

**THERMODYNAMIC ANALYSIS OF VISCOELASTIC FLUID IN A POROUS MEDIUM
WITH PRESCRIBED WALL HEAT FLUX OVER STRETCHING SHEET SUBJECTED TO
A TRANSITIVE MAGNETIC FIELD**

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An analysis is performed for entropy generation in a steady laminar boundary layer flow of an electrically conducting second grade fluid in a porous medium prescribed wall heat flux (PHF) subject to a transverse uniform magnetic field past a semi-infinite stretching sheet, The effects of viscous dissipation, internal heat generation of absorption due to deformation are considered in the energy equation. Kummer's functions are used to obtain temperature field. The velocity, temperature are used to compute the entropy generation number with a change in various dimensionless parameters.

Key words: Entropy generation; PHF case; Magnetic field; Porous medium; Second grade fluid; stretching sheet.

Introduction

The design of thermal systems can be achieved by optimization of entropy generation in the systems. Due to number of applications in industrial manufacturing process, the problem of boundary layer flow past a stretching plate has attracted considerable attention of researchers during the past few decades for examples heat exchangers, cooling of nuclear reactors, MHD power generators, geophysical fluid dynamics, energy storage systems and cooling of electronic devices, etc. A series of studies on heat transfer effects on viscoelastic fluid have been made by many authors under different physical situations including [1-8] have derived similarity solution of viscoelastic boundary layer flow and heat transfer over an exponential stretching surface. Cortell [9] have studied flow and heat transfer of a viscoelastic fluid over stretching surface considering both constant sheet temperature and prescribed sheet temperature. Abel et al. [10] carried out a study of viscoelastic boundary layer flow and heat transfer over a stretching surface in the presence of non-uniform heat source and viscous dissipation considering prescribed surface temperature and prescribed surface heat flux. I-Chung Liu [11] analysis the flow and the heat transfer of a steady laminar boundary layer flow of an electrically conducting fluid of second grade in a porous medium subject to a transverse uniform magnetic field past a semi-infinite stretching sheet with power-law surface temperature or power-law surface heat flux. Khan [12] studied the case of the boundary layer problem on heat transfer in a viscoelastic boundary layer fluid flow over a non-isothermal porous sheet, taking into account the effect a continuous suction/blowing of the fluid, through the porous boundary. The effects of a transverse magnetic field and electric field on momentum and heat transfer characteristics in viscoelastic fluid over a stretching sheet taking into account viscous dissipation and ohmic dissipation is presented by Abel et al. [13]. Hsiao [14] studied the conjugate heat transfer of mixed

convection in the presence of radiative viscous dissipation in viscoelastic fluid past a stretching sheet. The case of unsteady magnetohydrodynamic was carried out by Abbas et al.[15].Using Kummer's functions, Singh [16] carried out the study of heat source and radiation effects on magneto hydrodynamics flow of a viscoelastic fluid past a stretching sheet with prescribed power law surface heat flux. The effects of non-uniform heat source, viscous dissipation and thermal radiation on the flow and heat transfer in a viscoelastic fluid over a stretching surface was considered in Prasad et al.[17]. The case of the heat transfer in magneto hydrodynamics flow of viscoelastic fluids over stretching sheet in the case of variable thermal conductivity and in the presence of non-uniform heat source and radiation is reported in Abel &Mahesha [18]. Using the homotopy analysis, Hayat et al. [19] looked at the hydrodynamic of three dimensional flow of viscoelastic fluid over a stretching surface. The investigation of biomagnetic flow of a non-Newtonian viscoelastic fluid over a stretching sheet under the influence of an applied magnetic field is done by Misra & Shit [20]. Subhas et al.[21] analysed the momentum and heat transfer characteristics in a hydromagnetic flow of viscoelastic liquid over a stretching sheet with non-uniform heat source. Nandeppanavar et al. [22] analysed the flow and heat transfer characteristics in a viscoelastic fluid flow in porous medium over a stretching surface with surface prescribed temperature and surface prescribed heat flux and including the effects of viscous dissipation. Chen [23] studied the magneto-hydrodynamic flow and heat transfer characteristics viscoelastic fluid past a stretching surface, taking into account the effects of Joule and viscous dissipation, internal heat generation/absorption, work done due to deformation and thermal radiation. Nandeppanavar et al .[24] considered the heat transfer in viscoelastic boundary layer flow over a stretching sheet with thermal radiation and non-uniform heat source/sink in the presence of a magnetic field. Cortell [25] also reported the flow and heat transfer of a fluid through porous medium over a stretching surface with internal heat generation. Free convective heat and mass transfer for MHD fluid flow over a permeable vertical stretching sheet in the presence of the radiation and buoyancy effects has been investigated by Rashidi et al. [26, 27]. Although the forgoing research works have covered a wide range of problems involving the flow and heat transfer of viscoelastic fluid over stretching surface they have been restricted, from thermodynamic point of view, to only the first law analysis. The contemporary trend in the field of heat transfer and thermal design is the second law of thermodynamics analysis and its related concept of entropy generation minimization. Entropy generation is closely associated with thermodynamic irreversibility, which is encountered in all heat transfer processes. Different sources are responsible for generation of entropy such as heat transfer and viscous dissipation Bejan [28.29] The analysis of entropy generation rate in a circular duct with imposed heat flux at the wall and its extension to determine the optimum Reynolds number as function of the Prandtl number and the duty parameter were presented by Bejan [30] . Sahin [31] introduced the second law analysis to a viscous fluid in circular duct with isothermal boundary conditions. In another paper, Sahin [32] presented the effect of variable viscosity on entropy generation rate for heated circular duct. A comparative study of entropy generation rate inside duct of different shapes and the determination of the optimum duct shape subjected to isothermal boundary condition were done by Sahin [33]. Narusawa [34] gave an analytical and numerical analysis of the second law for flow and heat transfer inside a rectangular duct. In a more recent paper, Mahmud & Fraser [35.36.37] applied the second law analysis to fundamental convective heat transfer problems and to non-Newtonian fluid flow through channel made of

