FILM CONDENSATION GENERATED BY FREE CONVECTION IN A POROUS MEDIUM

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Abstract: *In this paper, the steady process of laminar film condensation on a vertical wall with the backside cooled by free convection in a fluid-saturated porous medium is analysed as a conjugate problem. In the analysis, neither the temperature nor the heat flux at the wall sides is assumed, but they are determined from the solution like other unknown parameters. The analysis is conducted in a dimensionless way to generalize the solution. The main variables controlling the film condensation process are revealed from the analysis, which define the relative importance of interactive wall conduction, free convection and film condensation modes. The study indicates that the conjugate solution of laminar-film condensation problem is different from a Nusselt-type solution.*

Key words: *Film condensation, Conjugate heat transfer, Analytical convection*

1. Introduction

Convection results are dependent mainly on applied boundary conditions. Therefore, solving a convection heat transfer process as a conjugate problem can yield physically more accurate results than that as a direct problem. This is because in the conjugate analysis, no thermal conditions are prescribed at the solid-fluid boundary, but determined from the solution. Dorfman [1] reported an inclusive review on the conjugate problems in convection heat transfer.

Numerous studies have been conducted since the pioneering Nusselt work [2] of laminar film condensation on an isothermal surface to investigate the effects of some factors neglected in the original Nusselt analysis, such as film convection [3], vapour superheat [4], vapor shear [5], film flow regimes [6], surface geometry [7,8], and non-isothermal surface [9] among others. In contrast to this class of studies, other studies solved the film condensation process as a conjugate problem. In these conjugate studies, no solid-fluid interface conditions are prescribed in the analysis, but obtained from the solution. Using a similarity technique, Patankar and Sparrow [10] did the first numerical conjugate solution of laminar film condensation outside a vertical cylindrical wall joined with the internal wall conduction. Afterward, Faghri and Sparrow [11] solved numerically the same problem for a vertical tube of negligible wall resistance by taking into account the thermal connection between the film condensation and the inside fluid flow. They stated that Nusselt-type models are not appropriate for the estimation of the required heat transfer surface area of a condenser. Chen and Chang [12] used the local non-similarity method to analyze film

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condensation on a vertical thick plate coupled with forced flow on the back plate side. They proved that the wall resistances has a considerable effect on the solution. Later, Méndez and Trevino [13] treated the same problem, and showed that the axial wall conduction is significant only through the lateral plate surface. Bautista et al. [14] used perturbation techniques to analyze the thermal coupling between laminar film condensation outside a vertical two-plate channel and the internal forced flow. They found that Nusselt number is much lower than that of a Nusselt-type model. Later, Luna and Méndez [15] solved the problem for a porous channel, and concluded similar results.

Thermal conjugation between film condensation and free convection is a heat transfer process happened in many engineering applications; such as heat exchangers, chemical processing and thermal insulation. Concerning this thermal interaction process, only few studies have been reported in the literature. The first analytical study on film condensation and free convection conjugating across a vertical conductive wall separating them was done by Poulikakos [16] neglecting the wall resistance effect. Poulikakos adopted the Nusselt-analysis for the condensate film, and used the Osceen technique for solving the free convection layer. Later, Char and Lin [17] used the cubic Spline method to solve the same problem, however, for a vertical wall separating two porous media. Mosaad [18] solved analytically the conjugate problem of laminar film condensation of a pure saturated vapour on the surface of a vertical plate cooled on the backside by free convection in a porous medium. He assumed the solid wall as a partition of negligible thermal resistance in order to reduce the mathematical analysis complexity. The author proved that the conjugate solution is different from that of a Nusselt-type analysis. However, to achieve a better modeling for the physical reality of this problem, the effect of wall conduction should be included in the solution. This is the main objective of the present study. As a conclusion of the present study, the free convection layer is solved by employing the Osteen technique, while the condensate film is analyzed using the boundary layer theory and Nusselt-analysis approximations. The wall conduction is assumed one-dimensional in the crosswise direction. The two analyses are matched at the wall sides to yield finally the conjugate solution. The analysis is conducted in a dimensionless form to generalize the results. The main advantage of such an analytical solution is that the role of the dimensionless parameters governing the conjugation process is more evident than in a numerical solution.

2. Analysis

The physical model is sketched in Fig. 1. The sketch indicates that a vertical conductive wall of height H, thickness w and thermal conductivity k separates two fluid media at different temperatures. The hot medium is a pure vapour at a saturation temperature T_s, while the cold medium is a fluid-saturated porous medium of bulk temperature $T_{c\infty} << T_s$. The wall is assumed thermally insulated at its ends. Because of the heat transfer between the two media, a thin condensate film with a downward flow is generated on the hot wall side while a free convection layer with an upward flow is induced on the cold side. In the next subsections, each boundary layer flow is analyzed separately. Then, the two analysis results are joined together by applying the matching conditions of the temperature and heat flux continuity at the wall sides to yield finally the conjugate solution. In the analysis, neither the temperature nor the heat flux at the wall sides is prescribed in the analysis, but they are determined from the solution. For clarity in the analysis, the subscripts "c", "*f*" and "*w*" are used to designate convection, condensate film and wall, respectively.

