A DELAYED FRACTIONAL MODEL FOR COCOON'S HEAT-PROOF PROPERTY

by

Fu-Juan LIU [a], ***Hong-Yan LIU*** [a,b]， ***Zheng-Biao LI*** [c]， ***Ji-Huan HE*** [*a]

[a] National Engineering Laboratory for Modern Silk, College of Textile and Clothing Engineering, Soochow University, 199 Ren-Ai Road, Suzhou, China

[b] School of Fashion Technology, Zhongyuan University of Technology, Zhengzhou, China

[c] College of Mathematics and Information Science, Qujing Normal University, Qujing, China

Silkworm cocoon is extremely insensitive to environment change, and a pupa can be survived in a harsh environment. This paper gives a mathematical explanation to this superior survival ability and an experiment is carefully carried out to verify the mechanism. The results are of great importance to design functional clothings for harsh environment, e.g., a moon suit.

Key words: *Fractional model, Heat-proof property, Silkworm cocoon*

Introduction

Silkworm cocoons have superior ability to survive in a harsh environment. It has already been revealed that their hierarchical structure plays an important role in heat-proof property [1, 2] and highly selective permeability for oxygen [3, 4]. Such natural phenomena have been caught much attention. In this paper we will give establish a thermal model to explain cocoon's thermal regulation for pupa.

Fractional Fourier's Law

The cocoon is proved to be a fractal porous hierarchy, the fractional partner of Fourier's law for fractal porous media can be written in the forms:

$$q = -D\frac{\partial^\alpha T}{\partial x^\alpha} \tag{1}$$

and

$$\frac{\partial T}{\partial t} = -\frac{\partial^\alpha q}{\partial x^\alpha} \tag{2}$$

Combining Eqs.(1) and (2), we have a fractional differential equation for heat transfer in cocoon's hierarchy:

∗ Corresponding author; e-mail: hejihuan@suda.edu.cn

$$\frac{\partial T}{\partial t}+\frac{\partial^\alpha}{\partial x^\alpha}\left(D\frac{\partial^\alpha T}{\partial x^\alpha}\right)=0 \quad (3)$$

with boundary conditions

$$T(0,t)=T_0, T(L,t)=T_L \quad (4)$$

and initial condition

$$T(x,0)=g(x) \quad (5)$$

where T is the temperature, D is the thermal conductivity of heat flux in the fractal medium, g(x) is the initial temperature distribution, α is the fractional dimensions of the fractal medium, $\partial^\alpha / \partial x^\alpha$ is the fractional derivative defined as [5, 6]

$$\frac{\partial^\alpha T}{\partial x^\alpha}=\frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dx^n}\int_{t_0}^{t}(s-x)^{n-\alpha-1}[T_0(s)-T(s)]ds \quad (6)$$

where $T_0(x)$ can be the solution of its continuous partner of the problem with the same boundary/initial conditions of the fractal partner.

Experimental

The experiment set-up to measure for temperature change in the inner cocoon is shown in Figure 1. T_o is the external temperature of the thermostat, and T_i represents the real-time temperature inside the cocoon. Generally, T_o is the room temperature. In the experiment, the cocoon with temperature of T_o was put into the thermostat with a certain temperature. The data of real-time temperature began to be collected by data acquisition unit. The time interval was 4 seconds. Finally, the complete data were obtained in data storage unit.