two parallel plates. The study of entropy generation in a falling liquid film along an inclined heated plate was carried out by Saouli & Aïboud-Saouli [48]. The effects of magnetic field and viscous dissipation on entropy generation in a falling film and channel were studied by Aïboud-Saouli et al. [39,40]. The application of the second law analysis of thermodynamics to viscoelastic magneto hydrodynamic flow over a stretching surface was carried out by Aïboud & Saouli [41,42]. Irreversibility analysis in a couple stress film flow along an inclined heated plate with adiabatic free surface has been studied by Adesanya and Makinde [43]. Entropy generation and energy conversion rate for the peristaltic flow in a tube with magnetic field has also been investigated by Akbar [44]. Makinde [45] has investigated entropy analysis for MHD boundary layer flow and heat transfer over a flat plate with a convective surface boundary condition. Entropy analysis for an unsteady MHD flow past a stretching permeable surface in nano-fluid has been studied by Abolbashari et al. [46]. Chemical reaction effect on MHD free convective surface over a moving vertical plane through porous medium has been studied by Tripathy et al. [47].

2. Formulation of the problem

In two-dimensional Cartesian coordinate system x, y we consider magneto-convection, steady, laminar, electrically conductor, boundary layer flow of a second grade fluid caused by a stretching surface through porous medium to prescribed wall heat flux in the presence of a uniform transverse magnetic field. As shown in Fig.1. the x -axis is taken in the direction of the main flow along the plate and the y -axis is normal to the plate with velocity components u, v in these directions under the usual boundary layer approximations, the governing equations are [11]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{\alpha_1}{\rho} \left[\frac{\partial}{\partial x} \left(u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2} + v \frac{\partial^3 u}{\partial y^3} \right] - \frac{\sigma B_0^2}{\rho} u - \frac{v}{k_1} u \quad (2)$$

The boundary conditions are given by:

$$y = 0, \quad B > 0, \quad u = Bx, \quad v = 0 \quad (3.a)$$

$$y \rightarrow \infty, \quad u \rightarrow 0, \quad \frac{\partial u}{\partial y} \rightarrow 0 \quad (3.b)$$

The energy equation [11], corresponding to the boundary layer analysis, with viscous dissipation, done by deformation and internal heat generation or absorption is given by:

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 + \alpha_1 \frac{\partial u}{\partial y} \frac{\partial}{\partial y} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + Q(T - T_\infty) \quad (4)$$

The relevant boundary conditions are:

$$\frac{\partial T}{\partial y} = qw = D \left(\frac{x}{l} \right)^2 ; y = 0 \quad (5.a)$$

$$T \rightarrow T_\infty ; \text{ as } y \rightarrow \infty \quad (5.b)$$

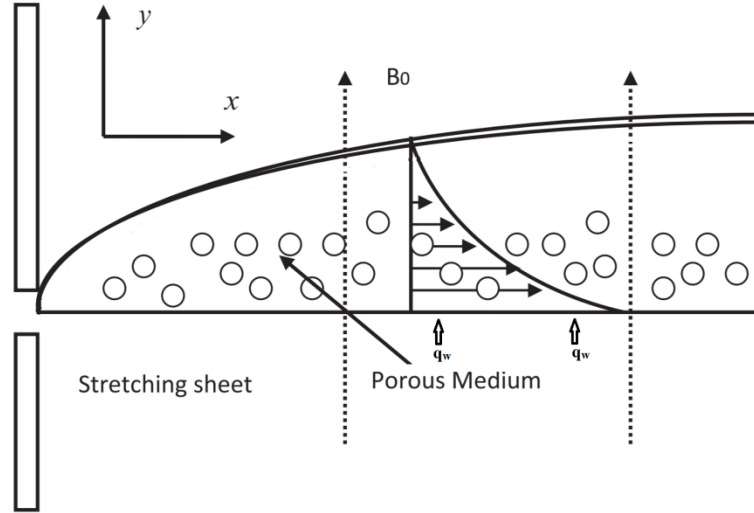


Fig.1. Physical model for the flow.