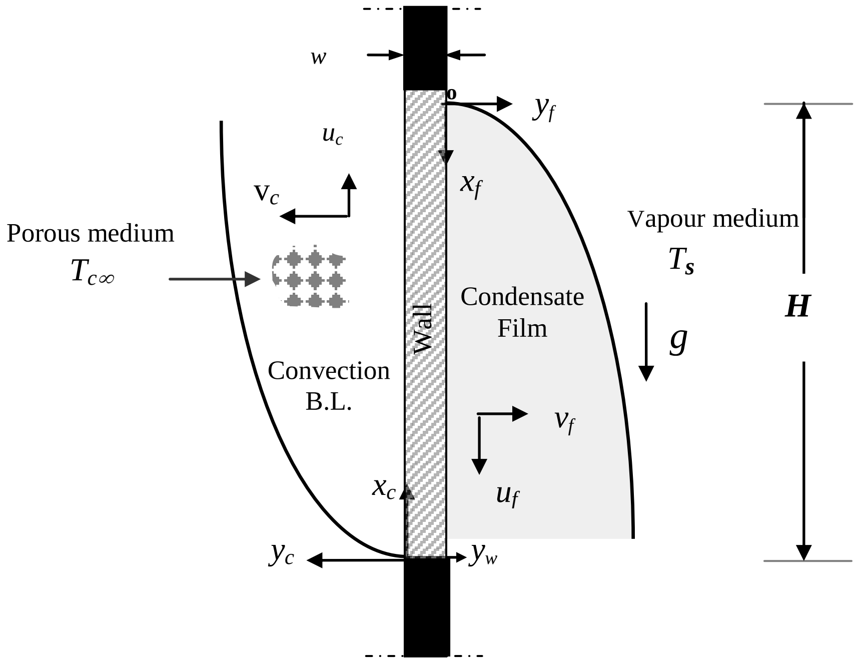


Fig. 1. Model illustration

2.1. *Film condensation*

For steady laminar condensate film of constant properties, the governing equations of mass, momentum and energy can be written, respectively, in dimensionless forms as:

$$\frac{\partial U_f}{\partial X_f} + \frac{\partial V_f}{\partial Y_f} = 0 \tag{1}$$

$$\frac{Ja}{\Pr_f}\left(U_f \frac{\partial U_f}{\partial X_f} + V_f \frac{\partial U_f}{\partial Y_f}\right) = -1 + \frac{\partial^2 U_f}{\partial Y_f^2} \tag{2}$$

$$Ja\left(U_f \frac{\partial \theta_f}{\partial X_f} + V_f \frac{\partial \theta_f}{\partial Y_f}\right) = \frac{\partial^2 \theta_f}{\partial {Y_f}^2} \tag{3}$$

The dimensionless variables introduced above are defined,

$$\begin{gathered} Y_f = y_f / l_f, \quad X_f = x_f / H, \quad U_f = u_f / (\mathrm{Ja}\ \alpha_f H / l_f^2), V_f = \mathrm{v}_f / (Ja\ \alpha_f / l_f), \\ \theta_f = (T_f - T_{c\infty}) / (T_s - T_{c\infty}),\ \Delta_f = \delta_f / l_f, \theta_{wf} = (T_{wf} - T_{c\infty}) / (T_s - T_{c\infty}). \end{gathered} \tag{4}$$

where θ_f is the dimensionless local condensate film temperature, θ_{wf} is the dimensionless local temperature of the wall side facing the condensate film, which is an unknown function to be determine as a part of the problem solution. The symbol Δ_f is the dimensionless local film thickness, and l_f is the film thickness scale defined by

$$l_f = HRa_f^{-1/4}. \tag{5}$$

Ra_f is the film Rayleigh number defined by

$$Ra_f = g H^3 (\rho_f - \rho_v) h_{fg} / (k_f \nu_f (T_s - T_{c\infty})) \tag{6}$$

Ja is the Jacob number defined by

$$Ja = Cp_f (T_s - T_{c\infty}) / h_{fg} \tag{7}$$

For the most practical fluids of $Pr \geq 1$, ignoring the inertia term in the film momentum equation has a negligible effect on the solution [5]. In addition, for most water applications, the Ja number is in the range: $10^{-4} < Ja < 10^{-2}$ [8]. Thus, for these practical ranges of Pr and Ja, the terms multiplied by Ja in Eqs. (2) & (3) can be omitted. Consequently, these two equations reduce, respectively, to

$$\frac{\partial^2 U_f}{\partial Y_f^2} - 1 = 0, \tag{8}$$

$$\frac{\partial^2 \theta_f}{\partial Y_f^{\,2}} = 0. \tag{9}$$

The appropriate boundary conditions are:

$$\begin{aligned} Y_f = 0; \quad & U_f = V_f = 0 \quad \text{and} \quad \theta_f = \theta_{wf}, \\ Y_f = \Delta_f; \quad & \partial U_f / \partial Y_f = 0, \quad \text{and} \quad \theta_f = 1, \\ x_f = 0; \quad & \Delta_f = 0 \quad \text{and} \quad \theta_f = 1. \end{aligned} \tag{10}$$

