THE MATHEMATICAL MODEL FOR TEMPERATURE CHANGE OF A JOURNAL BEARING

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In this work, a representative mathematical model has been developed, which reliably describes the heating and cooling of a journal bearing as a result of its malfunctioning, and the model has been further confirmed on a test bench. The bearing model was validated by using analytical modeling methods, i.e. the experimental results were compared to the data obtained by analytical calculations. The regression and variance analysis techniques were applied to process the recorded data, to test the mathematical model and to define mathematical functions for the heating/cooling of the journal bearing. This investigation shows that a representative model may reliably indicate the change in the thermal field, which may be a consequence of journal bearing damage.

Key words*: journal bearing, thermal effects, regression analysis, variance analysis*

1. Introduction

The accuracy and reliability of the technical systems and devices often depends on the functionality of their moving parts, of which journal bearings are certainly one of the most important elements. Plain bearings are widely used in heavy engineering with the mills, turbines, crushers, rolling mills and forging machines, presses, etc. The main advantage of journal bearings is reflected in their high load-carrying capacity and lifetime. Therefore, defects and failures of these vital elements may cause significant material losses. Accordingly, research into journal bearings malfunction are numerous and appear in the literature through different classifications. There are research into the causes, manifestations and corrective measures, research on the effect of lubricants, as well as diagnostic procedures and tests [1-4].

The main causes that lead to damage of journal bearings include many aspects of design, material selection, the imperfections in materials, production and processing, assembly, inspection, testing, storage, transport, maintenance, unexpected overload exposure, and direct mechanical or chemical damage during the bearing work [4]. Failures and damages are manifested most frequently as wear, fracture and plastic deformation of material.

One of important methods in the diagnosis and prediction of journal bearings malfunction is thermal analysis. Namely, the increase in bearing temperature may be a result of the friction in the bearing, its excessive load or high rotational speed. External heat sources and the bearing cooling system failure may also cause the temperature increase in the bearing. Accordingly, the bearing temperature is an indicator of the proper lubrication of the bearing, bearing damage, the adequacy of the load and rotation rate control, as well as the correctness of the cooling system for the bearing, or finally it may show on the presence of an unexpected external heat source [5-10]. For instance, it is known that, if the insufficient amount of lubricant is used or the lubricant's performance does not meet the requirements of the application, there is increased friction in the bearing due to which it is heated. The lubricant may contain various impurities, such as metal shavings, sand, water, etc., which in turn leads to the heating, caused by friction and furthermore damage and accelerated bearing wear [10]. In addition, the bearing heating may often come from the external source, and so it has to be forcedly cooled, by using circulating oil or by low ambient temperature in winter period.

To conclude, most of the bearing defects lead to the increased wear and consequently heating, so the increased temperature is usually a symptom of a bearing malfunction [11-14]. Of course, in the diagnosis the other potential causes of the increased temperature in bearing should be eliminated, such as the bearing overload [15, 16]. So, the compensation technique or other metrological methods should be applied in order to extract only the temperature change data related to the bearing malfunction. Certainly, any heating of the bearing above the set limit, regardless of its cause, is unacceptable.

The aim of this work was do develop mathematical models which will representatively describe the heating and the cooling process of the journal bearing, so that the models could be used in the future diagnostics of the bearing malfunction.

2. The mathematical model of the heating and cooling of the journal bearing

The following section offers mathematical functions which model the change in the bearing temperature during its operation.

The power loss in the bearing (*Pg*) due to the friction and the lubricant resistance can be approximately expressed by the formula:

$$Pg = Mg \cdot \omega \tag{1}$$

The formula is approximate because it is assumed that the friction resistance is linear, though in reality it has the characteristics of nonlinearity. If there is no flow of lubricant or forced cooling of the bearing, all the heat generated in the bearing is released into the environment through the bearing housing. The bearing angular velocity (angular frequency) is calculated using the formula:

$$\omega = \frac{\pi \cdot n}{30} \tag{2}$$

At the same time, the law that determines the heat taken from the bearing by the heat conduction process is:

$$Po = k \cdot Al \cdot (\theta_l - \theta_o) \tag{3}$$

In the absence of heat dissipation from the bearing via the lubricant or otherwise, and if the bearing is not heated by an external source, then the loss of power equals to the heat taken, as follows:

$$Po = Pg \tag{4}$$

Therefore

$$k \cdot Al \cdot (\theta_l - \theta_o) = Mg \cdot \omega \tag{5}$$

from which it is obtained that the increase in temperature is defined by:

$$\Delta\theta = \theta_l - \theta_o = \frac{1}{k \cdot Al} \cdot Mg \cdot \omega \tag{6}$$