1. Analytical solution

A similarity solution exists if we introduce a transformation:

$$u = Bx f'(\eta) , \quad v = -(B/v)^{1/2} f(\eta), \quad \eta = (B/v)^{1/2} y, \quad (6)$$

Where a prime denotes the differentiation with respect to η . Apparently eq. (8) has already satisfied the continuity equation. From eq. (2) and eq. (10), we have:

$$f'^2 - f f'' = f''' + K(2f' f''' - f''^2 - f f^{IV}) - (K0 + Mn) f' \quad (7)$$

Now let us seek a solution of eq. 7:

$$f(\eta) = \frac{1}{m}(1 - e^{-m\eta}) \quad (8)$$

Where $m = \sqrt{(1 + Mn + k0)/(1 + K1)}$ is the parameter associated with the viscoelasticity of the second grade fluid, permeability of the porous medium and the contribution of the magnetic field.

This is satisfied by the following boundary conditions

$$\eta = 0 , f = 0 \text{ et } f' = 1 \quad (9.a)$$

$$\eta \rightarrow \infty \quad f' \rightarrow 0, f'' \rightarrow 0 \quad (9.b)$$

On substituting eq. (8) into eq. (6) and using boundary conditions eq. (9a) and eq. (9b) the velocity

Components take the form:

$$u = Bx e^{-m\eta} \quad (10.a)$$

$$v = -(Bv)^{1/2}(1 - e^{-m\eta})/m \quad (10.b)$$

Defining the dimensionless temperature

$$\theta(\eta) = \frac{T - T_\infty}{T_S - T_\infty} \quad (11)$$

And using eq. (8), (10a), (10a), (11) and the boundary conditions eq. (5a) and (5b) can be written as:

$$\theta'' + \text{Pr} f \theta' + \text{Pr}(\beta - 2f') \theta = -\text{Pr} E[(f'')^2 + K f'(f' f'' - f f''')] \quad (12)$$

Introducing the variable:

$$\xi = -r e^{-m\eta} \text{ With } r = \text{Pr}/m^2 \quad (13)$$

Substituting eq. (13) into eq. (12) we find:

$$\xi \frac{d^2 \theta}{d\xi^2} + (1 - r - \xi) \frac{d\theta}{d\xi} + \left(2 + \frac{\beta r}{\xi}\right) \theta = -\frac{\text{Pr} E(1+K)\xi}{r^2} \quad (14)$$

And the corresponding boundary conditions become:

$$\theta'(-r) = \frac{-1}{rm}, \quad \theta(0) = 0 \quad (15)$$

The eq. (14) can be transformed into the standard confluent hyper geometric equation or the Kummer's equation [51].

The solution satisfies eq. (14) and eq. (15) is given by:

$$\theta(\xi) = \left(\frac{1}{m} + 2H\right) \left(\frac{\xi}{-r}\right)^{\frac{r+s}{2}} \frac{M\left(\frac{r+s-4}{2}, s+1, \xi\right)}{\frac{r+s}{2} M\left(\frac{r+s-4}{2}, s+1, -r\right) - \frac{r(r+s-4)}{2(s+1)} M\left(\frac{r+s-4}{2}, s+1, -r\right)} - H\left(\frac{\xi}{-r}\right)^2 \quad (16)$$

Where:

$$s = r \sqrt{1 - \frac{4\beta}{r}} \quad \text{et } H = \frac{EPr(1+K)}{4-2r+\beta r} \quad \text{and } M(a, b, \xi) = 1 + \sum_{n=1}^{\infty} \frac{(a)_n}{(b)_n n!} \xi^n \quad \text{is the Kummer's function.}$$

$$\begin{aligned} (a)_n &= a(a+1)(a+2) \dots (a+n-1) \\ (b)_n &= b(b+1)(b+2) \dots (b+n-1) \end{aligned} \quad (17)$$

The solution (16) can be rewritten, in terms of η as

$$\theta(\eta) = \left(\frac{1}{m} + 2H\right) e^{\frac{-(r+s)m\eta}{2}} \frac{M\left(\frac{r+s-4}{2}, s+1, -re^{-m\eta}\right)}{\frac{r+s}{2} M\left(\frac{r+s-4}{2}, s+1, -r\right) - \frac{r(r+s-4)}{2(s+1)} M\left(\frac{r+s-4}{2}, s+1, -r\right)} - He^{-2m\eta} \quad (18)$$

