NUMERICAL STUDY OF UNSTEADY AXISYMMETRIC FLOW AND HEAT TRANSFER IN CARREAU FLUID PAST A STRETCHED SURFACE

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Abstract: In this article, we present the numerical results for the unsteady axisymmetric flow and heat transfer of Carreau fluid induced by time dependent permeable radially stretching surface. Numerical results are demonstrated for both shear thinning and shear thickening fluids. The time dependent non-linear partial differential equations of the considered problem are reduced into non-linear ordinary differential equations with the aid of suitable transformations. An effective numerical technique namely bvp4c function in MATLAB is employed to construct the numerical solutions of the transformed non-linear ordinary differential equations for the velocity and temperature fields. Numerical computations of the local skin-friction coefficient and local Nusselt number are tabulated for steady and unsteady flows of shear thinning fluid as well as shear thickening fluid. It is worth mentioning that the magnitude of the skin friction coefficient and local Nusselt number for the steady flow is less than that for unsteady flow .

Keywords: Unsteady axisymmetric flow, Carreau fluid, heat transfer, numerical solutions.

Introduction

It is renowned fact that the study involving stretching surface flow is a subject of great interest for researchers due to its vast practical applications in industrial and manufacturing processes. These applications include design of extrusion of sheet materials, glass blowing, paper production, annealing of copper wires, purification of crude oil and so forth. The kinematics of stretching and simultaneously cooling/heating have great impact on the quality of final production which depends on the rate of heat transfer from the surface. Sakiadis [1] seems to be the first amongst the researchers to initiate the work on boundary layer flow over a solid surface and modeled boundary layer equations of two-dimensional axisymmetric flow. A large number of investigations have been made by the authors on this problem from different point of view. However, a literature survey reveals that less attention has been paid regarding the axisymmetric flows induced by radially stretching surface. Ariel [2] studied the problem of axisymmetric flow induced by a radially stretching surface and computed the exact, numerical, perturbation and asymptotic solutions of the problem. Again, Ariel [3] reported the impact of the partial slip on axisymmetric flow past a radially stretching sheet and found the exact and numerical solutions. Martins *et al.* [4] examined the influence of inertia and shear thinning on the axisymmetric flows of Carreau fluid by means of Galerkin least square method and they compared their numerical results with the existing literature and found to be in excellent agreement. Rashidi *et al.* [5] carried out the analytical and numerical studies of transient two-dimensional and axisymmetric squeezing flows. Sajid *et al.* [6] studied the problem of unsteady axisymmetric flow past a radially stretching surface and computed the series solutions by employing the homotopy analysis method. Sahoo [7] analyzed the impact of partial slip on axisymmetric flow of viscoelastic fluid over a radially stretched surface by using the finite difference method and obtained by Broyden's method. He also compared obtained numerical solutions with the solutions of Ariel [2] and found to be in outstanding agreement.

On the other hand, the study of non-Newtonian fluids is of paramount relevance in many fields of engineering and industrial applications. These fluids are encountered in many practical applications such as plastic sheet formation, paper production, oil recovery, petroleum drilling glass blowing and food processing. Newton's law of viscosity does not hold for such class of fluids and their rheological characteristics cannot be discussed in a comprehensive way by one constitutive relationship and they have nonlinear relationship between the shear stress and shear rate. The generalized Newtonian fluids are those in which viscosity of the fluid depends on the shear rate. The power-law, Bingham, Sisko, Cross, Ellis and Carreau models are the generalized Newtonian fluids. Carreau model is an important class of generalized Newtonian fluids proposed by Carreau [8]. Different simulations involving the flow of Carreau fluid have been made by the authors around spheres, stretching sheet, cylinder, pipes, cavities, and channels. This model possesses the ability of characterizing the rheology of different polymeric solutions such as 1% methylcellulose lylose in glycerol solution. The study of axisymmetric flow of Sisko fluid over a radially stretched surface was carried out by Khan and Shahzad [9]. They obtained the series solution of the considered problem by using analytical technique namely, homotopy analysis method. Additionally, they computed the exact solutions in special cases for the power law index $n=0$ and $n=1$. Makinde *et al.* [10] analyzed the impact of variable viscosity and thermal radiation on nanofluid flow over a stretched surface in the presence of convective boundary condition. They noticed that the heat transfer rate depreciates with nanofluids and viscosity parameters. Pantokratoras [11] conducted the comparative study of Blasius and Sakiadis flows of Carreau fluid. Fetecau *et al.* [12] presented the exact solutions for unidirectional flow of rate type fluids. Khan *et al.* [13] demonstrated a numerical study to examine the melting heat transfer in Carreau nanofluid flow over a wedge in the presence of heat generation/absorption. They observed that temperature and nanoparticles concentration profiles reduce for improving values of melting parameters. Singh and Makinde [14] considered the problem of mixed convection flow over a moving plate in the presence of partial slip. Recently, Ahmad *et al.* [15] carried out the analytical and numerical study of unsteady axisymmetric flow of power law fluid over a radially stretching surface. They found that both the analytical and numerical results are in good agreement. The study of stagnation point flow over a radially stretching heated surface was conducted by Shateyi and Makinde [16]. They noticed that the heat transfer rate increases for increasing values of Biot number. Vieru *et al.* [17] examined the effects of Newtonian heating and mass diffusion on free convection flow near the vertical plate. Makinde [18] presented computational modelling for nanofluid flow over an unsteady stretching surface in the presence of convective condition. The study of heat and mass transfer analysis in nanofluid flow over a convectively heated surface was conducted by Khan *et al.* [19]. They noted that suction depreciates the thermal and concentration boundary layer thickness. Recently, Khan and Azam [20] conducted a numerical study to examine the heat and mass transfer rates in Carreau nanofluid flow in the presence of magnetic field. They concluded that the local Nusselt and Sherwood numbers are decreasing functions of the thermophoresis parameters.

Motivated by the above literature and applications, the main theme of the current investigation is the numerical study of unsteady axisymmetric boundary layer flow and heat transfer characteristics of Carreau fluid induced by time dependent radially stretching surface. The non-linear time dependent partial differential equations regarding Carreau rheological model are reduced to non-linear ordinary differential equations by employing the suitable local similarity transformations. Then bvp4c routine in MATLAB is adopted to construct the numerical solutions of the considered problem.