At the same time, the torque loss is determined by:

$$Mg = r \cdot \eta \cdot P \tag{7}$$

Finally, the calculation of the bearing temperature increase is presented as:

$$\Delta\theta = \frac{r}{k \cdot Al} \cdot \eta \cdot P \cdot \omega \tag{8}$$

It can be concluded that the heating intensity of the bearing is determined by the friction coefficient, the rpm number and the bearing load. The heat accumulated in the bearing and the housing is determined by the formula:

$$Q = c \cdot m \cdot \Delta\theta \tag{9}$$

If there is a power loss of the bearing, for example due to the increased rpm, then the temperature of the bearing increases also and consequently an increase in the accumulated heat in the bearing occurs. And vice versa, the reduced power loss results in the cooling of the bearing. This dynamic process is determined by Fourier's law:

$$Pg = \frac{dQ}{dt} + Po = c \cdot m \cdot \frac{d\theta}{dt} + k \cdot Al \cdot \theta \tag{10}$$

When the time constant T replaces the relationship $c{\cdot}m/k{\cdot}Al$, then the Eq. (10) changes to:

$$\frac{Pg}{c \cdot m} = \frac{d\theta}{dt} + \frac{1}{T} \cdot \theta \tag{11}$$

It is important to note that the introduced heating time constant has higher value for the higher material mass and heat capacity, and smaller value for the lower heat conductivity coefficient and larger bearing surface area. The time constant *T* is expressed in time units, usually in seconds.

It is clear that the bearing heating or cooling process may be described by the differential equation of the first order. By solving this equation, with the condition for dynamic model of the first order (Eq. 11a)

$$\frac{d\theta}{dt} + \frac{1}{T} \cdot \theta = 0 \tag{11a}$$

the formula is obtained which models the cooling of the bearing, which may happen, for example, during the decline in rpm, according to the Newton's law of cooling [17]:

$$\theta(t) = \theta p \cdot \exp(-t / T) \tag{12}$$

The expression (12) may be corrected by introducing Θ_k, the final temperature measured when the steady-state is reached:

$$\theta(t) = (\theta p - \theta k) \cdot \exp(-t / T) + \theta k \tag{13}$$

On the contrary, when the heating process occurs, for example as a result of increased rpm, this process is described by the equation:

$$\theta(t) = (\theta k - \theta p) \cdot (1 - \exp(-t / T)) + \theta p \tag{14}$$

Figure 1. presents time diagrams of the cooling and heating of the journal bearing according to the equations (13) and (14).

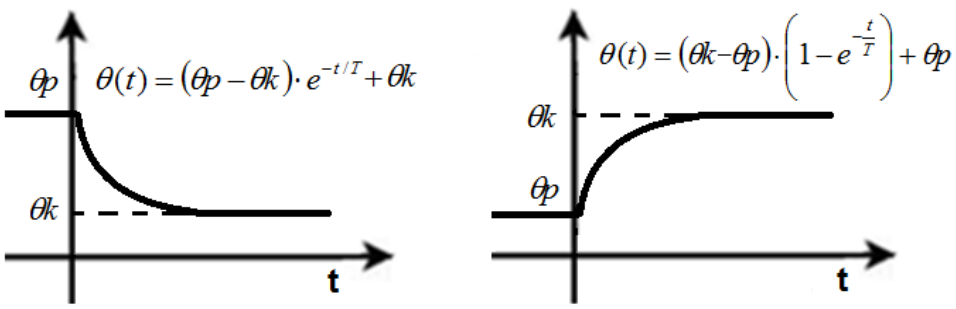


Figure 1. Time diagram of cooling (left) and heating (right) of the journal bearing

3. Experimental setup

In order to test the mathematical model presented in Section 2, the experimental part of this work was performed on a test bench, which contained a journal bearing, an electric drive motor and an rpm regulator, as shown in fig 2.