4. Second law analysis

According to Wood [48] the local volumetric rate of entropy generation in the presence of a magnetic field and porous medium is given by:

$$S_G = \frac{k}{T_0} \left[\left(\frac{\partial T}{\partial x}\right)^2 + \left(\frac{\partial T}{\partial y}\right)^2 \right] + \frac{\mu}{T_0} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\sigma B_0^2}{T_0} u^2 + \frac{\mu}{k_1 T_0} \left(\frac{\rightarrow}{V}\right)^2 \quad (19)$$

The eq. (19) clearly shows contributions of four sources of entropy generation. The first term on the right-hand side of eq. (19) is the entropy generation due to heat transfer; the second term is the local entropy generation due to viscous dissipation, whereas the third term is the local entropy generation due to the effect of the magnetic field and de fourth term is the entropy generation due to the Porous medium.

It is appropriate to define dimensionless number for entropy generation rate NS . This number is defined by dividing the local volumetric entropy generation rate S_{G_0} to a characteristic entropy generation rate S_G [48].

For prescribed boundary condition, the characteristic entropy generation rate is:

$$S_{G_0} = \frac{K(\Delta T)^2}{l^2 T_0^2} \quad (20)$$

Therefore, the entropy generation number is:

$$N_S = \frac{S_G}{S_{G_0}} \quad (21)$$

Using eqs. (10a), (10b), (18) and (19), the entropy generation number is given by:

$$N_S = \frac{4}{Re_l X^2} \theta^2 + \theta'^2 + \frac{Br Re_l}{\Omega} f''^2 + \frac{Br}{\Omega} (Ha^2 + K O Re_l) f'^2 + \frac{Br K O}{\Omega X^2} f^2 \quad (22)$$

5. Results and discussion

The Flow and heat transfer of an electrically conducting fluid of second grade in a porous medium with prescribed wall heat flux over a stretching sheet subject to a transverse magnetic field has been solved analytically using Kummer's functions after that the velocity and temperature have been used to compute the entropy generation.

In all the figures of dimensionless temperature profiles plotted we notice that temperature is maximum at the wall where both heat flux and magnetic field are imposed and minimum at the free surface whatever the values of the parameters studied so we conclude that transfer happened in surface extensible. In order to validate our study, the results were compared with those of previous studies as well as the series solution for several values of parameters. It can be seen from Table 1 that our results agree well with those reported in the literature.

It is clear from fig.2, 3 that dimensionless temperature increases slightly with an increase of both medium porous and magnetic parameters this can be explained by Lorentz force which creates by the influence of the vertical magnetic fields of electrically conducting fluid. The decrease of permeability

(augmentation of medium porous parameter) increases the friction between the molecule fluid this causes an increase of heat in the boundary layer.

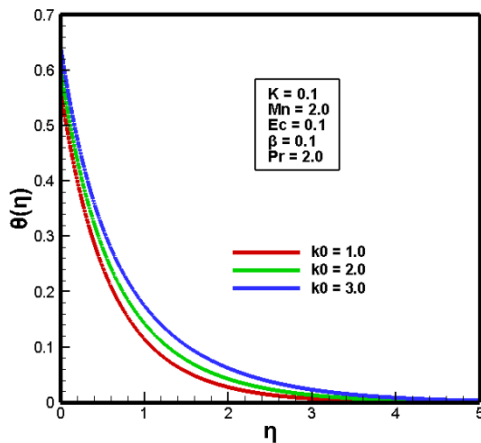


Fig. 2. Effect of the porous medium parameter on the temperature.

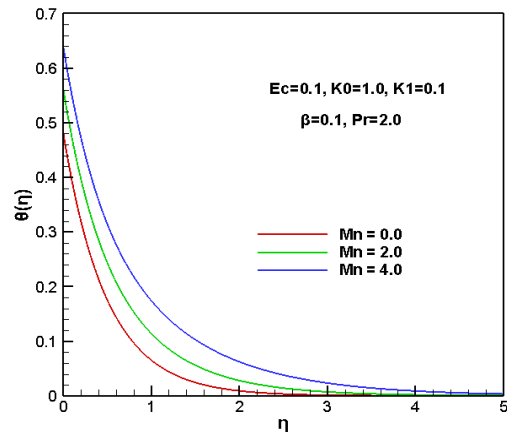


Fig. 3. Effect of the magnetic parameter on the temperature.