Mathematical formulation

Let us consider the unsteady axisymmetric two-dimensional flow of an incompressible Carreau fluid induced by a time dependent permeable radially stretching sheet which is stretched in the radial direction with stretching velocity U_w proportional to the distance r from the origin. The sheet is coinciding with the plane $z=0$ and the flow occurs in the upper half plane $z>0$. For mathematical description, we consider the cylindrical polar coordinate system (r,θ,z). In view of the rotational symmetry, all the physical quantities are independent of θ. Therefore the azimuthal component of velocity vanishes. It is also assumed that the temperature of the sheet is $T_w(r,t)$ and considered to be higher than the ambient temperature T_∞ $(T_w > T_\infty)$.

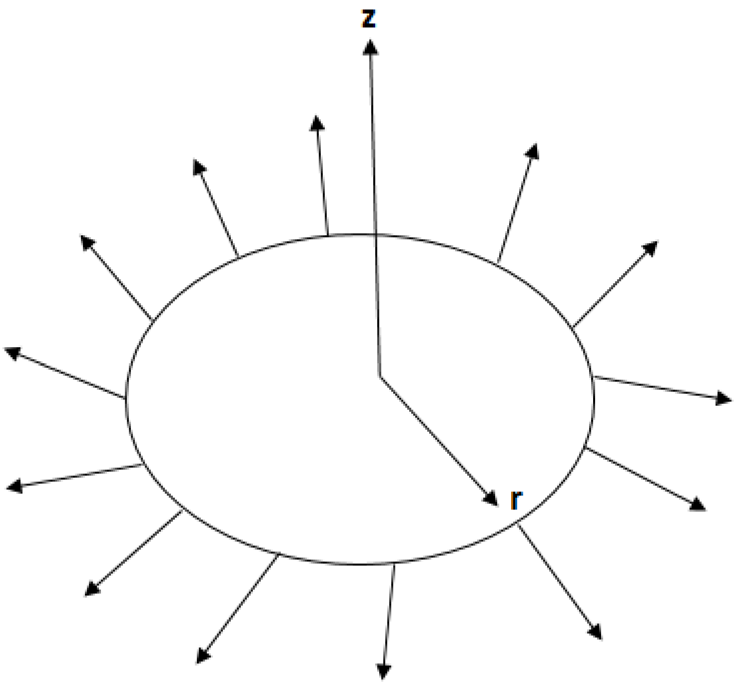

Fig. 1: Geometry of the problem.

The constitutive equation for a Carreau model is given by [21]

$$\mu = \mu_\infty + (\mu_0 - \mu_\infty)\left[1+\left(\Gamma \dot{\gamma}\right)^2\right]^{\frac{n-1}{2}}, \tag{1}$$

where μ the apparent viscosity, μ_0 and μ_∞ are the zero shear and infinite shear rate viscosities, respectively, n the power law index which corresponds to shear thinning for $(0<n<1)$, shear thickening for $(n>1)$ and Newtonian for $n=1$, $\Gamma=0$ the material time constant also known as relaxation time and $\dot{\gamma}$ rate of strain tensor. In the current formulation, we assumed that $\mu_\infty = 0$. Thus, the constitutive equation takes the form

$$\mu = \mu_0 \left[1 + \left(\Gamma \dot{\gamma} \right)^2 \right]^{\frac{n-1}{2}}. \qquad (2)$$

The velocity and temperature fields for the unsteady two-dimensional axisymmetric flow are assumed to be of the form

$$\mathbf{V} = [u(r,z,t),\ 0,\ w(r,z,t)], \qquad T = T(r,z,t). \qquad (3)$$

Under the aforesaid assumptions, the governing boundary layer equations for the axisymmetric unsteady flow along with the boundary conditions take the form

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \qquad (4)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = \nu \frac{\partial^2 u}{\partial z^2}\left[1 + \Gamma^2\left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{n-1}{2}} + \nu(n-1)\Gamma^2 \frac{\partial^2 u}{\partial z^2}\left(\frac{\partial u}{\partial z}\right)^2\left[1 + \Gamma^2\left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{n-3}{2}}, \qquad (5)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial z^2} + \frac{\mu_0}{\rho c_p}\left(\frac{\partial u}{\partial z}\right)^2\left[1 + \Gamma^2\left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{n-1}{2}}, \qquad (6)$$

$$u = U_w(r,t), \quad w = f_w(t), \qquad T = T_w(r,t) \qquad at \quad z = 0, \qquad (7)$$

$$u \to 0, \qquad T \to T_\infty \qquad as \quad z \to \infty, \qquad (8)$$

where u and w denote the velocity components along r and z directions, respectively, $t,\ \nu,\ \rho,\ k,\ c_p$ are the time, kinematic viscosity, fluid density, thermal conductivity of the fluid and the specific heat, respectively.

We assumed that the stretching velocity $U_w(r,t)$, surface temperature $T_w(r,t)$ and mass fluid velocity $f_w(t)$ are of the following form:

$$U_w(r,t) = \frac{cr}{1-\alpha t}, \quad T_w(r,t) = T_\infty + \frac{br}{1-\alpha t}, \quad f_w(t) = \frac{-W_0}{(1-\alpha t)^{\frac{1}{2}}}, \qquad (9)$$

where $\alpha t < 1$ with α and c are positive constants having dimensions reciprocal of time, W_0 is a uniform suction/injection velocity ($W_0 > 0$ for suction and $W_0 < 0$ for injection). The particular form for the mass fluid velocity $f_w(t)$, surface temperature $T_w(r,t)$ and stretching velocity $U_w(r,t)$ are chosen to employ the following suitable transformation:

$$\eta = \frac{z}{r}\mathrm{Re}^{1/2}, \qquad \Psi(r,z,t) = -r^2 U_w \mathrm{Re}^{-1/2} f(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \qquad (10)$$

where Ψ is the Stokes stream function having the property $u=-\frac{1}{r}\frac{\partial\Psi}{\partial z}, w=\frac{1}{r}\frac{\partial\Psi}{\partial r}$, θ the dimensionless temperature, $\mathrm{Re}\left(=\frac{rU_w}{\nu}\right)$ the local Reynolds number and η the independent variable, respectively. Thus the velocity components are

$$u=U_w f'(\eta), \qquad w=-2U_w \mathrm{Re}^{-1/2} f(\eta). \qquad (11)$$

In view of the above transformations, the governing equations (5) and (6) along with the boundary conditions (7) and (8) are reduced to the following non-dimensional form