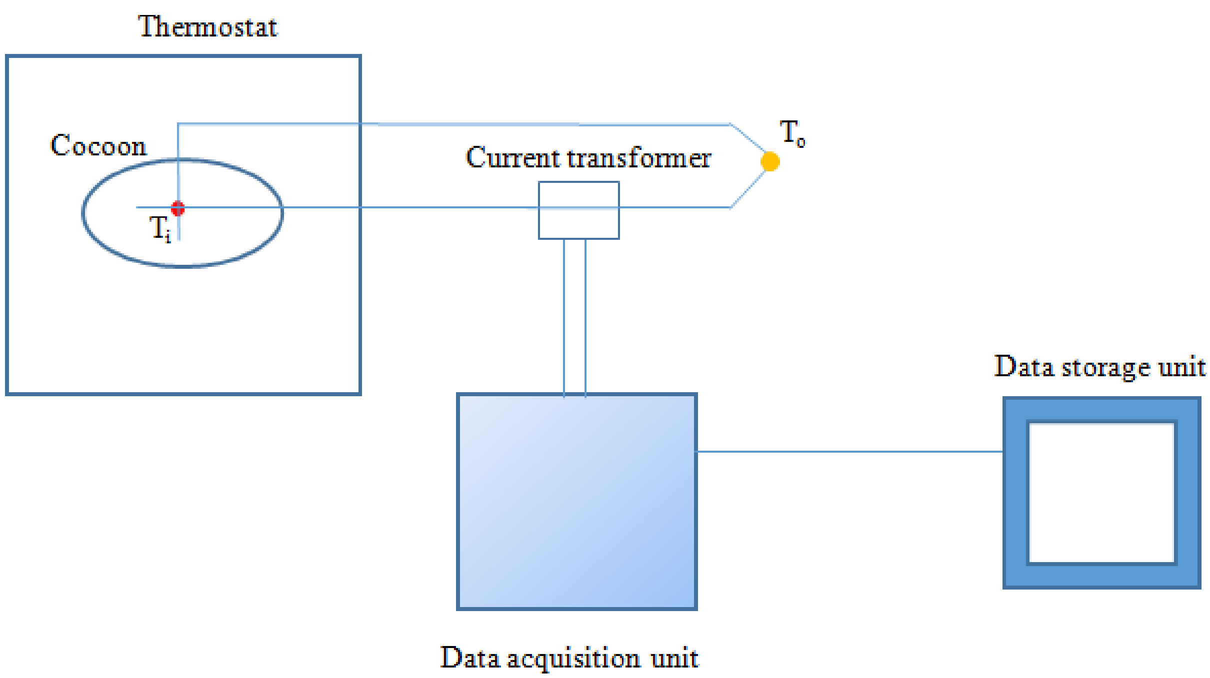

Figure 1. The experiment set-up for measuring temperature change in the inner cocoon

Due to the cocoon wall, the inner temperature will not change immediately, there is a delayed time Δt, Eq.(3) for our experimental study is modified as follows

$$\frac{\partial T(x,t+\Delta t)}{\partial t}+\frac{\partial^{\alpha}}{\partial x^{\alpha}}\left(D\frac{\partial^{\alpha}T(x,t)}{\partial x^{\alpha}}\right)=0 \tag{7}$$

and the initial condition becomes

$$T(x,t)=g(x), t\le \Delta t \tag{8}$$

where Δt is the delay time.

Fractional complex transform

The fractional complex transform [7-10] is to convert a fractional differential equation to a partial differential equation. Introducing a fractional complex transform [11-13]

$$s=\frac{x^{\alpha}}{\Gamma(1+\alpha)} \tag{9}$$

We can convert Eq.(7) to a partial differential equation, which is

$$\frac{\partial T(x,t+\Delta t)}{\partial t}+\frac{\partial}{\partial s}\left(D\frac{\partial T(x,t)}{\partial s}\right)=0 \tag{10}$$

Considering its steady case, Equation (10) becomes

$$\frac{\partial}{\partial s}\left(k\frac{\partial T}{\partial s}\right)=0 \tag{11}$$

Equation (11) has the solution

$$T=a+bs=a+\frac{bx^{\alpha}}{\Gamma(1+\alpha)} \tag{12}$$

After incorporating the boundary conditions, we have

$$T=T_0+\frac{(T_L-T_0)}{L^{\alpha}}x^{\alpha} \tag{13}$$

It is obvious that the solution has the following remarkable property:

$$\frac{dT}{dx}(x=0)=\begin{cases}0, & \alpha>1\\ \frac{(T_L-T_0)}{L}, & \alpha=1\\ \infty, & \alpha<1\end{cases} \tag{14}$$

