation or absorption column [7], etc. Approach to a single phase flow pressure drop calculations analogous to Erguns show valid results even in heat exchangers [8, 9].

The comparison of experimental (z_i) and correlated ($z_{c,i}$) data can be done by the statistical parameters like: maximal positive error (9), maximal negative error (10) and correlation ratio (11).

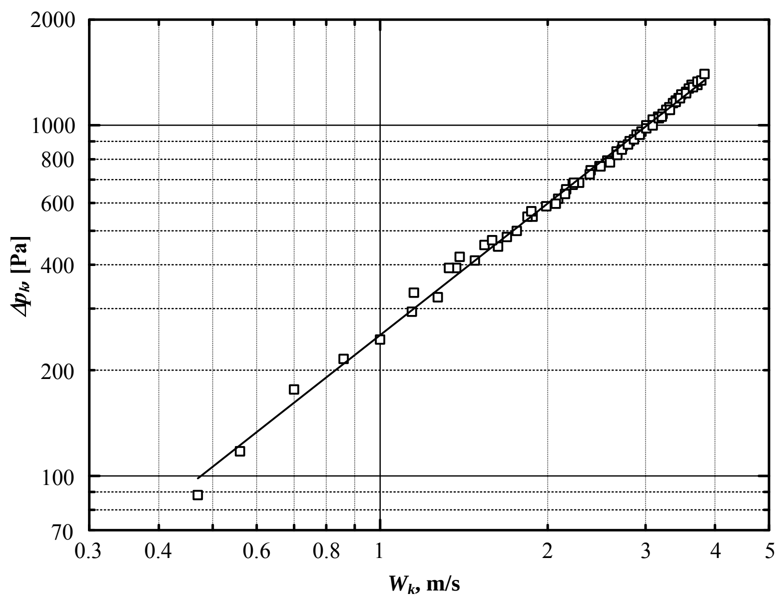

Figure 2. Pressure drop vs air velocity for empty column

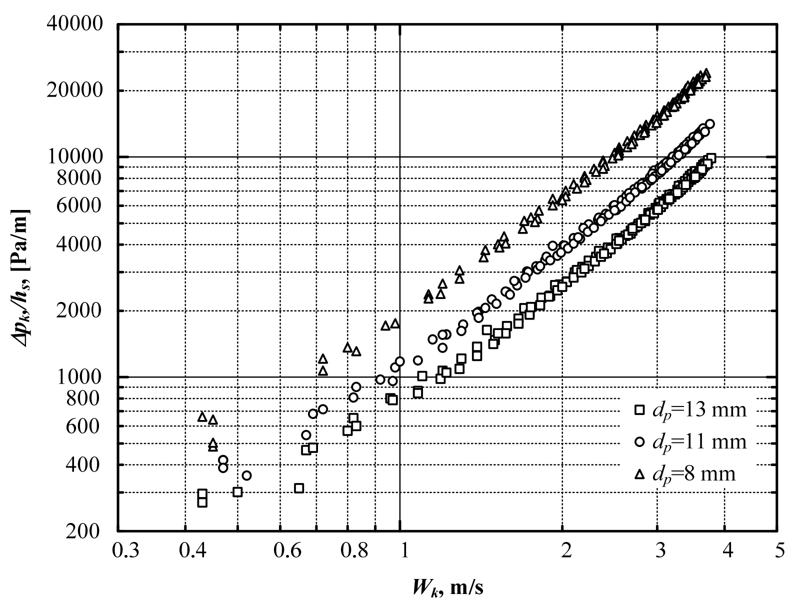

Figure 3. Pressure drop per unit height vs air velocity – original measurements

Maximal positive error

$$maxRE^{+} = \max\left(\frac{z_i - z_{c,i}}{z_i}\right) \tag{9}$$

Maximal negative error

$$maxRE^{-} = \max\left(\frac{z_{c,i} - z_i}{z_i}\right) \tag{10}$$

Correlation ratio

$$CR = \sqrt{1 - \frac{\sum_{i=1}^{n}\left(z_i - z_{c,i}\right)^2}{\sum_{i=1}^{n}\left(z_i - z_{av}\right)^2}} \tag{11}$$

where z_{av} is the average value for complete set of *n* experimental runs

$$z_{av} = \frac{\sum_{i=1}^{n} z_i}{n} \tag{12}$$

We have checked Ergun's equation (3) first, and we have obtained the following statistical parameters: *SD* = 36.3%, *maxRE*$^+$ = +31.4%, *maxRE*$^-$ = –59.5% and *CR* = 85.6%. High correlation ratio encouraged us to modify his equation to a following one

$$f_p = \left[150 + 1.3 \cdot \left(\frac{\mathrm{Re}_p}{1-\varepsilon}\right)\right] \cdot \frac{(1-\varepsilon)^2}{\varepsilon^3 \cdot \mathrm{Re}_p} \tag{13}$$

Correlation (13) shows significantly better statistics: *SD* = 12.2%, *maxRE*$^+$ = +47.0%, *maxRE*$^-$ = –19.4% and *CR* = 99.3%.