$$\left\{1+nWe^2(f'')^2\right\}\left\{1+We^2(f'')^2\right\}^{\frac{n-3}{2}} f'''+2ff''-(f')^2-A\left[f'+\frac{\eta}{2}f''\right]=0, \qquad (12)$$

$$\theta''+\Pr(2f\theta'-f'\theta)-\Pr\frac{A}{2}(\eta\theta'+2\theta)+Ec\Pr(f'')^2\left\{1+We^2(f'')^2\right\}^{\frac{n-1}{2}}=0, \qquad (13)$$

$$f(0)=S, \quad f'(0)=1, \quad \theta(0)=1, \qquad (14)$$

$$f'(\infty)\to 0, \quad \theta(\infty)\to 0, \qquad (15)$$

where prime denotes differentiation with respect to η, $We\left(=\sqrt{\frac{c^3\Gamma^2r^2}{\nu(1-\alpha t)^3}}\right)$ the local Weissenberg number, $\Pr\left(=\frac{\mu Cp}{k}\right)$ the Prandtl number, $A\left(=\frac{\alpha}{c}\right)$ the dimensionless parameter which measures the unsteadiness and $S\left(=\frac{W_0}{2\sqrt{\nu c}}\right)$ the constant mass transfer parameter with $S>0$ for suction and $S<0$ for injection and $Ec\left(=\frac{cU_w}{bC_p}\right)$ the Eckert number.

The physical quantities of prime engineering interest are the local skin friction coefficient C_f and the local Nusselt number Nu which are given by

$$C_f=\frac{\tau_w}{\rho U_w^2}, \qquad Nu=\frac{rq_w}{k(T_w-T_\infty)}, \qquad (16)$$

where τ_w and q_w are the wall shear stress and wall heat flux, respectively, having the following expressions

$$\tau_w=\mu_0\frac{\partial u}{\partial z}\left[1+\Gamma^2\left(\frac{\partial u}{\partial z}\right)^2\right]^{\frac{n-1}{2}}, \qquad q_w=-k\left(\frac{\partial T}{\partial z}\right)_{z=0}. \qquad (17)$$

Consequently, in view of Eqs. (10) and (16), Eq. (17) takes the dimensionless form as

$$\mathrm{Re}^{1/2}C_f=f''(0)[1+We^2(f''(0))^2]^{\frac{n-1}{2}}, \qquad \mathrm{Re}^{-1/2}Nu=-\theta'(0). \qquad (18)$$

Numerical procedure

The current numerical computations for solving the boundary value problem usually require a guess for the solution. Boundary value problems are much difficult to handle than initial value problems and any solver may fail even with good guesses. The bvp4c function is an effective solver of the boundary value problems which employs a collection method for determining the solution of boundary value problems of the form $y' = f(x, y, \mathbf{p}), a \le x \le b$ with boundary conditions $g(y(a), y(b), \mathbf{p}) = 0$ where $\mathbf{P}$ denotes a vector of unknown parameters. The transformed non-linear ordinary differential equations (12) and (13) along with boundary conditions (14) and (15) are solved numerically by employing the MATLAB boundary value problems solver bvp4c function. For this, we convert the given ordinary differential equations as a system of first order ordinary differential equations. As a result, the non-linear ordinary differential equations (12) and (13) are reduced to a system of first order ordinary differential equations as follows:

$$f = u_1, \quad f' = u_2, \quad f'' = u_{3,} \quad f''' = \frac{[A(u_2 + \frac{\eta}{2}u_3) + u_2^2 - 2u_1u_3]}{[1 + nW_e^2u_3^2][1 + W_e^2u_3^2]^{(\frac{n-3}{2})}}, \tag{19}$$

$$\theta = u_4, \quad \theta' = u_{5,} \quad \theta'' = \Pr\frac{A}{2}(\eta u_5 + 2u_4) - \Pr(2u_1u_5 - u_2u_4) - \Pr EcW_e^2u_3^2[1 + W_e^2u_3^2]^{(\frac{n-1}{2})}. \tag{20}$$

A comprehensive discussion of this method is explained in the reference [22].

Results and discussion

The main focus of present investigation is the study of the unsteady axisymmetric boundary layer flow and heat transfer characteristics of Carreau fluid over a permeable time dependent radially stretching sheet. The system of non-linear ordinary differential equations (10) and (11) subject to the boundary conditions (12) and (13) are solved numerically by an effective numerical technique namely bvp4c function in MATLAB. The comprehensive numerical results are computed for different values of the physical pertinent parameters of flow and heat transfer namely, unsteadiness parameter A, power law index n, Prandtl number Pr, local Weissenberg number We, mass transfer parameter S and Eckert number Ec. The influence of these physical parameters on the velocity and temperature fields is depicted graphically with comprehensive discussions. The variations of the local skin-friction coefficient $\mathrm{Re}^{1/2} C_f$ and the local Nusselt number $\mathrm{Re}^{-1/2} Nu$ are illustrated in tabular form through tables 1 and 2 for various values of the pertinent parameters for both shear thinning $(0 < n < 1)$ and shear thickening $(n > 1)$ fluids in the steady $(A = 0)$ and unsteady $(A \ne 0)$ flows. Note that the transformations in Eq. (10) give rise to some physical parameters which are not independent from spatial/temporal variables. It means that the present model is only a local approximation.

Table **1** shows that the magnitude of local skin friction coefficient is an increasing function of the mass transfer parameter S in shear thinning $(0 < n < 1)$ and shear thickening $(n > 1)$ fluids for both the cases of steady flow $(A = 0)$ as well as unsteady flow $(A \ne 0)$. It is also noticed that the magnitude of local skin friction coefficient diminishes by increasing the values of local Weissenberg number We in shear thinning fluid both for steady and unsteady