Solving Eqs. (8)& (9) for above boundary conditions (10) gives, respectively,

$$U_f = 0.5 Y_f (2\Delta_f - Y_f) \tag{11}$$

$$\theta_f = \theta_{wf} + (1 - \theta_{wf}) Y_f / \Delta_f \tag{12}$$

Integrating continuity Eq. (1) across the condensate film from Y_f = zero to Δ_f, this gives

$$V_f \Big|_{Y_f = \Delta_f} = \left(U_f \frac{d\Delta_f}{dx_f} \right)_{Y_f = \Delta_f} - \frac{d}{dx_f} \int_0^{\Delta_f} U_f \, dY_f \tag{13}$$

Applying the condition of the heat flux continuity at the condensate-vapour interface gives

$$\frac{\partial \theta_f}{\partial Y_f} \Bigg|_{Y_f = \Delta_f} = \left(U_f \frac{d\Delta_f}{dx_f} - V_f \right)_{Y_f = \Delta_f} \tag{14}$$

Combining Eqs. (11) - (14) yields,

$$\frac{\partial \Delta_f^4}{\partial X_f} = 4(1 - \theta_{wf}) \tag{15}$$

2.2 *Porous free convection*

For simplifying the free convection analysis, the following approximations are made: The boundary layer flow is steady, laminar, incompressible and two-dimensional; the Boussinesq approximation and Darcy's law are applicable; the porous medium is isotropic, homogeneous, fluid-saturated, and thermally equilibrium. According to these simplifications, the two-dimensional, conservation equations of mass, momentum and energy of the porous free convection layer can be expressed, respectively, in the dimensionless forms [18]:

$$\frac{\partial U_c}{\partial X_c} + \frac{\partial V_c}{\partial Y_c} = 0 \tag{16}$$

$$\frac{1}{Ra_c}\frac{\partial V_c}{\partial X_c} - \frac{\partial U_c}{\partial Y_c} = -\frac{\partial \theta_c}{\partial Y_c} \tag{17}$$

$$U_c\frac{\partial \theta_c}{\partial X_c} + V_c\frac{\partial \theta_c}{\partial Y_c} = \frac{1}{Ra_c}\frac{\partial^2 \theta_c}{\partial {X_c}^2} + \frac{\partial^2 \theta_c}{\partial {Y_c}^2} \tag{18}$$

The appropriate boundary conditions are

$$\begin{aligned} &Y_c = 0; \quad V_c = 0 \quad \text{and} \quad \theta_c = \theta_{wc}, \\ &Y_c \to \infty; \quad U_c = 0, \quad \text{and} \quad \theta_c = 0. \\ &X_c = 0; \quad \theta_c = 0. \end{aligned} \tag{19}$$

The above-introduced dimensionless parameters read as:

$$\begin{aligned} &Y_c = y_c/l_c,\ X_c = x_c/H \quad U_c = u_c/(\alpha_c H/l_c^2),\ V_c = v_c/(\alpha_c/l_c), \\ &\theta_c = (T_c - T_{c\infty})/(T_s - T_{c\infty}),\ \theta_{wc} = (T_{wc} - T_{c\infty})/(T_s - T_{c\infty}). \end{aligned} \tag{20}$$

θ_c is the local temperature in the cold porous medium, θ_{wc} is the local temperature of the wall side facing the cold porous side, which is unknown function to be determine as a part of the problem solution. The symbol l_c is the convection-layer thickness scale defined by

$$l_c = H/Ra_c^{1/2}; \tag{21}$$

Wherein, Ra_c is a modified Rayleigh number defined by

$$Ra_c = g\beta_c KH^3(T_s - T_{c\infty})/(\nu_c\alpha_c). \tag{22}$$

According to the boundary layer theory condition that the ratio l_c/H should be << 1.0, and subject to relation (21), Ra_c should be of moderate or high value. For this Ra_c limit, the terms divided by Ra_c in Eqs. (17) & (18) can be neglected. Hence, combining the simplified energy and momentum equations gives

$$\frac{\partial^2 U_c}{\partial {Y_c}^2} + [V_c]\frac{\partial U_c}{\partial Y_c} + \left[\frac{\partial \theta_c}{\partial X_c}\right]U_c = 0 \tag{23}$$

Following some previous relevant studies (e.g., [15, 18]), the analytic Oseen method can be used to solve analytically the above nonlinear differential equation. In this method, the velocity component V_c and the temperature gradient ($\partial\theta_c / \partial X_c$) in Eq. (23) are assumed as functions of Y_c-variable only. Consequently, the nonlinear differential equation (23) reduces to an ordinary differential equation, whose solution subject to boundary conditions (19) gives:

$$U_c = \theta_{wc} \,\mathrm{e}^{-Y_c/\eta} \tag{24}$$

$$\theta_c = \theta_{wc}(x_c) \,\mathrm{e}^{-Y_c/\eta} \tag{25}$$

The above velocity and temperature profiles have two unknown variables: The inverse Oseen function η and the wall side temperature θ_{wc}. Investigating Eqs. (24) & (25) indicates that η-parameter works the role of the convection layer thickness. Integrating the above energy equation across the entire boundary layer for the above velocity and temperature profiles yields the relation:

$$\frac{d}{dX_c}\left(\eta\,\theta_{wc}{}^2\right) = \frac{2\theta_{wc}}{\eta}. \tag{26}$$