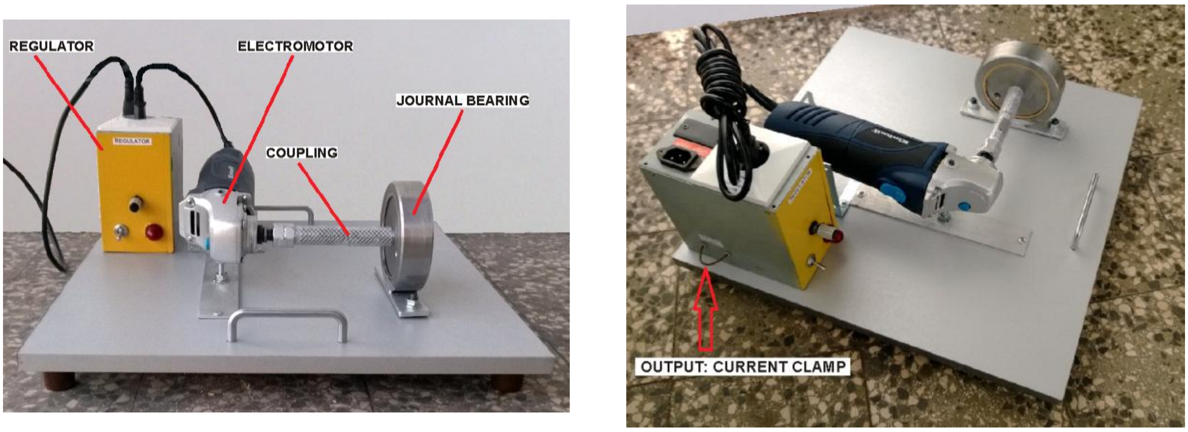


Figure 2. The test bench – front and side view [16]

A thyristor rpm regulator was used, produced by BOSCH, with the implemented technology of integrated circuits. The output power of the regulator is 1500W at 230 V voltage, so that it was compatible with the electromotor Einhell BT-AG 850, of 850W power. The schematic representation of the connection of the regulator and electromotor is given in fig. 3. The regulator contains a branch for connecting current clamps that can be used to measure the total current of the electromotor.

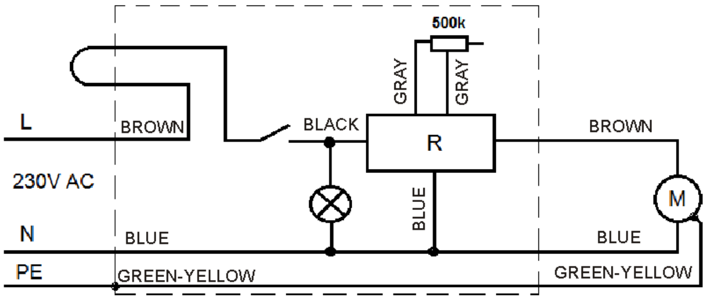


Figure 3. The connection of the rpm regulator and the electromotor

The rpm regulation is accomplished by using potentiometer in the range of 30 - 100 %. Table 1 gives the data describing electromotor Einhell BT-AG 850.

Table 1. The characteristics of the electromotor Einhell BT-AG 850

Voltage	230V – 50Hz	Power	850W	Shaft type	M14
Current intensity	4A	Nominal rpm	11000		

The power transmitting from the electromotor to the journal bearing was performed through the flexible coupling made of reinforced PVC hose, which significantly reduced the transfer of vibrations from the motor to the bearing. The bearing was lubricated by machining grease. The bearing sleeve contained an 8 mm hole where, if needed, the weight for the disturbing of the rotor balance could be placed.

The temperature change during the heating or the cooling was monitored with the use of non-contact infrared thermometer SKF CMAC4200, having very short response time, which enabled very precise measurement of temperature changes. The readings on this instrument where checked by using high accuracy contact thermometer SKF TMTP1, in order to adjust emissivity on the non-contact thermometer.