flows but opposite trend has been noticed in shear thickening fluid. Furthermore, the magnitude of skin friction coefficient $\left|\mathrm{Re}^{1/2} C_f\right|$ for the steady flow $(A=0)$ is less than that for the unsteady flow $(A \neq 0)$. Table **2** indicates that the Prandtl number Pr and mass transfer parameter S enhance the local Nusselt number $\mathrm{Re}^{-1/2} Nu$ in shear thinning $(0<n<1)$ and shear thickening $(n>1)$ fluids for both the cases of steady flow $(A=0)$ as well as unsteady flow $(A \neq 0)$. However, on incrementing the values of the local Weissenberg number We, the local Nusselt number $\mathrm{Re}^{-1/2} Nu$ diminishes in shear thinning fluid but enhances in shear thickening fluid for both the cases of steady flow $(A=0)$ as well as unsteady flow $(A \neq 0)$. Additionally, it is noted that the local Nusselt number $\mathrm{Re}^{-1/2} Nu$ for the steady flow $(A=0)$ is less than that for the unsteady $(A \neq 0)$ flow. It is also clear that the local Nusselt number is a decreasing function of the Eckert number Ec.
Figs. $2(a)$ and $2(b)$ depict the influence of unsteadiness parameter A on the velocity field $f'(\eta)$ and temperature field $\theta(\eta)$ for both the shear thinning $(0<n<1)$ and shear thickening $(n>1)$ fluids. It is noted that the velocity $f'(\eta)$ and temperature $\theta(\eta)$ are diminishing functions of unsteadiness parameter A for both cases. It is further observed that rising values of the unsteadiness parameter A depreciate the momentum boundary layer and thermal boundary layer thicknesses for both the shear thinning and shear thickening fluids. Moreover, it is observed that the momentum boundary layer thickness in case of shear thickening fluid is thicker in comparison with the shear thinning fluid. However, quite the opposite is true for the thermal boundary layer thickness. Figs. $3(a)$ and $3(b)$ are reported to illustrate the influence of mass transfer parameter S on the velocity field $f'(\eta)$ and the temperature field $\theta(\eta)$ for both the shear thinning and shear thickening fluids. It is evident from these Figs. that an increase in the mass transfer parameter S corresponds a decrease in the velocity field $f'(\eta)$ and temperature field $\theta(\eta)$ in shear thinning and shear thickening fluids. However, the momentum boundary layer and thermal boundary layer thicknesses decrease by uplifting the mass transfer parameter S in shear thinning and shear thickening fluids. In fact, resistance took place to the fluid flow and fluid velocity due to the suction. Figs. $4(a)$ and $4(b)$ portray that the enhancement in the value of power law index n improves the velocity profiles $f'(\eta)$ and depresses the temperature field $\theta(\eta)$. Physically, enhancement in the value of power law index n helps us to diminish the resistive force. It is also noticed that the momentum boundary layer thickness increases for large values of power law index n and quite the opposite is true for the thermal boundary layer thickness. The influence of the local Weissenberg number We on the velocity field $f'(\eta)$ and temperature field $\theta(\eta)$ is demonstrated in Figs. $5(a)$ and $5(b)$ for the case of shear thinning and shear thickening fluids. From these Figs., it can be seen that an elevation in the local Weissenberg number We is to boost the velocity profiles $f'(\eta)$ and temperature profiles $\theta(\eta)$ in shear thinning fluid while a quite opposite effects are noticed for shear thickening fluid. Further, the local Weissenberg number We has the tendency to enhance the momentum boundary layer thickness in shear thinning fluid. Fig. 6 reveals the impact of the Prandtl number Pr on temperature profiles of the flow for the shear thinning and shear thickening fluids. From this Fig., it is observed that inflation in the Prandtl number Pr lowers the temperature profiles and thermal boundary layer thickness for both the cases of shear thinning and shear thickening fluids. This is because of the fact that enhancement in the Prandtl number corresponds to low thermal conductivity and consequently diminishes the

conduction and the thermal boundary layer thickness. Fig. 7 represents the variation of temperature profiles for different values of Eckert number Ec. From this Fig., it can be seen that temperature is an enhancing function of the Eckert number Ec. Fig. 8 indicates the flow pattern of stream lines which are symmetric everywhere.

Table 1. Numerical computations of the local skin friction $\mathrm{Re}^{1/2} C_f$ for selected values of A, S, We and n.

$\mathrm{Re}^{1/2} C_f$					
Parameters		A = 0 (steady flow)		A = 0.2 (unsteady flow)	
S	We	n = 0.5	n = 1.5	n = 0.5	n = 1.5
0.2	2.0	-1.119129	-1.614836	-1.152785	-1.676683
0.4	-	-1.347501	-1.890583	-1.377471	-1.948162
0.6	-	-1.613982	-2.186952	-1.640031	-2.240451
0.8	-	-1.913899	-2.500332	-1.936121	-2.550076
0.2	2.0	-1.119129	-1.614836	-1.152785	-1.676683
-	4.0	-0.950219	-1.781116	-0.977609	-1.850448
-	6.0	-0.861721	-1.899110	-0.885892	-1.973780
-	8.0	-0.805068	-1.991355	-0.827130	-2.070245

Table 2. Numerical computations of the local Nusselt number $\mathrm{Re}^{-1/2} Nu$ for selected values of Pr, A, S, n and We.

$\mathrm{Re}^{-1/2} Nu$							
Parameters				A = 0 (steady flow)		A = 0.2 (unsteady flow)	
Pr	S	We	Ec	n = 0.5	n = 1.5	n = 0.5	n = 1.5
0.72	0.2	2.0	0.2	0.92432	1.06309	0.97643	1.10823
1.0				1.18143	1.33451	1.23418	1.38254
3.0				2.59035	2.76978	2.65366	2.83765
10.0				6.10549	6.28025	6.19358	6.38878
0.72	0.2	2.0	0.2	0.92432	1.06309	0.97643	1.10823
	0.4			1.05781	1.22960	1.10687	1.27112
	0.6			1.21218	1.41117	1.25748	1.44913
	0.8			1.38707	1.60545	1.42834	1.64011
0.72	0.2	2.0	0.2	0.92432	1.06309	0.97643	1.10823
		4.0		0.85435	1.09335	0.91293	1.13618
		6.0		0.80908	1.11055	0.87282	1.15204
		8.0		0.77587	1.12190	0.84381	1.16254
0.72	0.2	2.0	0.0	1.00350	1.16080	1.05616	1.20581
			1.0	0.60761	0.67225	0.65748	0.71791
			2.0	0.21173	0.18371	0.25879	0.23000
			3.0	0.18416	-0.30484	-0.13989	-0.25790

