IMPROVED GRAY-ENCODED EVOLUTION ALGORITHM BASED ON CHAOS CLUSTER FOR PARAMETER OPTIMIZATION OF MOISTURE MOVEMENT

by

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To improve computational precision for parameter optimization of the van Genuchten model in simulating moisture movement in environment protection, an improved gray-encoded evolution algorithm based on chaos cluster is proposed, in which an initial population is generated by chaotic mapping, and the searching range is automatically renewed with the excellent individuals by chaos cluster operation. Its efficiency is verified experimentally. The results indicate that the absolute error by the improved gray-encoded evolution algorithm based on chaos cluster decreases by 7.52% and 40.40%, respectively, and the relative error decreases by 12.65% and 49.95%, respectively, compared to those by the standard binary-encoded evolution algorithm, and the particle swarm optimization algorithm. Improved gray-encoded evolution algorithm based on chaos cluster has higher precision and it is good for the global optimization in the practical parameter optimization in environment system.

Key words: *parameter optimization, gray-encoded evolution algorithm, van Genuchten model, precision, moisture movement*

Introduction

Environment simulation is of great theoretical significance in environment protection [1, 2]. Van Genuchten model is often used in simulating the moisture movement based on environmental hydraulics theory in environment systems [3, 4]. The parameters of van Genuchten model are usually estimated using experimental data with methods such as standard binary-encoded evolution algorithm (SBEA), and particle swarm optimization algorithm (PSOA), but these methods may not obtain the global optimization efficiently when the objective function has local extreme points [5]. In this paper, an improved gray-encoded evolution algorithm based on chaos cluster (IGEACC) [6, 7] is introduced to reduce computational amount and to improve the parameter precision in the van Genuchten model for simulating moisture movement.

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Van Genuchten model for moisture movement

Van Genuchten model [8] for moisture movement based on environmental hydraulics theory can be described:

$$\theta = \theta_r + \frac{\theta_s - \theta_r}{(1+|\alpha h|^n)^m}$$

where θ [cm^{3}cm^{-3}] is the volumetric water content, h – the suction head, θ_r and θ_s [cm^{3}cm^{-3}] – the residual and saturated soil water content, respectively, α [cm^{-1}] – the scaling parameter that is inversely proportional to mean pore diameter, n – the soil water characteristic curve index, and $m = 1 - 1/n$. In order to estimate the parameters of the previous van Genuchten model, we adopt the following objective function:

$$\text{Min} f(\theta_r, \theta_s, \alpha, n) = \sum |\theta_o(i) - \theta_c(i)| \tag{1}$$

where $\theta_o(i)$ is the observed value, $\theta_c(i)$ – the calculated value, and θ_r, θ_s, α, n are the parameters, respectively.

Description of IGEACC

Here we construct an IGEACC [9] cluster for parameter optimization in van Genuchten model of moisture movement. Consider the following non-linear optimization problem:

$$\text{Min} \;\; f(x_1, x_2, \cdots, x_p) \;\; \text{s. t.} \;\; a_j \le x_j \le b_j, \text{for } j = 1, 2, \ldots, p \tag{2}$$

where $x = \{x_j, j = 1, 2,\ldots, p\}$, x_i is a parameter to be optimized, f – an objective function, and $f \ge 0$. A formal description of the algorithm is given further in the paper.

Step 1. Gray encoding. Suppose gray encoding [10, 11] length is e in every variable, the j^{th} variable range is the interval $[a_j, b_j]$, and then each interval is divided into $2^{\text{e}} - 1$ subintervals:

$$x_j = a_j + I_j c_j \tag{3}$$

where the length of sub-interval of the j^{th} variable $c_j = (b_j - a_j)/(2^{\text{e}} - 1)$ is constant. The searching location I_j is a decimal integer, and $0 \le I_j < 2^{\text{e}}$ for $j = 1, 2,\ldots, p$. The gray code array of the j^{th} variable is denoted by the grid points of $\{d(j, k) \,|\, k = 1, 2, ..., \text{e}\}$:

$$I_j = \sum_{m=1}^{\text{e}} [\oplus_{k=m}^{\text{e}} \text{d}(j, k)] 2^{m-1} \tag{4}$$

where $\oplus$ denotes the operator of addition modul 2 on {0, 1} (*i. e.*, $0 \oplus 0 = 0$, $0 \oplus 1 = 1 \oplus 0 =$ $= 1$, $\oplus 1 = 0$). The IGEACC operations direct to gray code array. The IGEACC uses a code of parameters instead of the individual parameters and works on a population of points and not on one single point.