2.3 *Interfacial conditions*

Assuming the wall thickness-to-height ratio (*w/H*) is much less than one, the wall conduction can be assumed only in the crosswise direction. Hence, applying the continuity condition of the temperature and heat flux at both wall sides gives:

$$\left.\frac{\partial\theta_f}{\partial Y_f}\right|_{Y_f=0} = -\Gamma \left.\frac{\partial\theta_c}{\partial Y_c}\right|_{Y_c=0} = \Gamma\,\frac{\left(\theta_{wf}-\theta_{wc}\right)}{\varepsilon_w} \tag{27}$$

The above-introduced dimensionless parameters are defined as,

$$Y_w = y_w / w, \qquad \varepsilon_w = \frac{w\,k_c}{H\,k_w} Ra_c^{1/2}, \;\; \Gamma = \frac{k_c}{k_f}\frac{Ra_c^{1/2}}{Ra_f^{1/4}}. \tag{28}$$

The wall parameter ε_w relates the thermal resistance of the solid wall to that of free convection layer. While the parameter Γ represents the thermal resistance ratio of condensate film to free convection layer.

Calculating the temperature derivatives in Eq. (27) by using Eqs. (12)& (25) yields:

$$\theta_{wf} = \frac{\eta+\varepsilon_w}{\eta+\varepsilon_w+\Gamma\Delta_f}. \tag{29}$$

$$\theta_{wc} = \frac{\eta}{(\eta+\varepsilon_w+\Gamma\Delta_f)} \tag{30}$$

Relation (29) for $\Delta_f = 0$ gives θ_{wf} =1, while relation (30) for η=0 gives θ_{cw}=0. This means that the two above relations validate the initial conditions: θ_{wf} = 1 at X_f =0, and θ_{cw}=0 at X_c= 0.

Inserting above θ_{wc} & θ_{wf} results into Eqs. (15) & (26) with substituting $X_f = 1 - X_c$, this gives

$$\frac{d\Delta_f^3}{dX_c} = \frac{-3\Gamma}{(\eta + \varepsilon_w + \Gamma\Delta_f)} \quad (31)$$

$$\frac{d\eta^3}{dX_c} = \frac{6(\eta + \varepsilon_w + \Gamma\Delta_f)^3\Delta_f^2 - 6\eta^3\Gamma^2}{\Delta_f^2(\eta + 3\varepsilon_w + 3\Gamma\Delta_f)(\eta + \varepsilon_w + \Gamma\Delta_f).} \quad (32)$$

The simulations solution of above Eqs. (29) - (32) will provide the distributions of Δ_f ,η, θ_{wc} and θ_{wf} along the wall as functions of ε_w and Γ parameters. Hence, the local Nusselt number of the porous convection side $Nu_x (= h_x\ x / k_c)$, is found by

$$\frac{Nu_x}{Ra_{c\,x}^{1/2}} = \frac{\sqrt{X_c}}{\eta}\theta_{wc} \quad (33)$$

Similarly, the local Nusselt number of the film condensation side is determine by

$$\frac{Nu_x}{Raf_x^{0.25}} = \frac{\sqrt[4]{X_f}}{\Delta_f}\left(1 - \theta_{wf}\right) \quad (34)$$

The mean Nusselt number over the entire wall height can be defined based on the average heat flux across the wall and the total temperature difference $(T_s - T_{c\infty})$ by

$$\frac{Nu}{Ra_{f\,x}^{0.25}} = \int_0^1 \left.\frac{\partial\theta_f}{\partial Y_f}\right|_{Y_f=0} dX_f \quad (35)$$

3. Solution

3.1 *Asymptotic results*

In this subsection, asymptotic results for the special problem case of $\varepsilon_w \rightarrow 0$ are found from the general problem analysis derived in the above section. In this case, the solid wall works as a partition of zero thermal resistance, whose temperature θ_w is a function of *x*-coordinate only. For this $\varepsilon_w \rightarrow 0$ limit, Eqs. (29) & (30) show that as $\Gamma \rightarrow \infty$, $\theta_{wf} \rightarrow 0$ & $\theta_{wc} \rightarrow 0$. I.e., the wall assumes the free porous-side temperature. This means disappearing the convection layer. Hence, the conjugate problem simplifies to the classical direct problem of film condensation on an isothermal vertical surface. Solving film condensation result (15) for θ_{wf} = 0, this yields $\Delta_f = \sqrt[4]{4X_f}$. Substituting this result into Eq. (34) yields the local Nusselt number by

$$Nu_x \big/ Ra_{f\,x}^{0.25} = 0.707 \quad (36)$$

Consequently, the mean Nusselt number is determined by

$$Nu \big/ Ra_f^{0.25} = 0.943 \quad (37)$$

The above result is the known Nusselt solution of film condensation on an isothermal vertical surface.