Next one was the Brauer's correlation (4) that has quite good statistics: *SD* = 12.9%, *maxRE*$^+$ = +35.4%, *maxRE*$^-$ = –31.1% and *CR* = 99.3%.

It can be concluded that Brauer's correlation (4) shows similar statistical parameters in comparison with modified Ergun's correlation (13).

Our idea was to transform Ergun's correlation to a significantly different form that can cover the experimental databank with greater certainty. After statistical analysis we came to a final correlation in the form:

$$f_p = \frac{1}{\dfrac{27.4 - 25700 \cdot d_h}{\mathrm{Re}_p} + 0.545 + 6.85 \cdot d_h} \tag{14}$$

accompanied with the following statistical parameters: *SD* = 5.4%, *maxRE*$^+$ = +16.2%, *maxRE*$^-$ = –32.7% and *CR* = 93.7%.

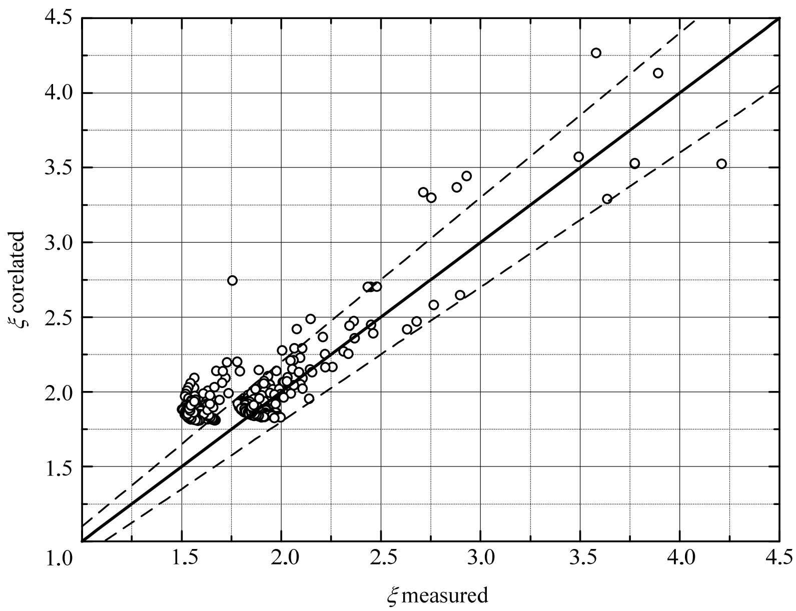

Figure 4. Original correlation

The form of correlation (14) is pretty simple and convenient for both hand and computer calculations, although it has to be said that more complex mathematical models can be applied [12]. Correlation (14) is shown in Figure 4, along with a ±10% correlation field.

Like some other models [13], hereby presented results are suitable for application in automated control systems for burners for low-calorific gaseous fuels.

Conclusions

Packed beds have wide application in variety of industrial systems. Pressure drop is considered as one of the most important parameters when it comes to the design of process equipment with packed beds. There are a large number of correlations for calculation of pressure drop for fluid flow through the packed bed. Still, there is no reliable universal equation for prediction of the pressure drop within packed beds of spheres. One of the most popular and widely used is Ergun's equation.

The correlation of Ergun (3) was found to provide the following statistical parameters: *CR* = 85.6% and *SD* = 36.3%, but simply modified Ergun's equation (13) showed significantly better statistics: *CR* = 99.3% and *SD* = 12.2%. Brauer's equation (4) was the subject of analysis and the statistical parameters are very similar to the previous correlation (13): *CR* = 99.3% and *SD* = 12.9%. Finally, significantly different novel correlation hereby proposed (14) covers the experimental databank with greater certainty expressed through *CR* = 93.7% and *SD* = 5.4%.

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Nomenclature

a – parameter, [-]
b – parameter, [-]
d_h – hydraulic diameter, [m]
d_k – column diameter, [m]
d_p – sphere diameter, [m]
f_p – friction factor, [-]
h_s – porous layer height, [m]
Δp – pressure drop through the layer of Al_2O_3 spheres, [Pa]
Δp_k – pressure drop through empty glass column, [Pa]
Δp_{uk} – total pressure drop, [Pa]
t_v – temperature of air, [°C]
Re_p – Reynolds number $\mathrm{Re}_p=(w_k \cdot d_p \cdot \rho_f)/\mu_f$, [-]
Re_m – modified Reynolds number $\mathrm{Re}_m=\mathrm{Re}_p/(1-\varepsilon)$, [–]
Re_1 – modified Reynolds number $\mathrm{Re}_1=\mathrm{Re}_p/6(1-\varepsilon)$, [–]

Greek symbols

ε – porosity, [-]
μ – viscosity, [Pa·s]
ρ – density, [kg·m^{-3}]

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