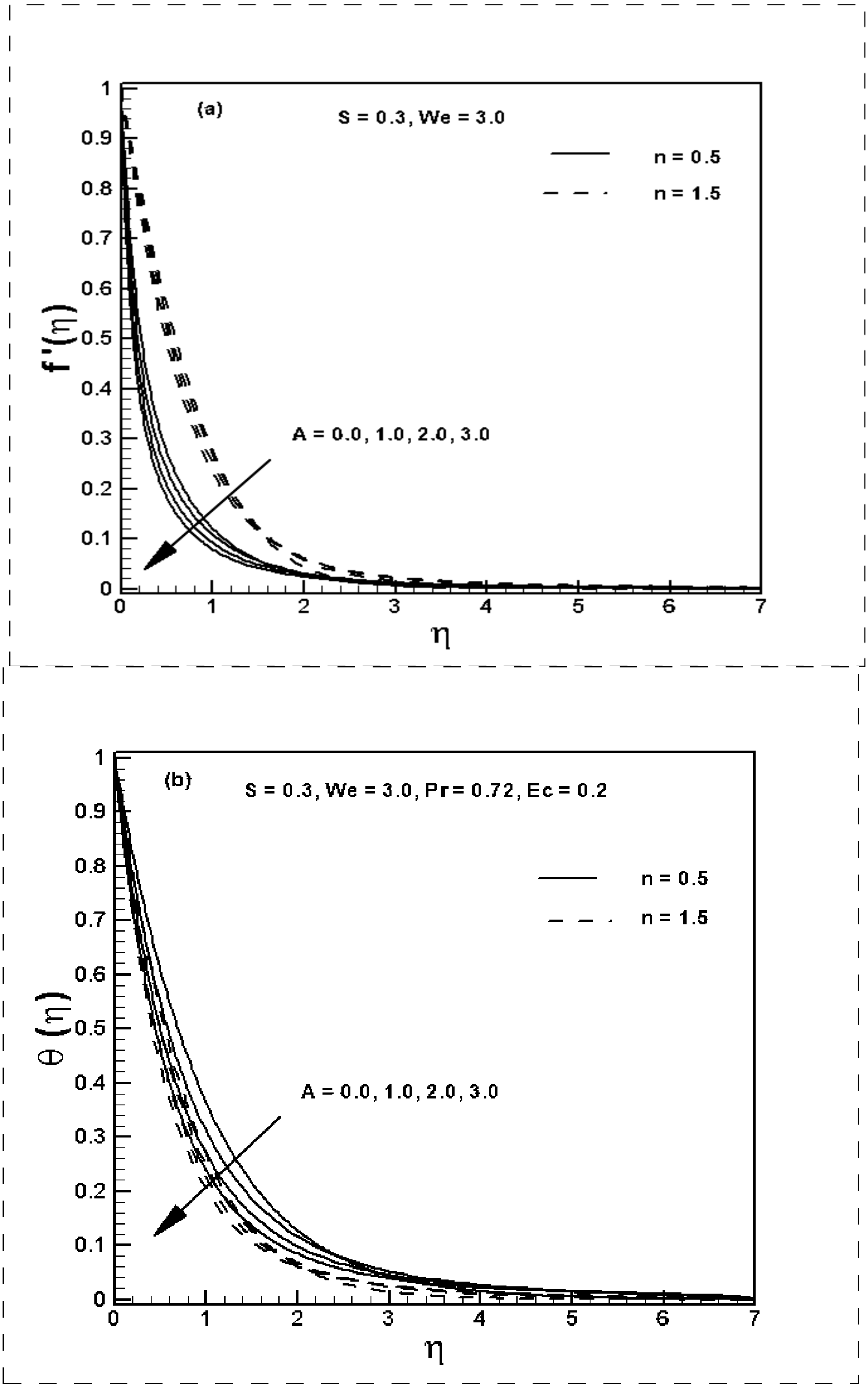

Fig. 2. Effects of the unsteadiness parameter A on velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles.

