EXPERIMENTAL RESEARCH OF PRESSURE DROP IN PACKED BEDS OF MONOSIZED SPHERES A NOVEL CORRELATION FOR PRESSURE DROP CALCULATION

by

Mirjana S. STAMENIĆ*

Department of Process Engineering, Faculty of Mechanical Engineering, University of Belgrade, Belgrade, Serbia

Original scientific paper
https://doi.org/10.2298/TSCI161025327S

Flow through packed beds of spheres is a complex phenomenon and it has been extensively studied. Although, there is many different correlations there is still no reliable universal equation for prediction of pressure drop. The paper presents the results of experimental research of pressure drop in packed bed of monosized spheres of three different diameters, 8, 11, and 13 mm set within cylindrical vessel of diameter d_k = 74 mm, and two different heights of packed bed, h_s = 300 and 400 mm. It has been proposed modification of widely used Ergun's equation in the form of $f_p = [150 + 1.3(Re_p/(1 - \varepsilon))](1 - \varepsilon)^2/(\varepsilon^3 Re_p)$ and new correlation $f_p = 1/[(27.4 - 25700d_h)/Re_p + 0.545 + 6.85d_h]$ for pressure drop calculation in simple and convenient form for hand and computer calculations. For total number of 362 experimental runs the correlation ratio of the modified Ergun's relation was CR = 99.3%, and standard deviation SD = 12.2%, while novel relation has CR = 93.7% and SD = 5.4%.

Key words: *packed bed, monosized spheres, friction factor, pressure drop, bed porosity, laminar and turbulent flow*

Introduction

Packed bed columns and reactors have wide application in process industries. Packed bed is typically used to improve contact between two phases during mass and/or heat transfer. It is usually used as a catalyst carrier in chemical reactors, as a packing in separation processes – absorption, stripping/distillation, as filter filler and as heat storage in regenerative heat exchangers. Recently, packed beds are used in porous ceramic burners for combustion of low calorific gaseous fuels [1, 2].

Porosity, specific surface of packed bed and mean pressure drop across it are the most significant for operating performance of apparatus with packed beds. The variables affecting pressure drop through packed bed can be classified into two groups: (1) variables related to the fluid – viscosity, density, velocity and (2) variables related to the bed – size, shape and orientation of particles, bed porosity, particle surface roughness, and bed geometric aspect ratio, d_k/d_p.

* Author's e-mail: mstamenic@mas.bg.ac.rs

One of key parameters to be assessed during the design is pressure drop through the packed bed. Flow through packed beds of spheres, as a very complex phenomenon, has been extensively studied, but there is still no reliable universal equation for prediction of the pressure drop [3]. Only few studies investigated packed bed pressure drop at elevated temperatures [4]. There is large number of correlations for calculation of pressure drop for fluid flow through the packed bed. For laminar water flow through the bed of sand Darcy observed that pressure drop through the bed is proportional to the superficial fluid velocity, w_k. On the other hand, the pressure drop for laminar fluid flow through a randomly packed bed of monosized spheres with diameter d_p can be calculated according to Carman-Kozeny equation:

$$\frac{\Delta p_k}{h_s} = 180 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu_f w_k}{d_p^2} \tag{1}$$

Still, one of the most popular and widely used is Ergun's equation [5]:

$$\frac{\Delta p_k}{h_s} = f_p \frac{\rho_f w_k^2}{d_p}, \quad \frac{\Delta p_k}{h_s} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu_f w_k}{d_p^2} + 1.75 \frac{1-\varepsilon}{\varepsilon^3} \frac{\rho_f w_k^2}{d_p} \tag{2}$$

Two parts of equation describe viscous (laminar) and inertial (turbulent) pressure losses.

Comprehensive review of widely used correlations coverred in relevant literature, sistematized using a uniform notation for mutual comparison is presented in [6]. Table 1 shows correlations for particle friction factors tested in this paper.