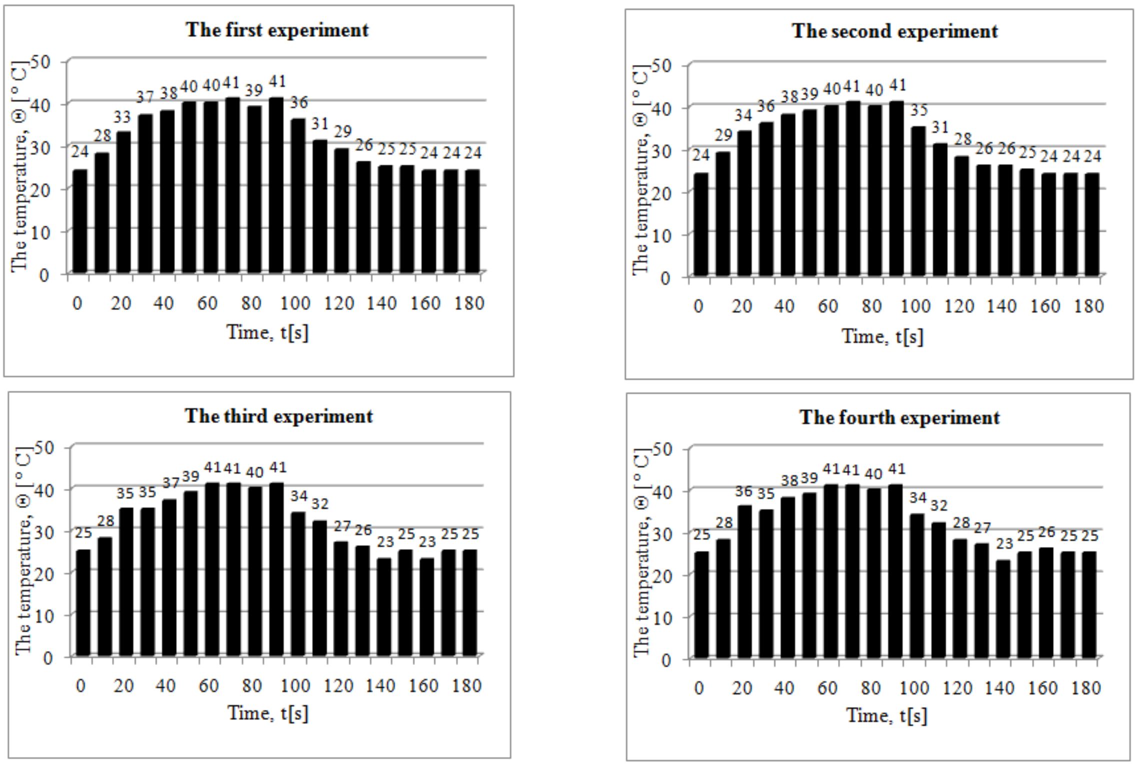
4. Results and discussion

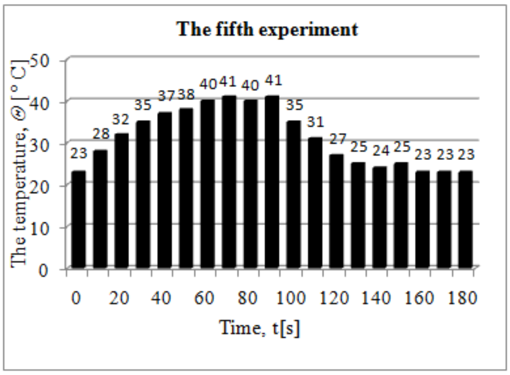
4.1. Results obtained on the test bench

The tab. 2 and fig. 4 represent the bearing temperature values recorded on the test bench during 180 s. The experiment was repeated five times under the identical conditions, to enable more accurate statistical data analysis. The temperature was recorded immediately after the bearing started to rotate with 8000 rpm. Each time, the steady state was achieved after approximately 50 s, and it was observed until 90 s of the bearing operation, after which the electromotor was powered off. Nevertheless, the temperature measurement continued until the next steady state was achieved, this time due to the cooling of the bearing, which was approximately after new 40 s. So, in each of the five tests, the maximum bearing temperature was reached in the approximately the same time (period 0-50 s), and also, nearly the same time was needed to cool the bearing to the room temperature in each test (period 90-130 s).

Table 2. The bearing temperature records on the test bench

Time, t [s]	Temperature, Θ [°C]				
	1. test	2. test	3. test	4. test	5. test
0	24	24	25	25	23
10	28	29	28	28	28
20	33	34	35	36	32
30	37	36	35	35	35
40	38	38	37	38	37
50	40	39	39	39	38
60	40	40	41	41	40
70	41	41	41	41	41
80	39	40	40	40	40
90	41	41	41	41	41
100	36	35	34	34	35
110	31	31	32	32	31
120	29	28	27	28	27
130	26	26	26	27	25
140	25	24	23	23	24
150	25	25	25	25	25
160	24	24	23	26	23
170	24	24	25	25	23
180	24	24	25	25	23





The additional experimental validation of the proposed dynamic heating/cooling model was performed in the manner that the rotation of the journal bearing sleeve was driven by the electromotor of a lathe, as illustrated in fig. 5. The bearing housing was fixed to the lathe support. The lathe was POTISJE PA-B 30 trademark, and its rotational rate was adjusted to 910 rpm. The temperature data collected in such experimental setup are presented in tab. 3 and fig. 6.