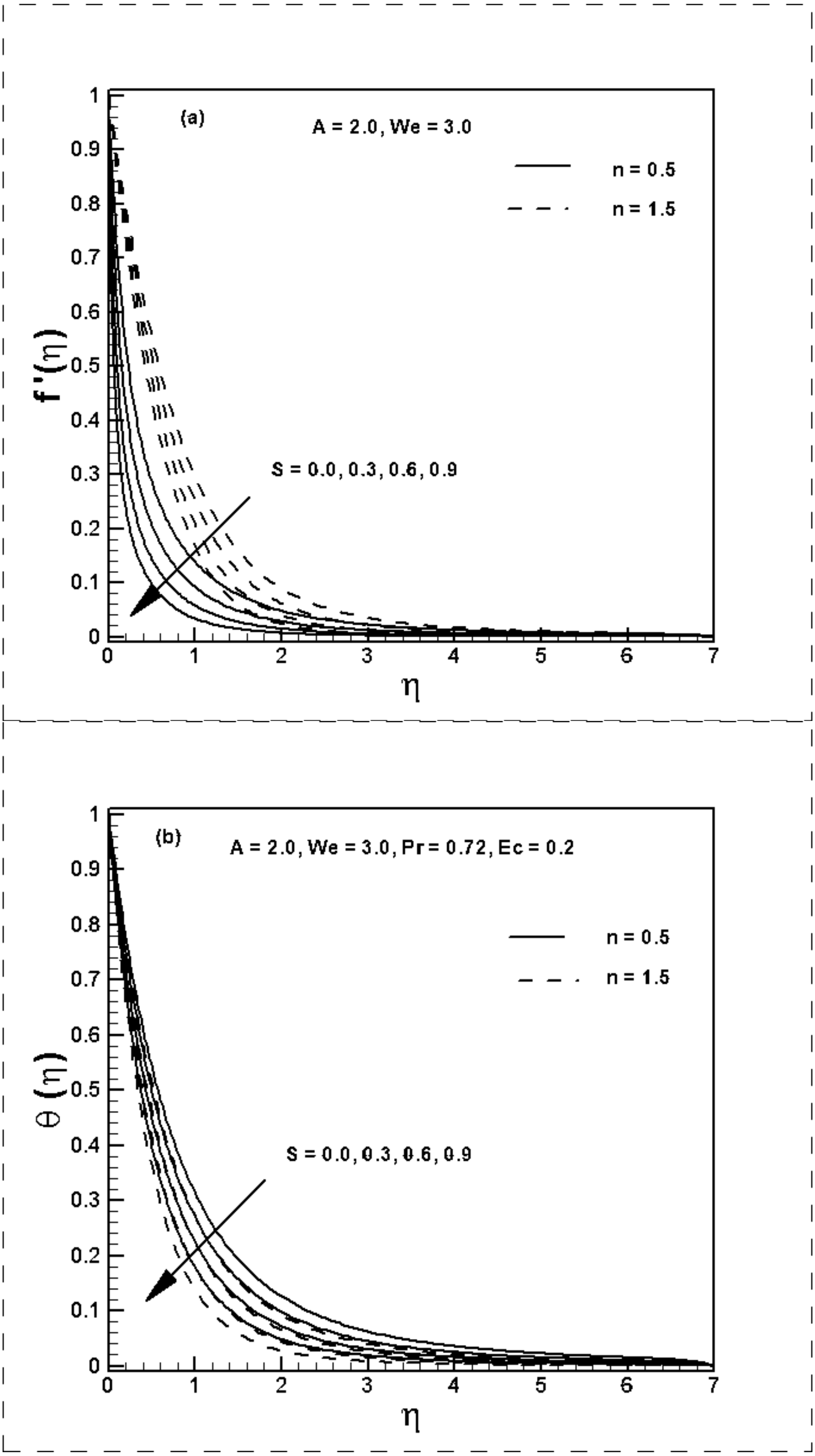

Fig. 3. Effects of the mass transfer parameter S on velocity $f^{\prime}(\eta)$ and temperature $\theta(\eta)$ profiles.

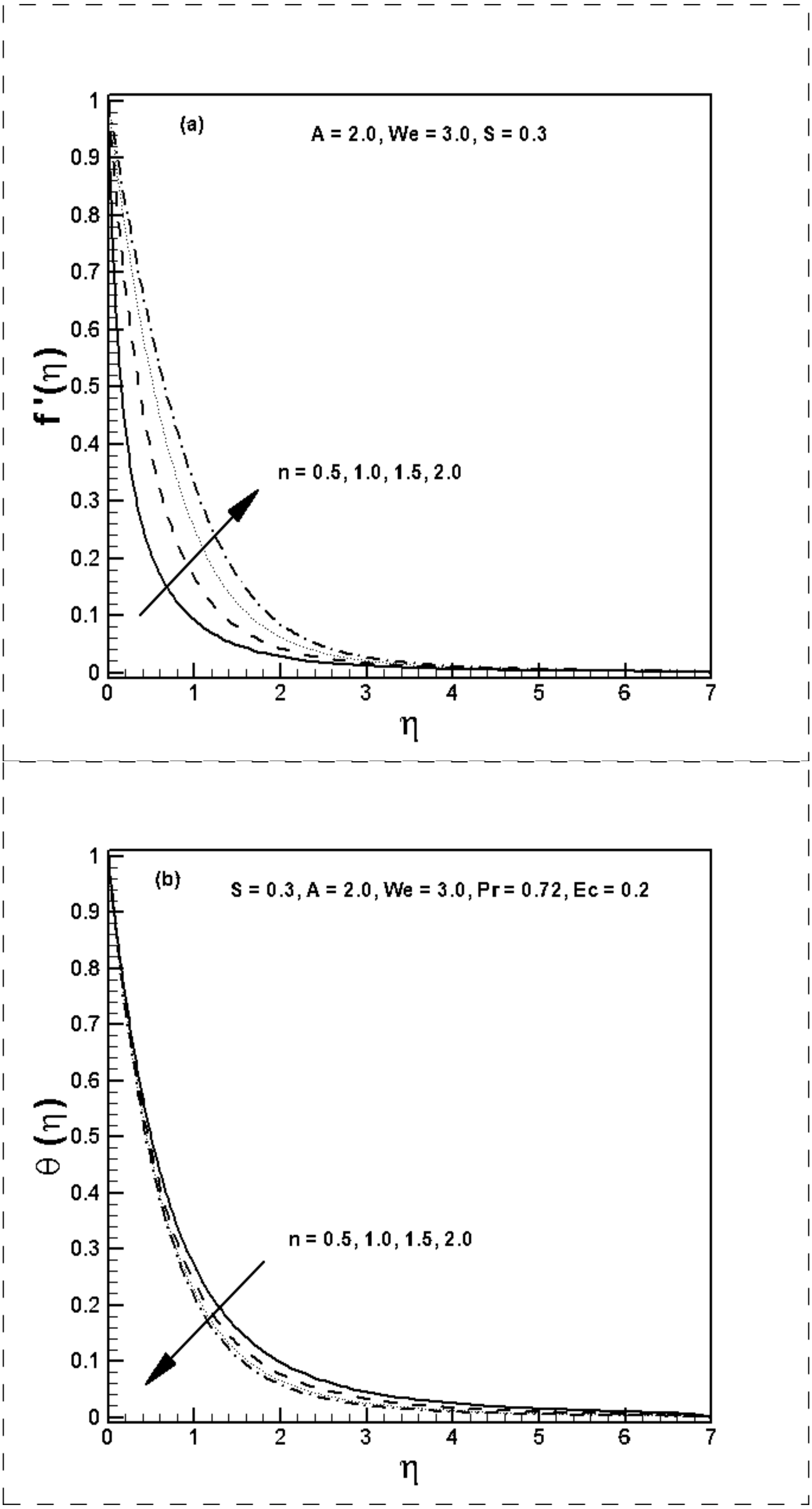

Fig. 4. Effects of the power law index n on velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles.