On the opposite limit of $\Gamma \to 0$, Eqs. (29) & (30) indicate, respectively, that $\theta_{wh} \to 1$ & $\theta_{wc} \to 1$. This mean that the wall assumes the extreme condensation-side temperature. This means disappearing the condensate film, and consequently, reducing the conjugate problem to the classical problem of free convection on an isothermal vertical surface in a fluid-saturated porous medium. Solving convection result (26) for $\theta_{wc} = 1$ yields $\eta = 2\sqrt{X_c}$. Substituting this result into Eq. (33), yields the local Nusselt number by,

$$Nu_x / Ra_x^{1/2} = 0.5 \qquad (38)$$

Consequently, the mean Nusselt number is found by

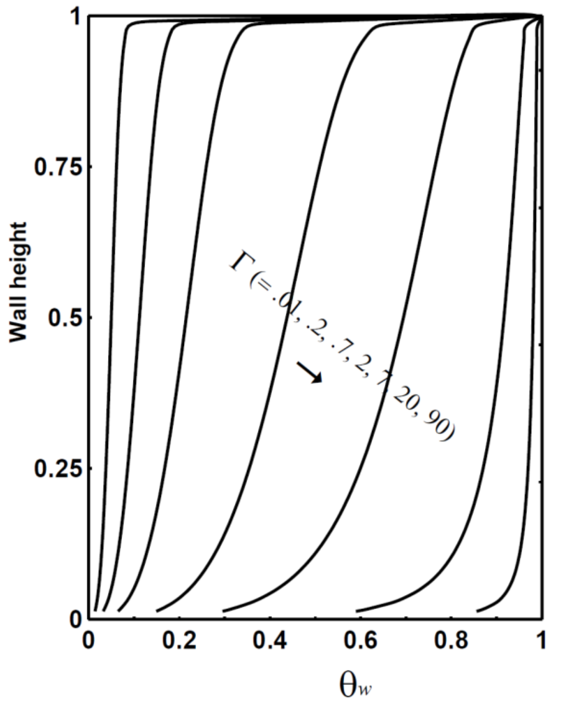
$$Nu / Ra^{1/2} = 1 \qquad (39)$$

The above result differs from the exact similarity solution [19] of porous free convection on an isothermal vertical surface by less than 12%. The constant coefficient in the similarity solution is 0.888 instead 1 in the above result. The asymptotic solutions (36)-(39) prove the present approach validity.

3.2 Numerical results

The two dependent differential relations (31) & (32) should be solved numerically to determine the distributions of Δ_f and η along the wall as functions of Γ and ε_w parameters. Δ_f is the dimensionless condensate film thickness, and η represents the dimensionless convection layer thickness. The fourth-order Runge-Kutta numerical method was used to solve simultaneously Eqs. (310 & (32). At the solution start point of X_f =0, one finds that Δ_f = 0 while the maximum value of η (cf., Fig. 1) is unknown. Therefore, this value of η_{max} is assumed at the solution start point. Then, the solution advances in small steps of ΔX_f until X_f =1. When the calculated η-value at X_f =1 is found different from zero, the solution is repeated by using a new adjusted value of η_{max}. The solution trial is stopped once the predicted η-value at X_f =1 is found very close to zero, namely less than 10^{-6}**.** In the preliminary tests, asymptotic results (36)-(39) were used as a reference to adjust the accuracy of the numerical solution as well as to prove its correctness**.** It was found that the solution with ΔX_f = 0.001 yields stable and accurate results. One the distributions of Δ_f and η along the wall have been determined for certain values of Γ and ε_w, the corresponding distribution of θ_{wf} & θ_{wc} can be calculated by Eqs. (29) & (30), respectively.

At first, the numerical results of the special problem case of $\varepsilon_w \to \mathbf{0}$ are discussed. In this case, the wall temperature θ_w(x) varies only in the longitudinal direction. Figure 2 shows the longitudinal distribution of θ_w(x) as a function of Γ – parameter. The plotted results indicate that for a fixed Γ -value, θ_w(x) varies almost linearly along the wall except near its ends. It is also noted that θ_w(x) approaches the extreme temperature of the film-condensation side as Γ →0, while approaches the low extreme temperature of the porous side as Γ → ∞. This behavior can be clearly seen from the results plotted in Fig. 3 in terms of the wall mid-height temperature versus Γ–parameter. The displayed results indicate also



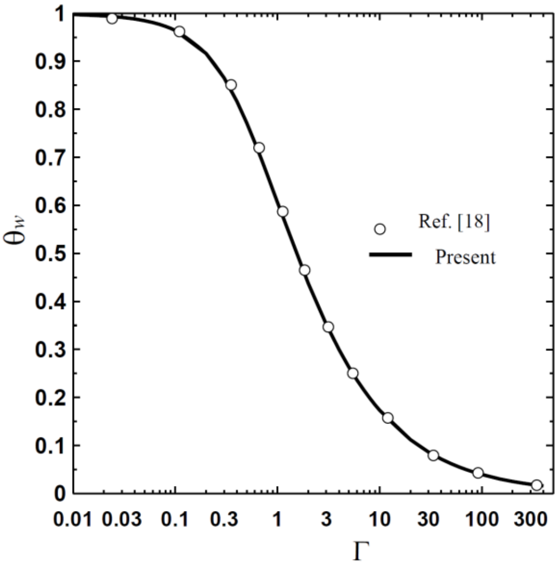


Fig. 2. Wall temperature profile at different Γ- parameter; for $\varepsilon_w \to 0$.