Figure 4. The bearing temperature change with time

Figure 5. The bearing driven by the lathe electromotor [16]

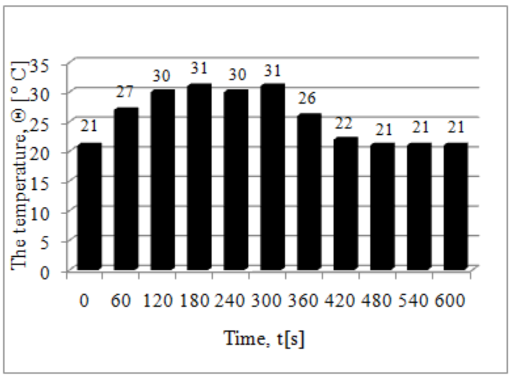


Figure 6. The temperature change of the bearing driven by the lathe electromotor

Table 3. The bearing temperature records on the lathe support

Time, t [s]	0	60	120	180	240	300	360	420	480	540	600
Temperature, Θ [°C]	21	27	30	31	30	31	26	22	21	21	21

4.2. The regression analysis of the results obtained on the test bench

The mathematical model given in Eq. (14), which describes the heating of the journal bearing, was validated by regression analysis as follows. The transformation of the expression (14) gives:

$$\theta k - \theta(t) = \left(\theta k - \theta p\right) \cdot e^{-\frac{t}{T}} \tag{15}$$

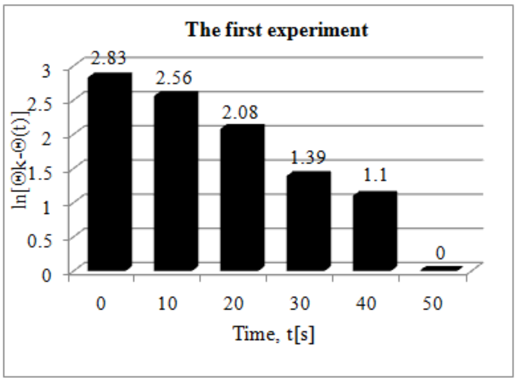
Logarithm of Eq. (15) gives the Eq. (16) with linear form, which enables the application of a simple, linear regression analysis:

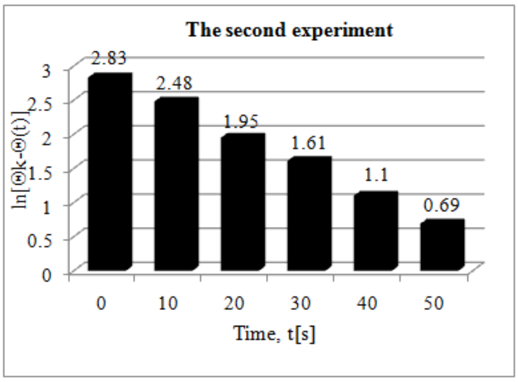
$$\ln\left(\theta k - \theta(t)\right) = \ln\left(\theta k - \theta p\right) - \frac{1}{T} \cdot t \tag{16}$$

The initial temperature Θ_p is the bearing temperature at the beginning of the experiment (0 s), and Θ_k is maximum temperature in the moment of reaching a steady state condition, which was shown to be 50 s. The time dependence of the expression ln[$\Theta_k - \Theta(t)$], determined in five experiments on the test bench model, as well as on the lathe support, is displayed in the tab. 4 and fig. 7.