Step 2. Create chaotic initial population. Chaotic initial population is created in this algorithm by chaotic mapping.

Step 3. Evaluate fitness value of each individual. The smaller the value $f(i)$ is, the higher the fitness of its corresponding i^{th} chromosome is. So the fitness function of i^{th} chromosome is $F(i) = 1/[f(i)^2 + \varepsilon]$.

Step 4. Selection. Chromosome pairs are randomly selected using roulette wheel method from the initial population according to the fitness function.

Step 5. Crossover. Perform crossover on each chromosome pair according to probability p_c to generate one offspring.

Step 6. Mutation. A new offspring can be computed by a random adaptive mutating probability p_m.

Step 7. Chaos cluster and memory operation. Some better points in the previous phase are clustered, which will be memorized and further searched by chaos algorithm [5]. The new better points will be inserted to replace the worst ones in the previous phase. Repeat *step 3* to *step 7* until the evolution times Q is met.

Step 8. Accelerated cycle. The parameter ranges of m excellent individuals obtained by Q-times of the pattern search evolution alternating are regarded as the new ranges of the values, and then the whole process back to the real valued-encoding [11]. The IGEACC computation is over until the algorithm running time gets to the design T times or there exists a chromosome c_{fit} whose fitness satisfies a given criterion. In the former case the c_{fit} is the fittest chromosome or the most excellent chromosome in the population. The chromosome c_{fit} represents the solution.

The parameter design of IGEACC is: $n = 600$, $p_c = 0.5$, $p_m = 0.05$, $\varepsilon = 0.0001$, $m = 10$, and $Q = 2$.

Case study

Example. The observed value of the soil moisture for practical events in Beijing is shown in fig. 1.

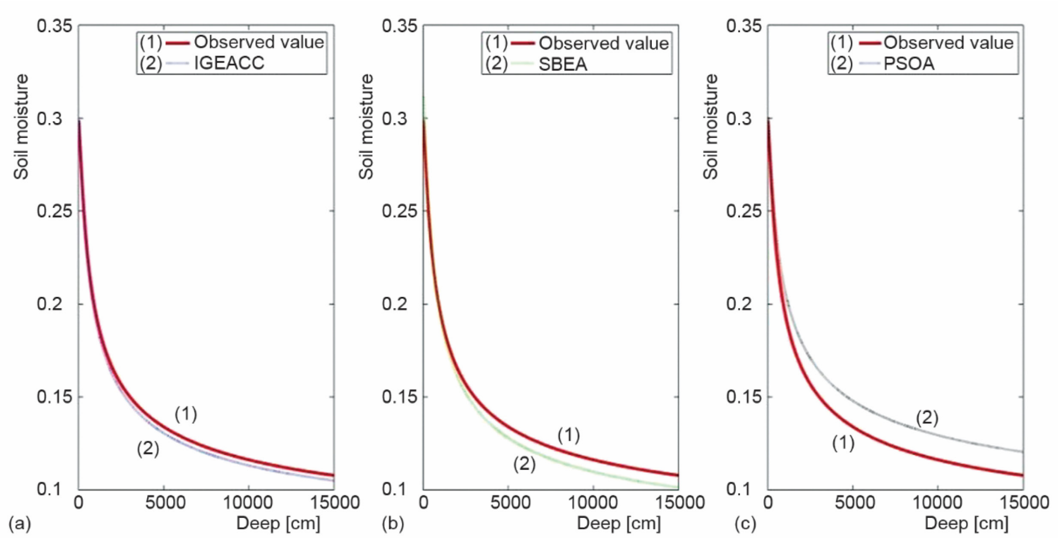

Figure 1. The observed and simulation values of the soil moisture, (a) observed and IGEACC value, (b) observed and SBEA value, and (c) observed and PSOA value

Figure 2 gives the results of absolute error (AE) and relative error (RE) values that change with the change of parameters x_2 and x_3.