Table 1. Particle friction factors

Author(s)	Relation	Range of applicability
Ergun [5]	$f_p = \left[150 + 1.75 \left(\frac{\mathrm{Re}_p}{1-\varepsilon}\right)\right] \frac{(1-\varepsilon)^2}{\varepsilon^3 \mathrm{Re}_p}$ (3)	$0.2 < \mathrm{Re}_1 < 700$
Brauer [7]	$f_p = \left[160 + 3.1 \left(\frac{\mathrm{Re}_p}{1-\varepsilon}\right)^{0.9}\right] \frac{(1-\varepsilon)^2}{\varepsilon^3 \mathrm{Re}_p}$ (4)	$2 < \mathrm{Re}_m < 20{,}000$

Experimental set-up

Experimental study has been performed in Laboratory for process engineering, energy efficiency and environmental protection of the Faculty of Mechanical Engineering, University of Belgrade as a first part of research work on Ph. D. thesis on working parameters of combustion the low calorific gaseous fuels and waste industrial gases in porous ceramic burner [1].

Experimental set-up for research of pressure drop in porous layer of Al_2O_3 spheres (tabular alumina, Almatis Iwakuni, Japan) is presented in fig. 1. We have used atmospheric air as a working fluid. There were three dimensions of Al_2O_3 spheres: d_p = 8, 11, and 13 mm.

Glass column (1) was filled with porous layer (6) with heights of h_s = 300 mm and h_s = 400 mm. Air flow rate was provided by a blower (10), and valves (7) and (8) were used for flow rate regulation.

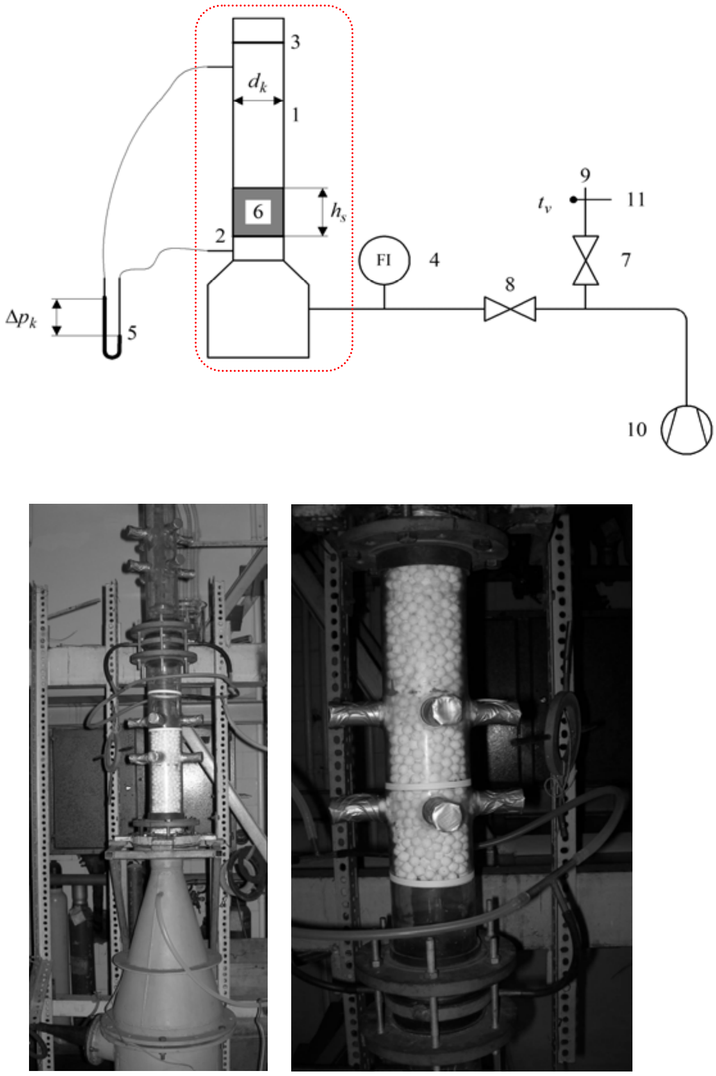

Figure 1. Experimental set-up

1 – glass column with internal diameter, d_k = 74 mm,
2, 3 – porous partition wall with filter cloth,
4 – anemometer,
5 – differential manometer (U tube)
6 – porous bed of Al_2O_3 spheres (h_s is layer height),
7, 8 – valves,
9 – atmospheric pipeline,
10 – blower,
11 – thermometer (t_v).