Table 4. The dependence of (ln[Θ_k–$\Theta(t)$]) on time

Time, t [s]	0	10	20	30	40	50	60	120
Test 1	2.83	2.56	2.08	1.39	1.10	0.00		
Test 2	2.83	2.48	1.95	1.61	1.10	0.69		
Test 3	2.77	2.56	1.79	1.79	1.39	0.69		
Test 4	2.77	2.56	1.61	1.79	1.10	0.69		
Test 5	2.89	2.56	2.20	1.79	1.39	1.10		
Test on the lathe support	2.3						1.4	0





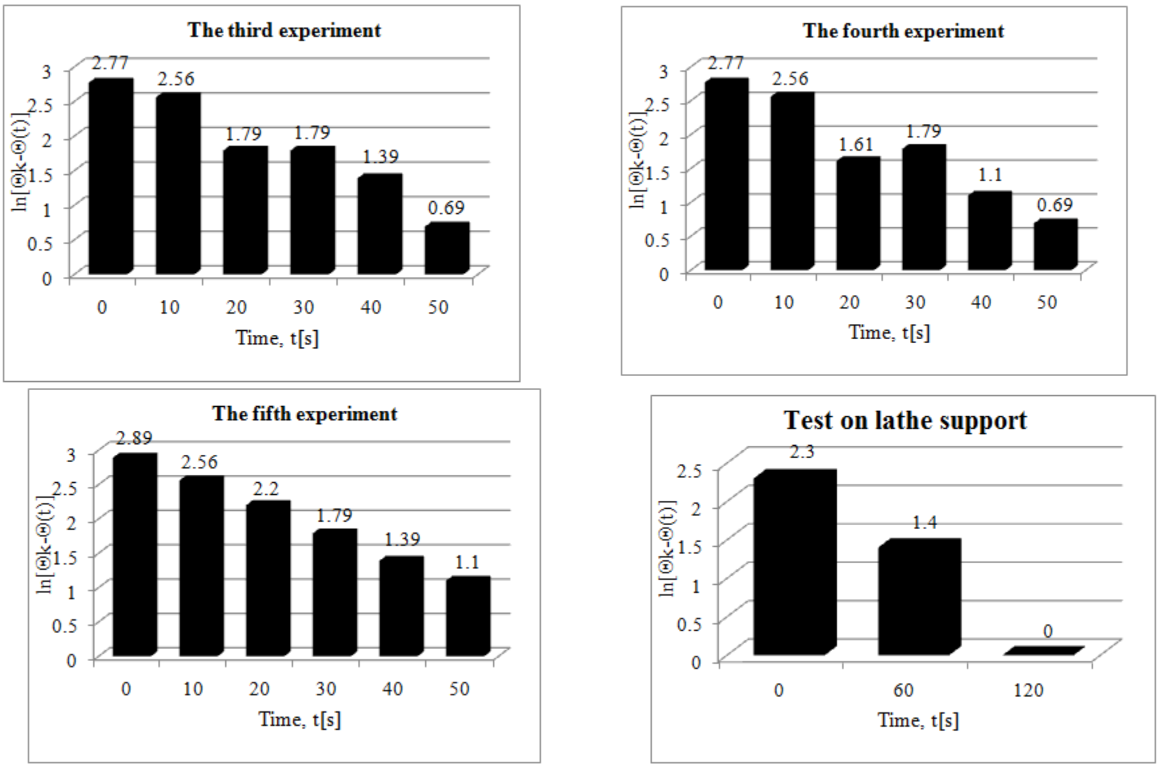


Figure 7. The dependence of (ln[$\Theta_k - \Theta(t)$]) on time in five experiments and on the bearing driven by the lathe electromotor

With the aim of assessing the representativeness of the mathematical model applied in this work, the results obtained were used to calculate the Pearson's coefficient of linear correlation, *r*, for each of the conducted measurements. The coefficient *r* may take values between -1 and +1, and the model is more representative when the value is closer to -1 for a descending function, or to +1 for an increasing linear function. The calculated *r* values are listed in tab. 5, and the average *r* value was 0.8 (for five experiments on the test bench), and 0.93 (for the experiment conducted on the lathe support), which gives evidence that the proposed mathematical model is representative.