In order to elucidate the temperature change in inner cocoon, we write Eq.(7) in the form

$$\frac{\partial T(x,t+\Delta t)}{\partial t}+\frac{\partial^{\alpha}q(x,t)}{\partial x^{\alpha}}=0 \tag{15}$$

Our aim focuses on the temperature change at x=0, and we assume that

$$\frac{\partial T(0,t+\Delta t)}{\partial t}=Q \tag{16}$$

where Q is assumed to be an approximate constant: $Q=-\frac{\partial^{\alpha} q(0,t)}{\partial x^{\alpha}}$. Eq.(16) can be approximately written in the form

$$\frac{\partial T}{\partial t}(0,t)+\Delta t\frac{\partial^2 T}{\partial t^2}(0,t)=Q \tag{17}$$

Its solution reads

$$T(0,t)=T_0+Qt+\frac{Q}{\Delta t}\exp(-\frac{t}{\Delta t}) \tag{18}$$

Note that Eq.(18) is valid for smaller time. It is obvious that

$$\frac{\partial T}{\partial t}(0,0)=0 \tag{19}$$

Eq. (19) implies unchanged temperature of inner cocoon at initial stage, this agrees well with the experiment data shown in Figs. 2 and 3.

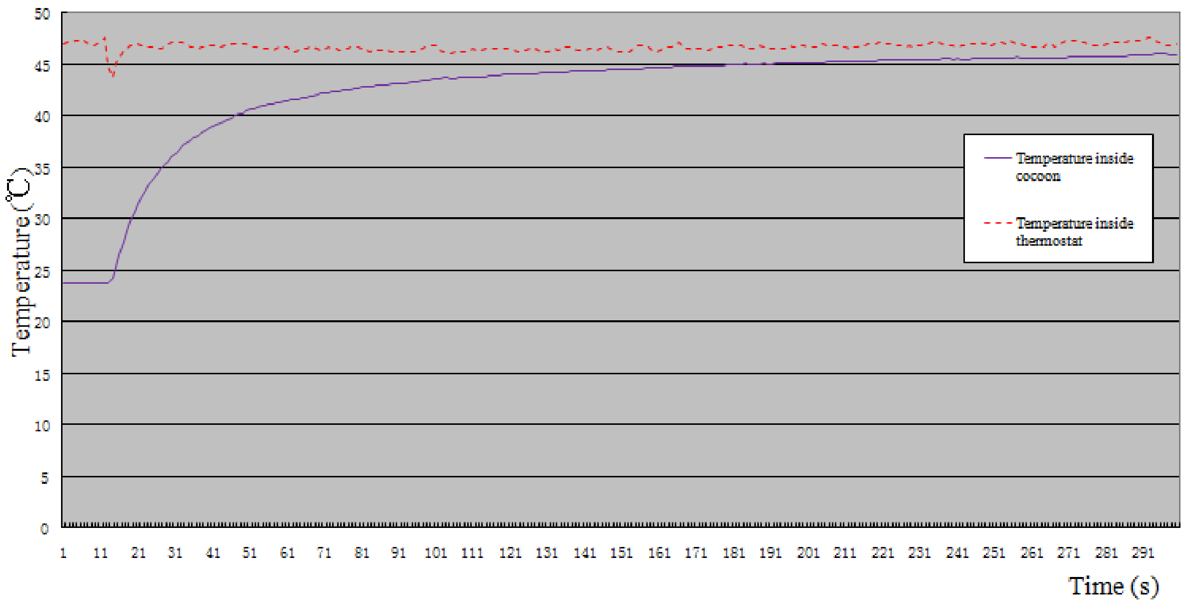

Figure 2. The real-time temperature change of inner cocoon in a high temperature environment