Fig. 3. Wall temperature at midheight as a function of Γ- parameter; for $\varepsilon_w \to 0$.

that the previous model [18] validates the present model.

The dependence of the local Nusselt number on the convection conjugation variable Γ is presented in Fig. 4. It is clear that Nu_x approaches the Nusselt film condensation solution as $\Gamma \to \infty$, while it approaches the free-convection convection solution of isothermal surfaces as Γ goes to 0. Similar conclusion on the effect of Γ – parameter on the mean Nusselt number can be decided from the results plotted in Fig. 5. For comparison with other study, corresponding results from ref. [18] are plotted in the same graph. The comparison indicates that the present model verifies the previous simple model [18] of negligible wall resistance case.

Next, results obtained for the case of $\varepsilon_w > \mathbf{0}$ are demonstrated in Figs. 6-7. The fluid temperature gradient at wall sides is plotted as a function of ε_w-parameter in Fig. 6. The displayed results indicate that the fluid temperature gradient at wall assumes a lower value on both sides for a higher value of ε_w. Thus, the wall relaxes the thermal interaction between the two interactive thermal media, i.e., it works as a thermal insulator between them. In Fig. 7, the mean overall Nusselt number is plotted as a function of ε_w and Γ parameters; for $0 \le \varepsilon_w \le 20$ & $0.01 \le \Gamma < 300$. In the plot, the upper curve of ε_w=0 is limited by the two dashed lines representing, respectively, the two exact results (37) & (39) of free convection and film condensation on isothermal vertical surfaces. The displayed results indicate that the mean Nusselt number rises with an increase in Γ-parameter and/or a decrease in ε_w-parameter. The plotted results indicate also that at certain ε_w and Γ values, *Nu* increases as *Pr* and/or *Re* increases.

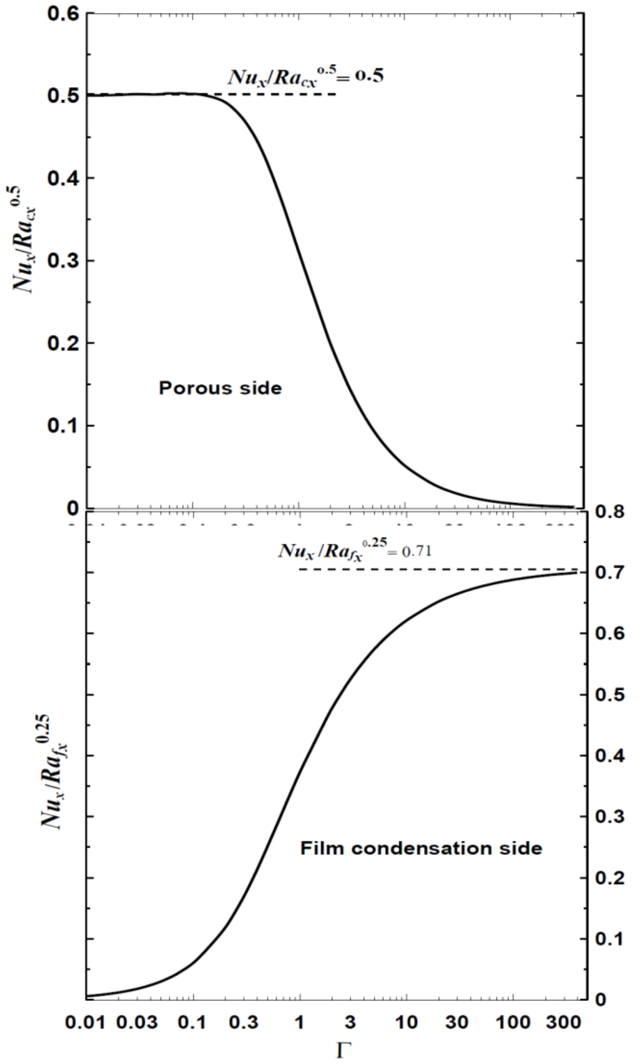


Fig. 4. Local Nusselt number at wall midheight as a function of Γ; for $\varepsilon_w \rightarrow 0$.

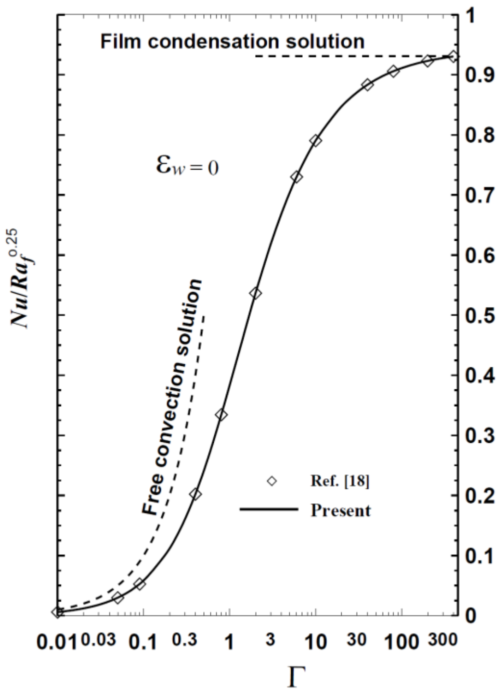


Fig. 5. Variation of mean Nusselt number with Γ-parameter; for $\varepsilon_w \rightarrow 0$.