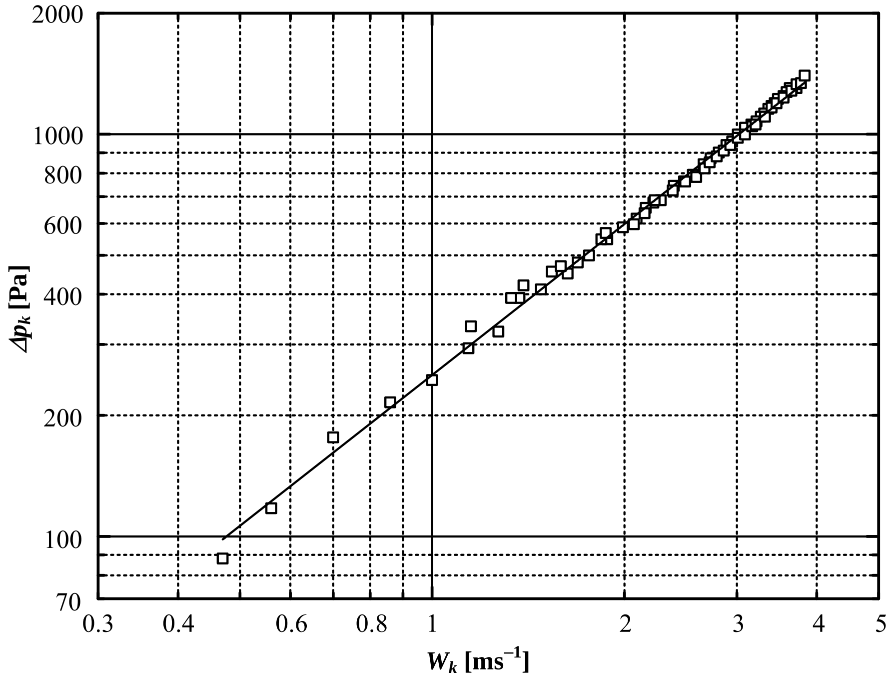

Figure 2. Pressure drop vs air velocity for empty column

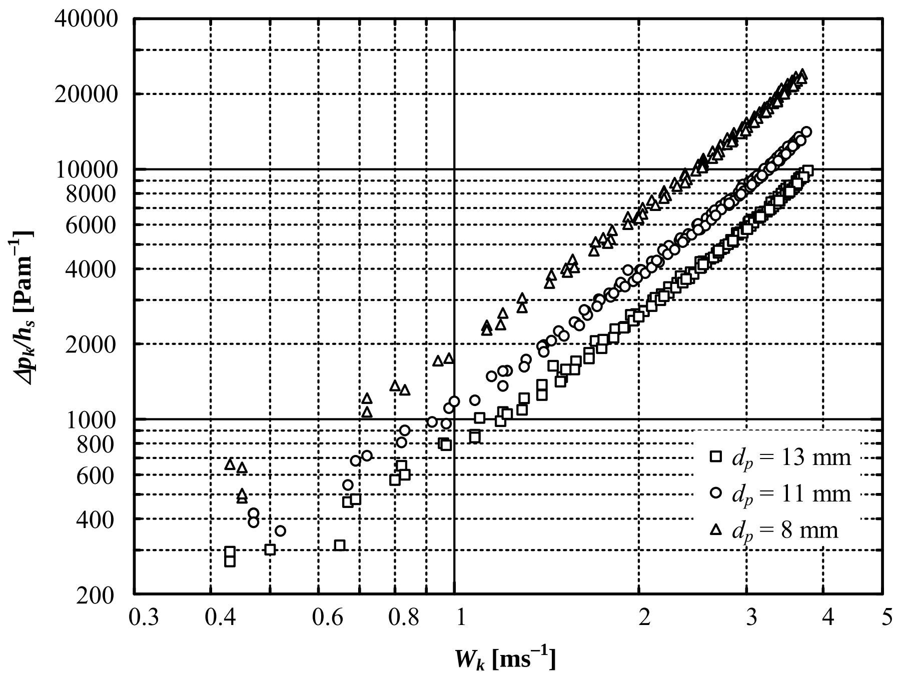

Figure 3. Pressure drop per unit height vs air velocity – original measurements

Air flow was measured using anemometer (4) as well as its temperature and pressure for correction.

First measurements were done with empty glass column in order to establish correlation:

$$\Delta p_k = a w_k^b \tag{5}$$

thus, taking into account all of the friction losses and minor pressure losses due to contractions, enlargements, swirl flows, etc. [8].

After measuring pressure drop on the column filled with Al_2O_3 spheres, pressure drop due to porous layer was calculated using:

$$\Delta p = \Delta p_{uk} - \Delta p_k \tag{6}$$

where Δp [Pa] is the pressure drop through the layer of Al_2O_3 spheres, Δp_{uk} [Pa] – the total pressure drop, and Δp_k [Pa] – the pressure drop through empty glass column.

Results and discussion

Statistical analysis of the results of measurements provided us the following equation for pressure drop of an empty column:

$$\Delta p_k = 252 w_k^{1.25} \tag{7}$$

with the following statistical parameters: CR = 99.8% and SD = 3.5% (fig. 2).