Figure 4 shows the increase of dimensionless temperature profiles as a function of η for different values of the viscoelastic parameter. As it can be noticed, an increase of viscoelastic parameter produces a reduction in the wall temperature this implies that the increase of viscoelastic parameter increases the flow velocity the contact time fluid-surface decreases this will cause reduction of heat transfer surface-fluid. We notice also from the fig.5 that an augmentation of the Prandtl number reduces the dimensionless temperature which implies the thermal boundary layer thickness decreases when the Prandtl number increases it physically means that the flow with large Prandtl presents spreading of heat in the fluid.

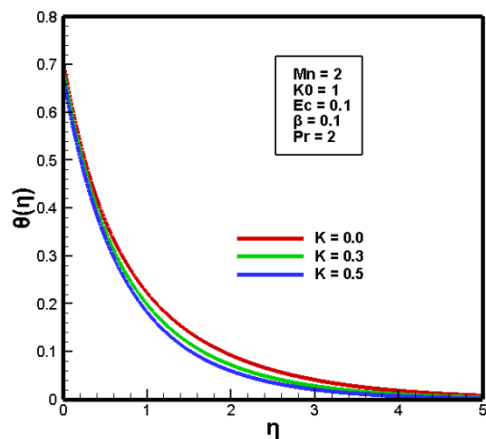


Fig. 4. Effect of the viscoelastic parameter on the temperature.

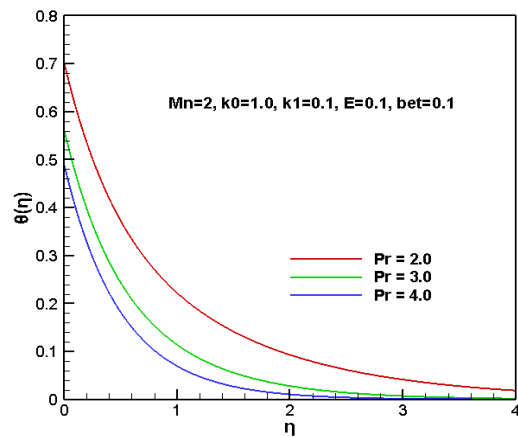


Fig.5. Effect of the Prandtl number on the temperature.

Figure 6 demonstrates the effect of the heat source/sink parameter. For a fixed value of η , the dimensionless temperature $\Theta(\eta)$ increases with an increase in heat source/sink. This is due to the increase of the heat generation inside the boundary layer leading to higher temperature profile.

The graph (7) presents a variation of dimensionless temperature with Eckert number; we notice that an increase of Eckert number augments the dimensionless temperature in the flow region it confirms that the energy is stocked in the fluid region dissipation due to the viscosity and elastic deformation.

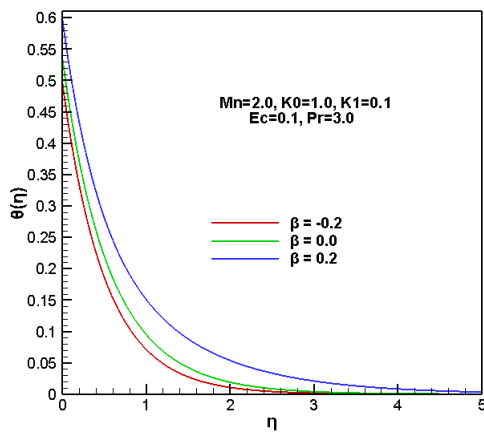


Fig.6. Effect of the heat source/sink parameter on the temperature.

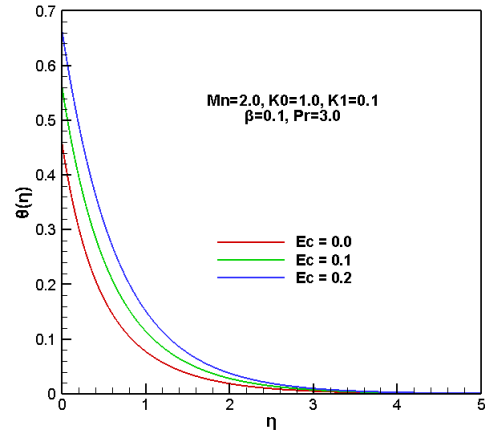


Fig.7. Effect of the Eckert number on the temperature.

Table 1: Comparison with previously published data for the values of surface temperature $\theta(0)$.

| Pr | M | Elbashbeshy [49] | Liu [50] | Present study |
|------|-----|------------------|----------|---------------|
| 0.72 | 0 | 1.2253 | / | 1.2367 |
| 1 | 0 | 1 | / | 1 |
| 10 | 0 | 0.2688 | / | 0.2688 |
| 6.7 | 0 | / | 0.333303 | 0.333303 |
| | 0.5 | / | 0.339715 | 0.339715 |
| | 1 | / | 0.345377 | 0.345377 |
| | 5 | / | 0.380930 | 0.380930 |

The results of the entropy generation are presented in function of many parameters such as the viscoelastic, magnetic, heat source/heat sink parameters and dimensionless group, Hartman, Reynolds, Prandtl, Eckert numbers

All the results depict that the entropy decreases with a decrease of mentioned parameters moreover the entropy is higher near the surface where the velocity and magnetic field are in their maximum values also where the heat flux is imposed this means that the surface acts a strong source of irreversibility.