Table 5. Correlation coefficient values for the heating process of the bearing

Experiment No.	1	2	3	4	5	$\bar{r}$
r_c	– 0.73	– 0.35	– 0.97	– 0.99	– 0.95	– 0.80

The expression (13), which is the mathematical model for the cooling of the bearing, was processed in the same way as that for the heating process, i.e. by the regression analysis. The algebraic transformation and the logarithm of the Eq. (13) gives the expression (17), and according to this equation, the term ln[$\Theta(t)$ - Θ_k] was plotted versus time, as presented in tab. 6 and fig. 8. This set of experimental data refers to the cooling process which started when the rotation of the bearing stopped, i.e. 90 s after the beginning of the test. This moment is labeled as the initial time (t = 0), and the cooling was finished when the steady state was reached, in t = 40 s. In analogy with the heating test, the cooling of the bearing was also monitored on the lathe support, and the ln[$\Theta(t)$ –Θ_k] versus time dependence for this case, is given also in tab. 6 and fig. 8.

$$\ln\left(\theta(t)-\theta k\right)=\ln\left(\theta p-\theta k\right)-\frac{1}{T}\cdot t \qquad (17)$$

Table 6. The dependence of (ln[$\Theta(t)$ –Θ_k]) on time

Time, t [s]	0	10	20	30	40	60	120
Test 1	2.83	2.48	1.95	1.61	0.69		
Test 2	2.83	2.40	1.95	1.39	0.69		
Test 3	2.77	2.20	1.95	0.69	0.00		
Test 4	2.77	2.20	1.95	1.10	0.69		
Test 5	2.89	2.48	2.08	1.39	0.69		
Test on the lathe support	2.3					1.6	0