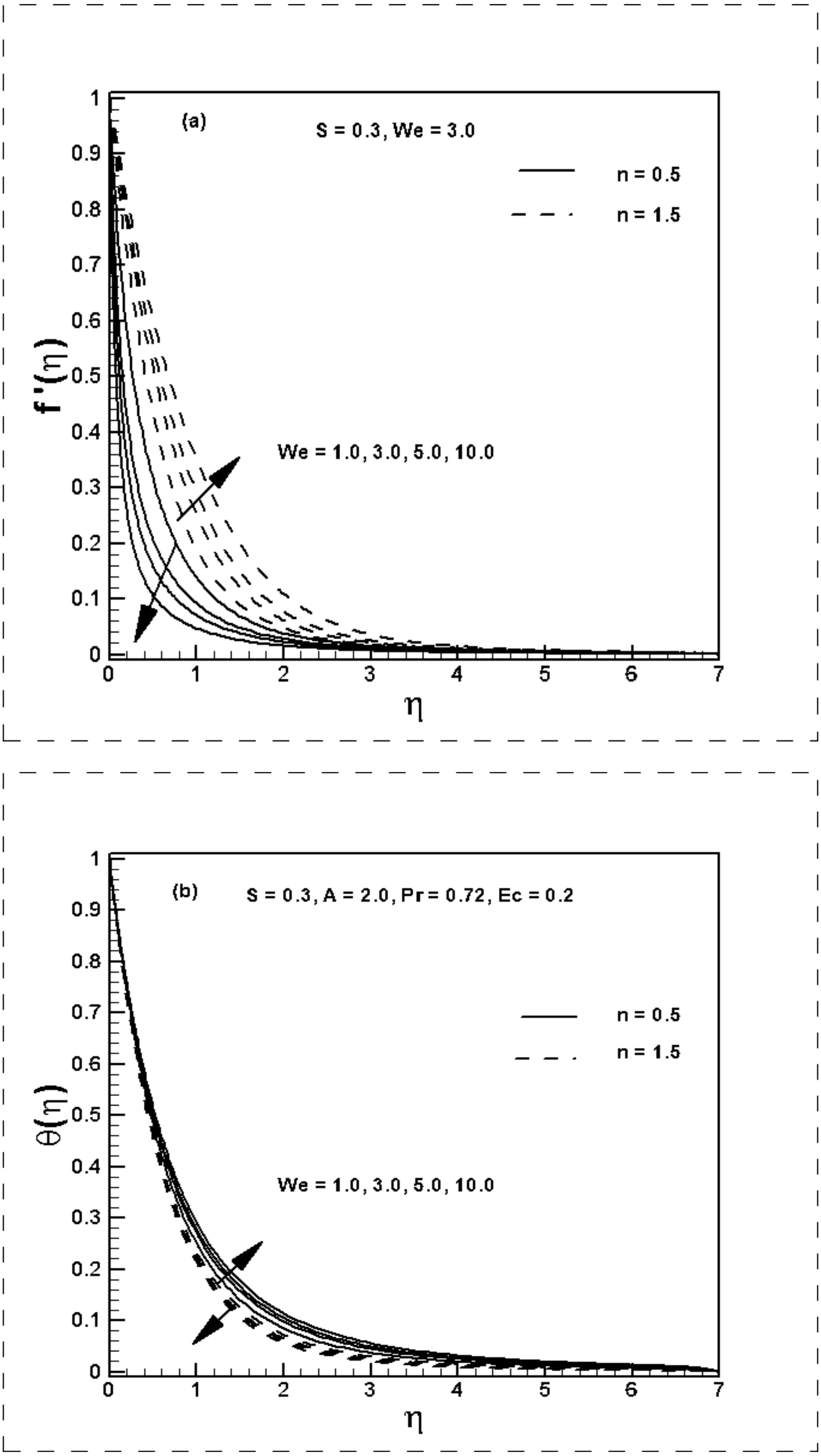

Fig. 5. Effects of the Weissenberg number *We* on velocity $f'(\eta)$ and temperature $\theta(\eta)$ profiles.

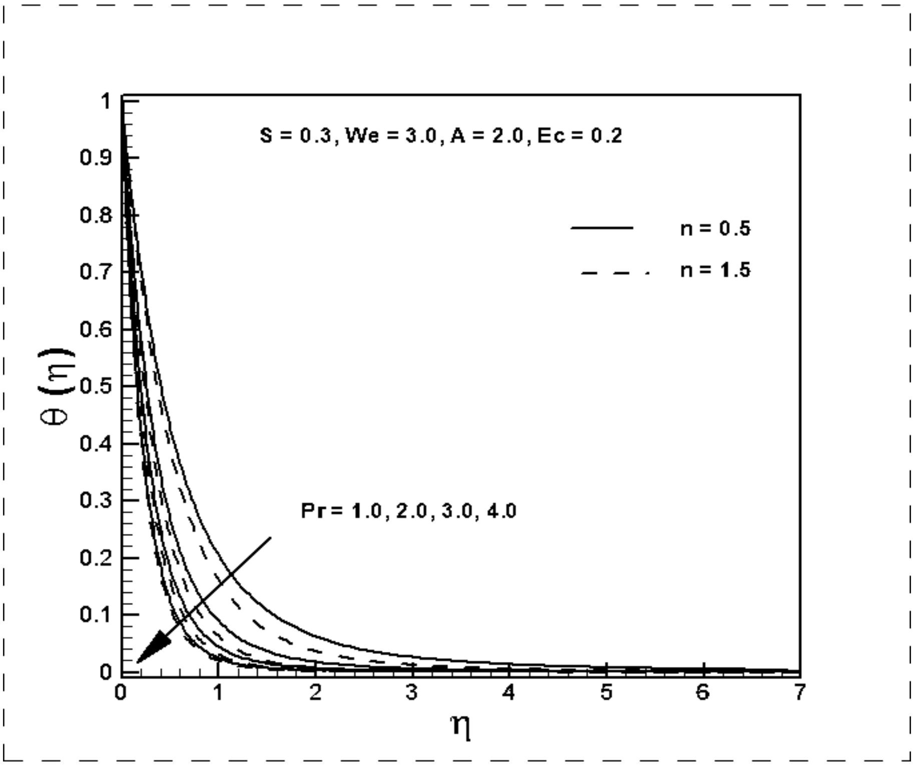

Fig. 6. Effects of the Prantdl number Pr on temperature $\theta(\eta)$ profiles.