The fig.8 shows the variation of the entropy with magnetic parameter as we can see for fixed value of the entropy increases with an increase in magnetic parameter because the magnetic field creates more entropy, the figure also displays that the increase of magnetic parameter the entropy increases up to certain distance from the sheet after that point the behavior mentioned is reverse. The figure 9 displays the variation of the entropy with medium porous parameter, the entropy production number increases with a diminution of permeability of porous medium, whenever the medium is more restrictive the particles of the fluid are more and more messy.

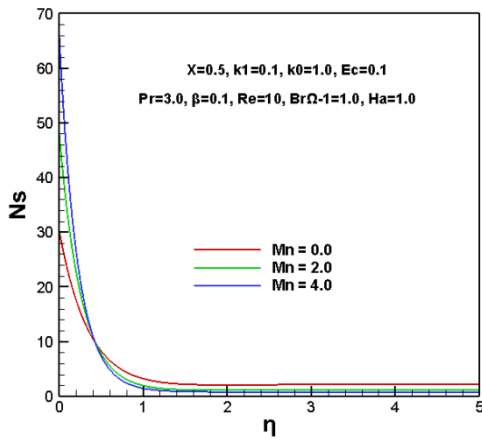


Fig. 8. Effect of the magnetic parameter on the entropy generation number.

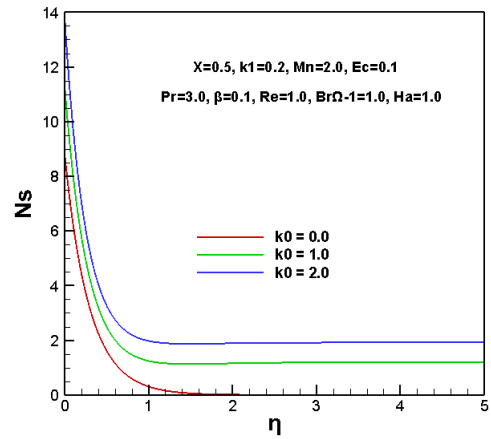


Fig. 9. Effect of the porous medium parameter on the entropy generation number.

From fig.10 and 11 we conclude that an augmentation of viscoelastic and heat source/ heat sinks parameters has a small effect on the entropy generation.

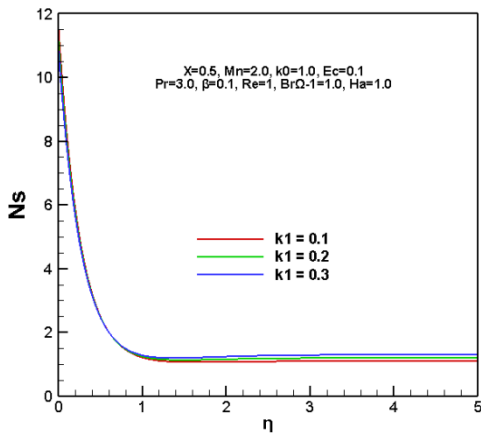


Fig.10. Effect of the viscoelastic parameter on the entropy generation number K_1 .

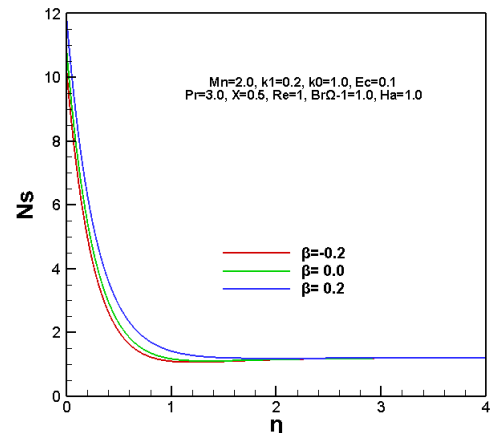


Fig.11. Effect of the heat source/ heat sink parameter on the entropy generation number.

The fig.12 depicts that the production of the entropy increases with an increase of Eckert number this behavior is attributes to the increase of the viscous dissipation that is a source of entropy generation. The variations in the number of entropy production depending on η for different values of Prandtl number is shown in fig.13. For a given thickness, the entropy production decreases with the increase of the Prandtl number. This is due to the fact that the temperature decreases with increasing Prandtl number.