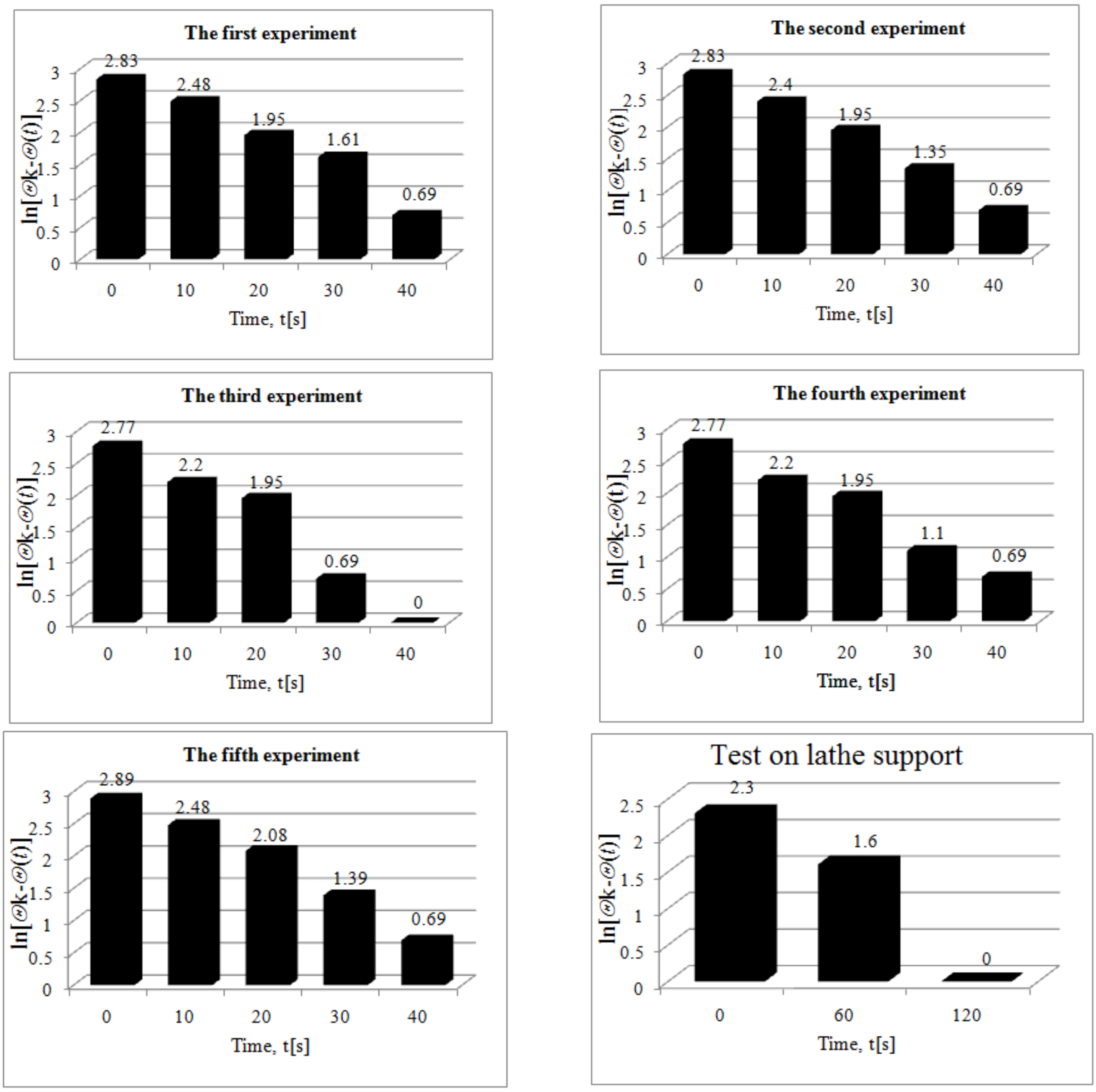


Figure 8. The dependence of (ln[$\Theta(t)$ – Θ_k]) on time in five experiments and on the bearing driven by the lathe electromotor, for the cooling process of the bearing

The calculated values of the Pearson's linear correlation coefficient for five independent experiments of the bearing cooling, are shown in tab. 7. It is seen that the mathematical model is representative for the cooling process, since the average coefficient value was –0.97. At the same time, the coefficient calculated for the experiment conducted at the lathe support, was –0.91. The joint average coefficient for the heating and cooling process may be easily calculated as –(0.80+0.97)/2 = –0,89.

Table 7. Correlation coefficient values for the cooling process of the bearing

Test No.	1	2	3	4	5	$\bar{r}$
r_c	–0.95	–0.98	–0.98	–0.97	–0.99	–0.97

4.3. Analysis of variance of the measurement results on the test bench

The additional validation method of the representativeness of the models presented in this work, was performed by applying the analysis of the variance of the measurement results. For this purpose, the first tests of heating and cooling were used. According to the data listed in tab. 8, the determination coefficient r^2 was calculated to be 0.88 for the heating process, and the obtained coefficient for the cooling process was 0.91. Finally, the joint average coefficient for this case was (0.88 + 0.91)/2 = 0.89, proving the validity of the model for the heating and cooling of the bearing.

Table 8. The data collected from the first test with the heating/cooling process of the bearing

Heating process						
Time, *t* [s]	0	10	20	30	40	50
$y_i=\ln[\Theta_k - \Theta(t)]$	2.83	2.56	2.08	1.39	1.10	0.00
$\hat{y}i$	2.83	2.42	2.01	1.60	1.19	0.78
Cooling process						
Time, *t* [s]	0	10	20	30	40	
$y_i= \ln[\Theta(t) - \Theta k]$	2.83	2.48	1.95	1.61	0.69	
$\hat{y}i$	2.83	2.42	2.01	1.60	1.19	

5. Conclusions

One of important parameters in the diagnosis and prediction of malfunction of plain bearings is the thermal analysis. Heating of the bearing is caused by friction in the bearing, load of the bearing and its rotation rate. Increased friction in the bearings is caused by inadequate lubrication or due to bearing damage, which inevitably leads to heating. External heat sources and cooling system failure may also cause the heating of the bearing.

In this work, a new mathematical model is suggested, which describes the heating and the cooling process of plain bearings. The results obtained by the theoretical model were compared to the data experimentally recorded on the test bench as well as on the bearing driven by a lathe electromotor. The correlation coefficients, determined by regression and variance analysis, confirm that the proposed models are representative for the heating/cooling process of the bearing.

Nomenclature

Pg –	power loss in the bearing, [W]	*r* –	radius of the sliding contact, [m]
Mg –	torque loss, [Nm]	η –	friction coefficient, [-]
ω –	angular velocity, [rad s^{-1}]	*P* –	bearing load, [N]
n –	rpm, rotations per minute	*c* –	specific heat capacity of material, [J K^{-1}]
k –	coefficient of conduction, [W m^{-2} K^{-1}]	*m* -	material mass, [g]
Al –	surface area of the bearing housing, [m^2]	*T* –	time constant which replaces the relationship *c·m/k·Al*
Θ –	bearing temperature, [^{o}C]	Θ_p –	initial temperature, [^{o}C]
Θ_o –	ambient temperature, [^{o}C]	*t* –	time, [s]

P_o –	heat flux, [W]	r_c –	correlation coefficient, [-]
Θ_k –	steady state temperature, [^oC]		

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