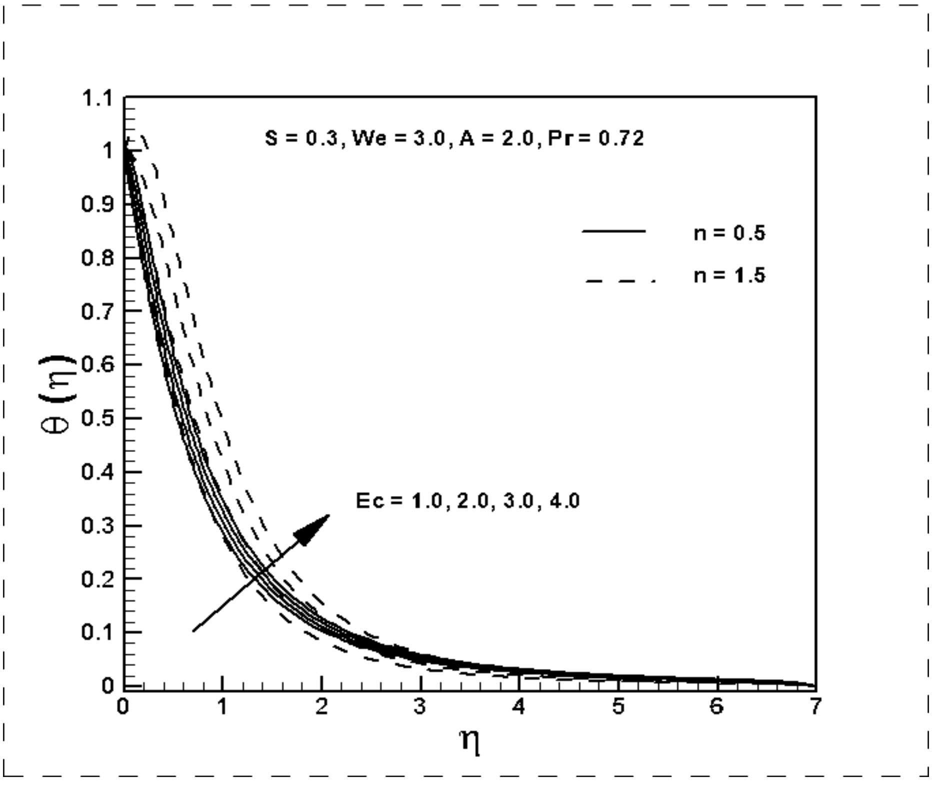

Fig. 7. Effects of the Eckert number Ec on temperature $\theta(\eta)$ profiles.

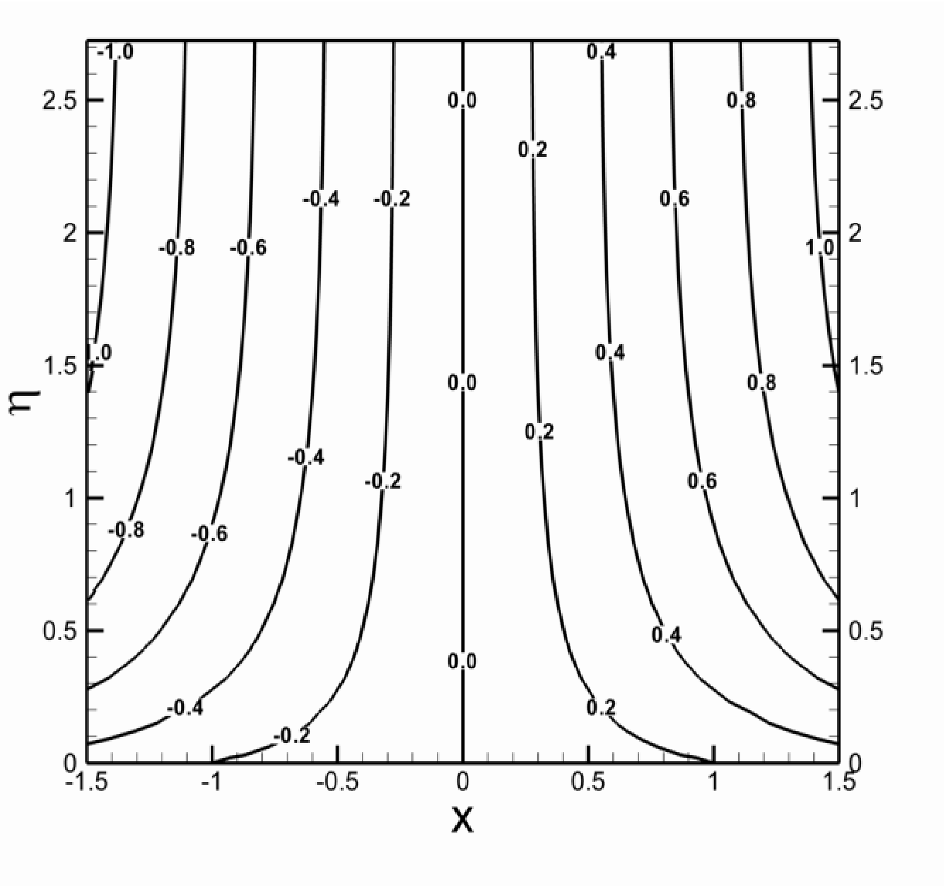

Fig. 8. Stream lines.

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Conclusions

In the present article, we constructed the numerical solutions for the problem of unsteady axisymmetric boundary layer flow and heat transfer of Carreau fluid past a time dependent permeable radially stretching sheet by employing the MATLAB routine bvp4c. Numerical results were computed for both shear thinning and shear thickening fluids. The significanc of pertinent parameters namely, the unsteadiness parameter A, mass transfer parameter S, power law index n, local Weissenberg number We and Prandtl number $\Pr$ was analyzed. The main observations of the present article are summarized as:

- The velocity profiles $f'(\eta)$ and temperature profiles $\theta(\eta)$ were diminished with an increment in the unsteadiness parameter A and mass transfer parameter S for both shear thinning $(0<n<1)$ and shear thickening $(n>1)$ fluids.
- Enhancement in the unsteadiness parameter A and the mass transfer parameter S depreciated the momentum boundary layer and thermal boundary layer thicknesses for both shear thinning $(0<n<1)$ and shear thickening $(n>1)$ fluids.
- Elevation in the local Weissenberg number We was to boost up the velocity profiles $f'(\eta)$ and momentum boundary layer thickness for the shear thinning fluid while a quite opposite effects were noticed for shear thickening fluid.
- Larger values of the power law index n improved the velocity profiles $f'(\eta)$ and momentum boundary layer thickness but depressed the temperature profiles $\theta(\eta)$.
- The magnitude of local skin friction coefficient $\left|\mathrm{Re}^{1/2} C_f\right|$ was increasing function of the mass transfer parameter S in shear thinning $(0<n<1)$ and shear thickening $(n>1)$ fluids for both the cases of steady flow $(A=0)$ as well as unsteady flow $(A\neq 0)$.
- The local Nusselt number $\mathrm{Re}^{-1/2} Nu$ and the magnitude of skin friction coefficient $\left|\mathrm{Re}^{1/2} C_f\right|$ for the steady flow $(A=0)$ were less than that for the unsteady flow $(A\neq 0)$.

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