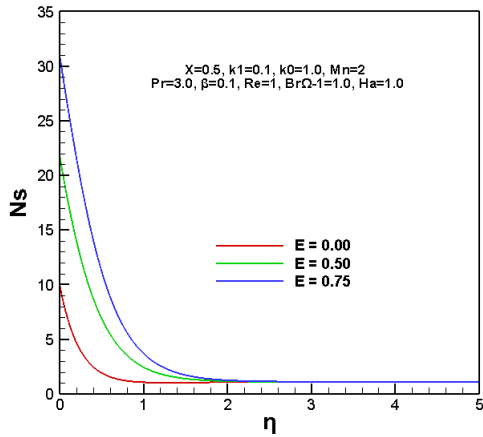


Fig.12. Effect of the Eckert number on the temperature.

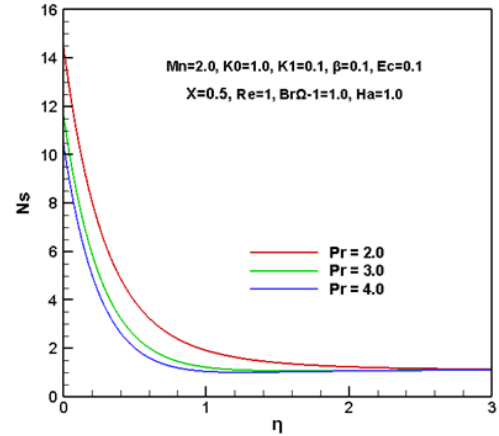


Fig.13. Effect of the Prandtl number on the entropy generation number.

The influence of the Reynolds number on the entropy generation number is plotted in Fig.14. For a given η , the entropy generation number increases as Reynolds number. The augmentation of the Reynolds number increases the contribution of the entropy generation number due to fluid friction. The variation of the entropy generation with different values of characteristic length is shown on the Fig.15, for fixed value an augmentation of characteristic length decreases the entropy generation this behavior can be interpreted by the energy lost inside the fluid flow.

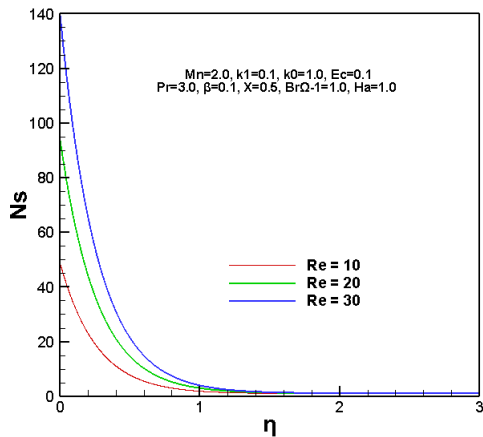


Fig.14. Effect of the Re_L on the entropy generation number.

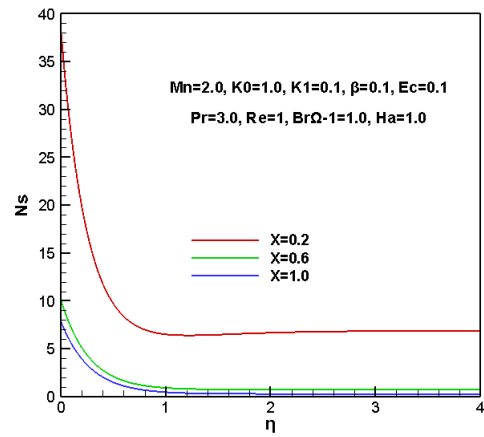


Fig. 15. Effect of the characteristic length on the entropy generation number.

The effect of the dimensionless group parameter $Br\Omega-1$ on the entropy generation number is depicted in Fig.16 for a given η , the entropy generation number is higher for higher dimensionless group. This is due to the fact that for higher dimensionless group, the entropy generation numbers due to the fluid friction. The effect of the Hartman number on the entropy generation number is plotted in Fig.17. For a given η , as the Hartman number increases, the entropy generation number increases. The entropy generation number is proportional to the Hartman number which is proportional to the magnetic field. The presence of the magnetic field creates additional entropy.

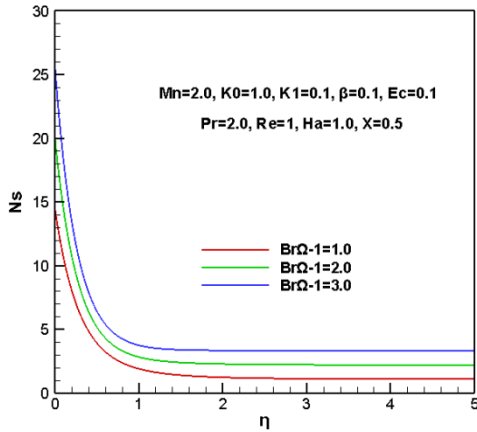


Fig.16. Effect of the dimensionless group on the entropy generation number.