There were 362 working regimes gathered as original measurements on experimental set-up and the range of working conditions were:t_v = 17.9-28.4 °C, w_k = 0.47-3.83 m/s. Porosity of packed bed of monosized spheres was in the range 0.42-0.45. Re_p was in the range 218-3188. Raw results are presented in fig. 3.

As stated before Ergun [5] was the first one who made the analysis of gathered laminar and turbulent flow of fluid through the porous layer. His model was analogously implemented in many cases of two phase flow, like flow of fluid through the packed distillation or absorption columns, adsorption columns with granular bed of activated carbon or other adsorbents, two phase flow in froth in trayed distillation or absorption column [9], etc. Approach to a single phase flow pressure drop calculations analogous to Erguns show valid results even in heat exchangers [10, 11].

The comparison of experimental (z_i) and correlated ($z_{c,i}$) data can be done by the statistical parameters like: maximal positive error (8), maximal negative error (9) and correlation ratio (10).

Maximal positive error:

$$maxRE^+ = \max\left(\frac{z_i - z_{c,i}}{z_i}\right) \tag{8}$$

Maximal negative error:

$$maxRE^- = \max\left(\frac{z_{c,i} - z_i}{z_i}\right) \tag{9}$$

Correlation ratio:

$$CR = \sqrt{1 - \frac{\sum_{i=1}^{n} \left(z_i - z_{c,i}\right)^2}{\sum_{i=1}^{n} \left(z_i - z_{av}\right)^2}} \tag{10}$$

where z_{av} is the average value for complete set of n experimental runs:

$$z_{av} = \frac{\sum_{i=1}^{n} z_i}{n} \tag{11}$$

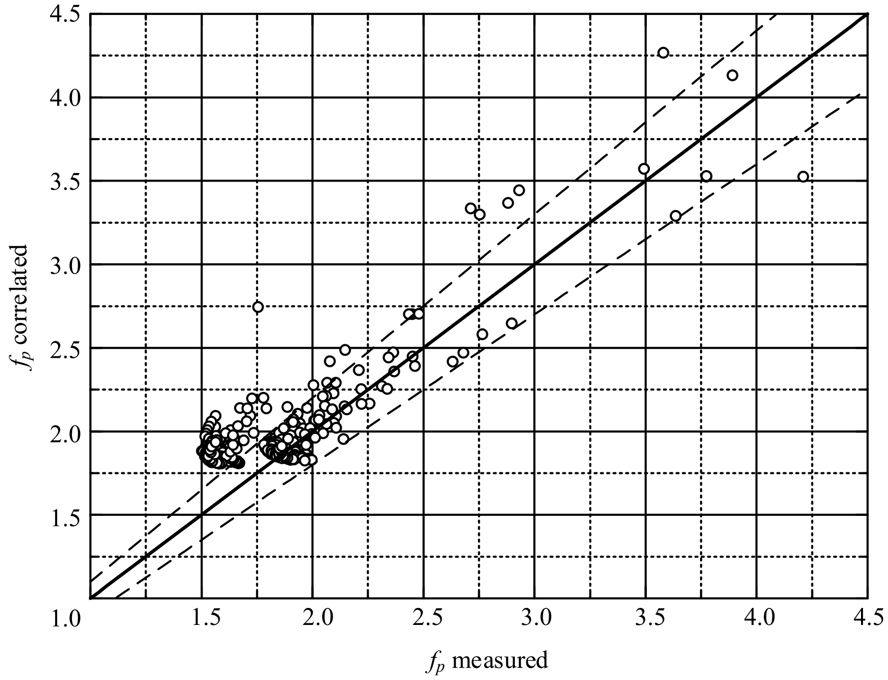

Figure 4. Correlation field according to equation (13)

We have checked Ergun's eq. (3) first, and we have obtained the following statistical parameters: SD = 36.3%, $maxRE^{+}$ = +31.4%, $maxRE^{-}$ = –59.5% and CR = 85.6%. High correlation ratio encouraged us to modify his equation to a following one:

$$f_p = \left[150 + 1.3\left(\frac{\mathrm{Re}_p}{1-\varepsilon}\right)\right]\frac{(1-\varepsilon)^2}{\varepsilon^3 \,\mathrm{Re}_p} \tag{12}$$

Correlation (12) shows significantly better statistics: SD = 12.2%, $maxRE^{+}$ = +47.0%, $maxRE^{-}$ = –19.4% and CR = 99.3%.

Next one was the Brauer's correlation (4) that has quite good statistics: SD = 12.9%, $maxRE^+$ = +35.4%, $maxRE^-$ = –31.1% and CR = 99.3%.

It can be concluded that Brauer's correlation (4) shows similar statistical parameters in comparison with modified Ergun's correlation (12).

Our idea was to transform Ergun's correlation to a significantly different form that can cover the experimental databank with greater certainty. After statistical analysis we came to a final correlation in the form:

$$f_p = \frac{1}{\dfrac{27.4 - 25700 d_h}{\mathrm{Re}_p} + 0.545 + 6.85 d_h} \tag{13}$$

accompanied with the following statistical parameters: SD = 5.4%, $maxRE^+$ = +16.2%, $maxRE^-$ = –32.7% and CR = 93.7%.

The form of correlation (13) is pretty simple and convenient for both hand and computer calculations, although it has to be said that more complex mathematical models can be applied [12]. Correlation (13) is shown in fig. 4, along with ±10% correlation field.

Like some other models [13], hereby presented results are suitable for application in automated control systems for burners for low-calorific gaseous fuels.

Conclusions

Packed beds have wide application in variety of industrial systems. Pressure drop is considered as one of the most important parameters when it comes to the design of process equipment with packed beds. There are a large number of correlations for calculation of pressure drop for fluid flow through the packed bed. Still, there is no reliable universal equation for prediction of the pressure drop within packed beds of spheres. One of the most popular and widely used is Ergun's equation.

The correlation of Ergun (3) was found to provide the following statistical parameters: CR = 85.6% and SD = 36.3%, but simply modified Ergun's equation (12) showed significantly better statistics: CR = 99.3% and SD = 12.2%. Brauer's equation (4) was the subject of analysis and the statistical parameters are very similar to the previous correlation eq. (12): CR = 99.3% and SD = 12.9%. Finally, significantly different novel correlation hereby proposed eq. (13) covers the experimental databank with greater certainty expressed through CR = 93.7% and SD = 5.4%.