4. Model Validation

In this context, the validity of the proposed model is discussed. In the above subsection 3.1, it has been shown that that for the case of negligible wall resistance of $\varepsilon_w \rightarrow 0$, the model gives as special solutions, the well-known analytical results (37) & (39) of laminar porous natural convection and laminar film condensation on isothermal vertical surfaces, respectively. In addition, it has been proved that the model verifies the previous Mosaad' model [18] of negligible wall resistance (cf., Figs. 3 & 5). However, for the $\varepsilon_w > 0$ case of considerable wall resistance, unfortunately, there is no experimental data available in the literature to be used for doing more comparisons with to prove of the solution validity in this ε_w range. An acceptable alternative way to test the model validity for this limit of ε_w -parameter is to construct a special model problem, which is solved numerically by employing the well-known Fluent software (V. 14.5). Hence, the Fluent results can be compared with the model predictions to check its validity. For this purpose, a special model problem has been constructed according to Fig. 1. In this problem, the separating

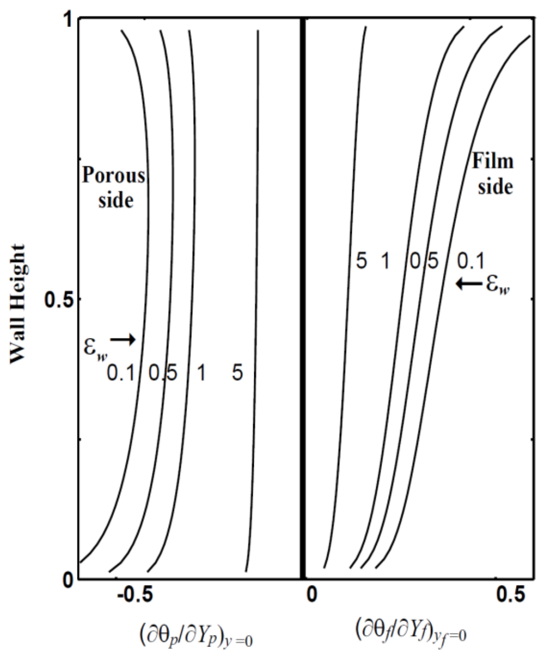


Fig. 6. Fluid temperature gradient at wall for different ε_w and Γ= 1.

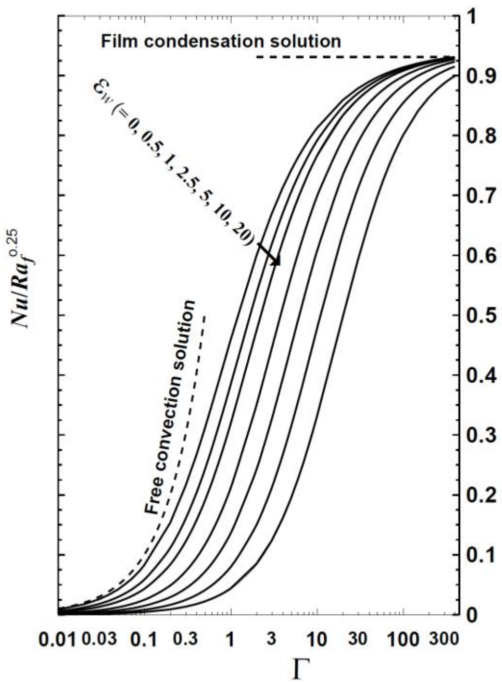


Fig. 7. Variation of mean Nusselt number with ε_w and Γ parameters; for $\varepsilon_w > 0$.

solid wall is assumed of 0.1-mm thickness and 0.3-m length, which is made from a stainless steel of thermal conductivity k = 16.2 W/ (m ^{o}C). The vapour medium is assumed steam of a saturation temperature T_s=100 ^{o}C. The porous medium is considered water-saturated fiberglass (CoFAB-A0108 of permeability K= 0.12x10^{-6} m^2, porosity=0.6 and thermal conductivity k = 0.035 W/ (m ^{o}C)). The numerical Fluent solution of this problem has been calculated for different free porous-medium temperature $T_{c\infty}$ = 10, 30, 50, 70, and 90 ^{o}C. The corresponding values of Γ-parameter (calculated by using Eq. (28)) are 0.180, 0.991, 1.232, 2.044 and 2.644, while those of ε_w-parameter are 0.075, 0.312, 0.342, 0.52 and 0.633, respectively. These values of Γ and ε_w are used to calculate the corresponding model predictions. Here, it is important to point out that the same simplifications applied in the present model were also been applied in the Fluent solution. As conclusion of the Fluent solution procedure applied, the main governing equations of mass, momentum and energy in both convection boundary layer and film condensate were solved numerically by using a control-volume discretization scheme. Discretizing the energy and momentum equations was done by using a second-order upwind scheme. The discretized equations were solved simultaneously by adapting a segregated solver. Under-relaxation factors were used to adjust the solution convergence. A rectangular cell with consecutive ratio of 1.023 was used in the two fluid domains, while equally spaced nodes were used in the solid plate. As a convergence condition, the variation in the temperature and velocity in all solid and fluid grids was limited to be less than 10−6. The comparison between the model and Fluent solutions is presented in Fig. 8. This comparison shows a reasonable agreement between the two solutions. The absolute relative discrepancy between the two results is about 5% at Γ=0.180, which reduces to be less than 2% as F increases 2.6. This deviation may attributed to numerical calculating errors.