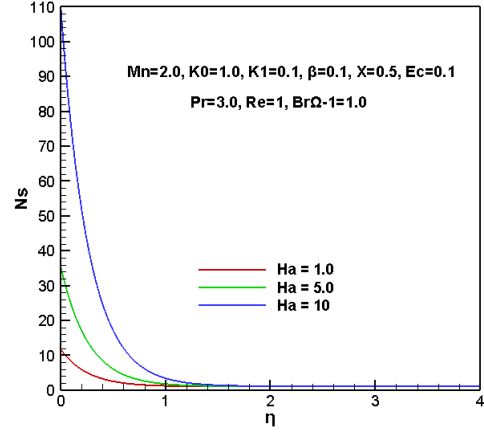


Fig.17. Effect of the Hartmann number on the entropy generation number.

6. Conclusions

The velocity and temperature profiles are obtained analytically and used to compute the entropy generation number in a viscoelastic fluid over a stretching sheet prescribed wall heat flux subject to a transverse magnetic field. The influences of the Prandtl number, the magnetic parameter and the heat source/sink parameter on the temperature profiles are presented. As far as the entropy generation number is concerned, it is dependent on the magnetic parameter, the Prandtl, the Reynolds number, the dimensionless group, the Hartman number; From the results the following conclusions could be drawn:

- The temperature increases as the porous medium, the magnetic and heat source sink parameters increase, but it decreases as the viscoelastic parameter and Prandtl number increase.
- The entropy generation number increases as Hartman number, dimensionless group parameter and Reynolds number increase.
- The entropy generation number is slightly influenced by Prandtl number, magnetic parameter.
- The surface acts as a strong source of irreversibility.

Nomenclature

| | | | |
|----------------|--------------------------------------------------------------------------|----------------|-------------------------------------------------------------------------|
| A | Constant, K | T | Temperature, [K] |
| B | Proportional. Constant | u | Axial velocity, [m.s ⁻¹] |
| B ₀ | Uniform magnetic field strength, [Wb.m ⁻²] | u _l | Plate velocity based on the characteristic length, [m.s ⁻¹] |
| Br | Brinkman number, $Br = \frac{\mu u_x^2}{k\Delta T}$ | u _p | Plate velocity, [m.s ⁻¹] |
| C _p | Specific heat of the fluid, [J.kg ⁻¹ .K ⁻¹] | v | Transverse velocity, [m.s ⁻¹] |
| Ec | Eckert number, $E = B^2 l^2 / AC_p$ | x | Axial distance, [m] |
| f | Dimensionless function | X | Dimensionless axial distance, $X = x/l$ |
| Ha | Hartman number, $Ha = B_0 l \sqrt{\frac{\sigma}{\mu}}$ | y | Transverse distance, [m] |
| K | Thermal conductivity of the fluid, [W.m ⁻¹ .K ⁻¹] | α | Positive constant |
| K ₀ | Medium porous parameter $K_0 = \frac{v}{k_1 B}$ | β | Heat source/sink parameter, $\beta = Q/B\rho C_p$ |
| K ₁ | Viscoelastic parameter $K_1 = \frac{\alpha_1 B}{\mu}$ | η | Dimensionless variable, |
| l | Characteristic length, [m] | ξ | Dimensionless variable, $\xi = -re^{-m\eta}$ |
| M | Kummer's function | μ | Dynamic viscosity of the fluid, [kg.m ⁻¹ .s ⁻¹] |

| | | | |
|------------|-----------------------------------------------------------------------------------|-------------------------------|---------------------------------------------------------------------------------|
| Mn | Magnetic parameter, $Mn = \frac{\sigma B_0^2}{\rho B}$ | ν | Kinematic viscosity of the fluid, [$m^2 \cdot s^{-1}$] |
| N_s | Entropy generation number, | Ω | Dimensionless temperature difference, $\Omega = \frac{\Delta T}{T_\infty}$ |
| Pr | Prandlt number, $Pr = \mu C_p / K$ | Θ | Dimensionless temperature, $\theta(\eta) = \frac{T - T_\infty}{T_p - T_\infty}$ |
| Q | Rate of internal heat generation or absorption, [$W \cdot m^{-3} \cdot K^{-1}$] | ρ | Density of the fluid, [$kg \cdot m^{-3}$] |
| Re_1 | Reynolds number based on the characteristic length, $Re_1 = \frac{u_1 l}{\nu}$ | σ | Electric conductivity, [$\Omega^{-1} \cdot m^{-1}$] |
| S_G | Local volumetric rate of entropy generation, [$W \cdot m^{-3} \cdot K^{-1}$] | | |
| Subscript | | | |
| P : plate. | | ∞ : far from the sheet | |

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