Acknowledgment

The research work presented in this paper was funded by Ministry of Education, Science and Technological Development of Republic of Serbia through Technological Development Project No. 33049.

Nomenclature

- a – parameter, [–]
- b – parameter, [–]
- d_h – hydraulic diameter, [m]
- d_k – column diameter, [m]
- d_p – sphere diameter, [m]
- f_p – friction factor, [–]
- h_s – porous layer height, [m]
- Δp – pressure drop through the layer of Al_2O_3 spheres, [Pa]
- Δp_k – pressure drop through empty glass column, [Pa]
- Δp_{uk} – total pressure drop, [Pa]

Re_p – Reynolds number [= $(w_k d_p \rho_f)/\mu_f$], [–]
Re_m – modified Reynolds number [= $\mathrm{Re_p}/(1-\varepsilon)$], [–]
Re_I – modified Reynolds number [= $\mathrm{Re_p}/6(1-\varepsilon)$], [–]
t_v – temperature of air, [°C]

Greek symbols

ε – porosity, [–]
μ – viscosity, [Pa·s]
ρ – density, [$\mathrm{kgm^{-3}}$]

References

[1] Stamenić, M. S., Research on Working Parameters of Combustion the Low Calorific Gaseous Fuels and Waste Industrial Gases in Porous Ceramic Burner (in Serbian), Ph. D. thesis, University of Belgrade, Belgrade, 2014
[2] Stamenić, M. S., *et al*., Results of Experimental Research on Parameters that Determine Stable Operating Limits of Ceramic Burner with Packed Bed of Uniform Spheres for Combustion of Low Calorific Gaseous Fuels, *Proceedings*, 3rd International Symposium on Environmental Friendly Energies and Applications, EFEA 2014, Paris, 2014
[3] Montillet, A., *et al.*, About a Correlating Equation for Predicting Pressure Drops through Packed Beds of Spheres in a Large Range of Reynolds Numbers, *Chem. Eng. Process.*, *46* (2007), pp. 329-333
[4] Pešić, R., *et al*., Pressure Drop in Packed Bed of Spherical Particles at Ambient and Elevated Air Temperatures, *Chem. Ind. Chem. Eng. Q.*, *21* (2015), 3, pp. 419-427
[5] Ergun, S., Fluid Flow through Packed Columns, *Chem. Eng. Process.*, *48* (1952), 2, pp. 89-94
[6] Erdim, E., *et al.*, A Revisit of Pressure Drop-Flow Rate Correlations for Packed Beds of Spheres, *Powder Technology*, *283* (2015), pp. 488-504
[7] Stephan, P., *et al.*, *VDI Heat Atlas*, Spriger-Verlag, Berlin, Heidelberg, Germany, 2010
[8] Lelea, D., The Microtube Heat Sink with Tangential Impingement Jet and Variable Fluid Properties, *Heat and Mass Transfer*, *45* (2009), 9, pp. 1215-1222
[9] Jaćimović, B., Genić, S., Froth Porosity and Clear Liquid Height in Trayed Columns, *Chemical Engineering and Technology*, *23* (2000), 2, pp. 171-176
[10] Genić, S., *et al.*, Research on Air Pressure Drop in Helically-Finned Tube Heat Exchangers, *Applied Thermal Engineering*, *26* (2006), 5-6, pp. 478-485
[11] Genić, S., *et al.*, Experimental Research of Highly Viscous Fluid Cooling in Cross-Flow to a Tube Bundle, *International Journal of Heat and Mass Transfer*, *50* (2007), 7-8, pp. 1288-1294
[12] Mitrović, Z., Arandjelović, I., Existence of Generalized Best Approximations, *Journal of Nonlinear and Convex Analysis*, *15* (2014), 4, pp. 787–792
[13] Salemović, D., *et al.*, A Mathematical Model and Simulation of the Drying Process of Thin Layers of Potatoes in a Conveyor-Belt Dryer, *Thermal Science*, *19* (2015), 3, pp. 1107-1118

Paper submitted: October 25, 2016
Paper revised: November 25, 2016
Paper accepted: December 19, 2016