From fig. 2, we can see that the result of AE and RE will present a non-linear change tendency with the change of parameters x_2 and x_3. The parameters $\theta_r(x_1)$, $\theta_s(x_2)$, $\alpha(x_3)$, and $n(x_4)$ are required in this study. The significance of these parameters can be seen in

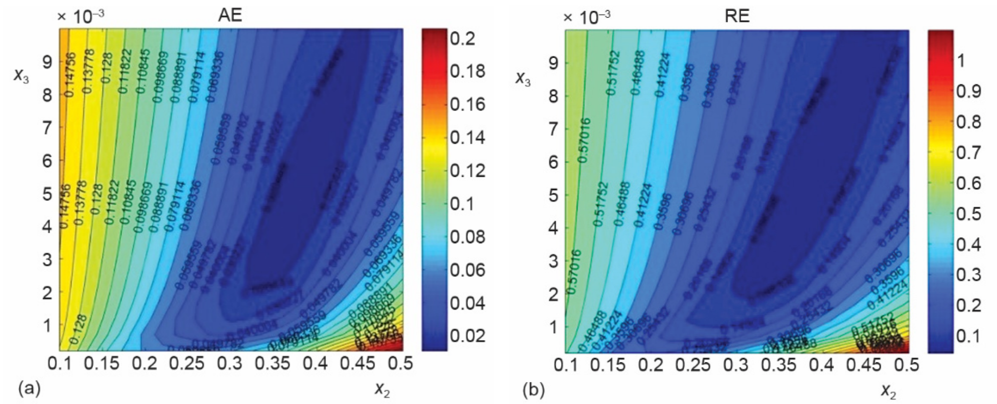

Figure 2. The error values with the change of parameters x_2 and x_3, (a) AE value, (b) RE value
(*for color image see journal web site*)

eq. (1). In this work, the four parameters are estimated with respect to one criterion, namely the sum of AE. The form of the objective function is described as eq. (1). The IGEACC is used for the parameter optimization of van Genuchten model of moisture movement. The optimal parameters are $\theta_r = 0.0638$, $\theta_s = 0.2937$, $\alpha = 0.0038$, and $n = 1.4127$, the sum of AE is 0.0924, and the sum of RE is 0.3729 with IGEACC. For SBEA, the optimal parameters are $\theta_r = 0.0573$, $\theta_s = 0.3019$, $\alpha = 0.0038$, and $n = 1.4108$, the sum of AE is 0.0997, and the sum of RE is 0.4269. For PSOA, the optimal parameters are $\theta_r = 0.0606$, $\theta_s = 0.3005$, $\alpha = 0.0036$, and $n = 1.3479$, the sum of AE is 0.1547, and the sum of RE is 0.8088.

The computational results of the previous model are given in tab. 1 and fig. 1. The results indicate that the AE by the IGEACC decreases by 7.52% and 40.40%, respectively, and the RE decreases by 12.65% and 49.95%, respectively, compared to those by SBEA and PSOA. The results show that IGEACC has a higher precision in parameter optimization of the van Genuchten model. Table 1 gives the errors comparison of several methods for parameter optimization of the van Genuchten model.

From tab. 1, we can see that the results achieved with our IGEACC are satisfactory for the parameter optimization of the van Genuchten model. In terms of minimizing the objective function, IGEACC has shown to be capable for van Genuchten model.

Table 1. Comparison with several methods for parameter optimization of van Genuchten model

Methods	Absolute error	Relative error	AEDR*	REDR*
IGEACC	0.0922	0.3729	–	–
SBEA	0.0997	0.4269	7.523	12.649
PSOA	0.1547	0.8088	40.401	49.952

* The AEDR is the absolute error decreasing rate by IGEACC, and REDR – the relative error decreasing rate by IGEACC

Conclusion

In this paper, an IGEACC is presented for the parameter optimization of van Genuchten model of moisture movement. The circulating mechanism of IGEACC has been studied. Because the operations of gray-encoded evolution algorithm, chaos initial population, chaos cluster, and memory are adopted, the efficiency and accuracy of IGEACC are higher than methods of SBEA and PSOA in a case study of van Genuchten model in Beijing. The results show that AE by the IGEACC decreases by 7.52% and 40.40%, respectively, and RE decreases by 12.65% and 49.95%, respectively, compared to those by SBEA and PSOA. This paper provides a good optimal algorithm for the parameter optimization of moisture movement in environment systems.

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