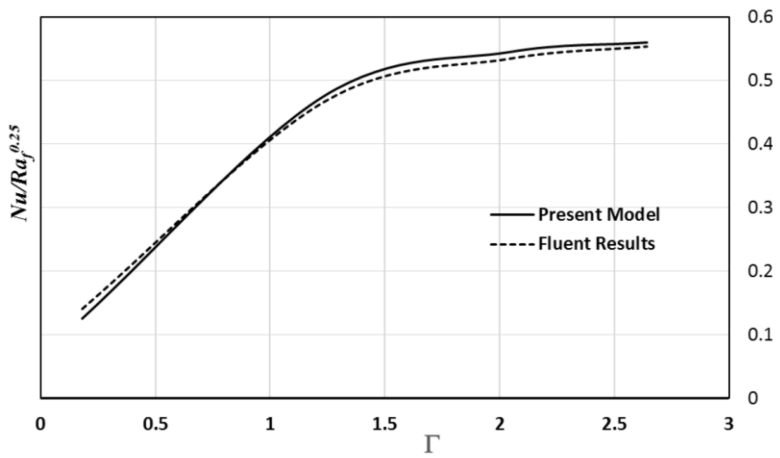


Fig. 8. Comparing model prediction with Fluent solution

5. Conclusions

A semi-analytical conjugate model has been developed for the problem of steady laminar film condensation on a vertical conductive wall with the backside cooled by free convection in a porous medium. In the analysis, the analytic Oseen technique has been used for solving the free convection layer, while the film condensation has been analyzed based on the boundary layer theory and Nusselt-analysis approximations. The analysis proved that the thermal interaction process is controlled mainly by two dimensionless variables Γ and ε_w. The variable Γ represents the thermal resistance ratio of film condensation to free convection layer. While ε_w-variable gives a measure of the thermal resistance ratio of the solid wall to free convection layer. Asymptotic analytical solutions have been derived for the special problem case of negligible wall resistance, which proves the model validity. Numerical solution to the main governing differential equations has also been obtained by using the fourth-order Runge-Kutta technique. Mean Nusselt number results have been obtained for the variables range: $0 \le \varepsilon_w \le 20$ & $0.01 \le \Gamma < 300$. The obtained results show that the mean *Nu* number rises with an increase in Γ-parameter and/or a decrease in wall parameter ε_w. The present model verifies the previous simple model [18] of negligible wall resistance. Furthermore, for the case of wall resistance $\varepsilon_w > 0$, comparing the model predictions with the Fluent results shows a satisfactory agreement.

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Nomenclature

g	gravitational acceleration, [ms^{-2}]
l_c	scale of convection layer thickness (cf. Eq. (21)), [-]
l_f	scale of condensate film thickness (cf. Eq. (5)), [-]
H	wall height, [m]
Ja	Jacob number (= $Cp_f(T_s - T_{c\infty})/h_{fg}$), [-]

k thermal conductivity, [$Wm^{-1}K^{-1}$]

K permeability of porous medium, [-]

Nu mean overall Nusselt number (cf. Eq. (35)), [-]

Nu_x local Nusselt number (cf. Eq. (33)), [-]

Pr Prandtl number (= ν / α) , [-]

Ra_c Rayleigh number of porous natural convection (cf. Eq. (22)), [-]

Ra_f Rayleigh number of condensate film (cf. Eq. (6)), [-]

T temperature, [K]

$T_{c\infty}$ free temperature of cold porous medium, [K]

T_s saturation temperature, [K]

u v dimensional velocity components, [ms^{-1}]

U ,V dimensionless velocity components (cf. Eqs. (4, 20)), [-]

x, y dimensional vertical and horizontal coordinates, [m]

X,Y dimensionless vertical and horizontal coordinates (cf. Eq. (4, 20)), [-]

w wall thickness, [m]

Greek letters

Δ_f dimensionless thickness of condensate film, [-]

α thermal diffusivity, [m^2s^{-1}]

β thermal expansion coefficient, [K^{-1}]

Γ thermal resistance ratio of condensate film to convection layer (cf. Eq. (28)), [-]

η inverse Oseen function (cf. Eq. (24)), [m]

ν kinematic viscosity, [m^2s^{-1}]

θ dimensionless temperature (cf. Eq. (28)), [-]

δ_f dimensional thickness of condensate film, [m]

ε_w thermal resistance ratio of wall to convection layer (cf. Eq. (28)), [-]

Subscripts

c convection

f condensate film

w wall

wc wall side facing convection layer

wf wall side facing condensate film.