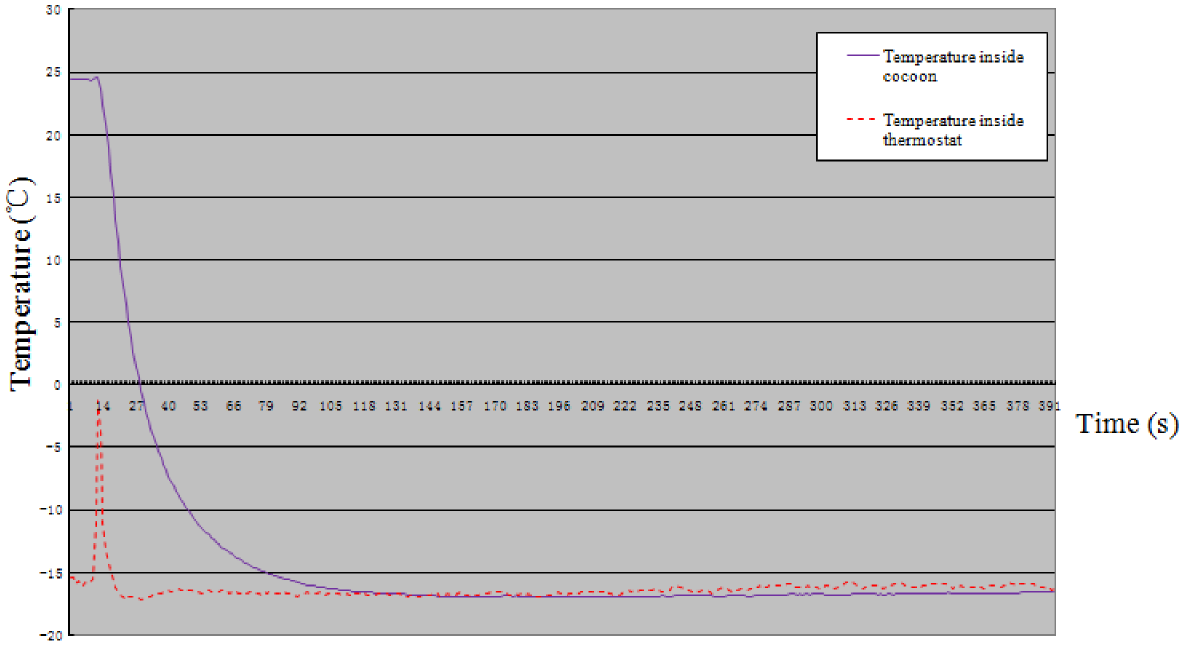

Figure 3. The real-time temperature change of inner cocoon in low temperature environment

Conclusions

This paper gives a theoretical explanation of silkworm cocoon's thermal property for a harsh environment using a fractional model. The obtained result can be used for biomimetic design of advanced clothings for harsh environment, e.g., a moon suit, where a hierarchically porous structure is of great importance for air permeability and thermal protection.

Acknowledgments

The work is supported by Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD), National Natural Science Foundation of China under grant Nos. 11372205, 51403143, 51463021 & 11402155, and Science & Technology Pillar Program of Jiangsu Province under grant No. BE2013072, Natural Science Foundation of Jiangsu Province under Grant Nos. BK20140398, BK20140396 & BK20140399, China Postdoctoral Science Foundation under grant No.2015M571806 and 2016T90495, China National Textile And Apparel Council Project under grant No.2015011,Key Scientific Research Projects of Henan Province under grant No.16A540001.

References

[1] Liu, F. J., Wang, P., Zhang, Y., Liu, H. Y., He, J. H., A fractional Model for Insulation Clothings with Cocoon-like Porous Structure, *Thermal Science*, *20* (2016), 3, pp. 779-784

[2] Liu, F. J., Li, Z.B., Zhang, S., Liu, H.Y., He's Fractional Derivative for Heat Conduction in a Fractal Medium Arising in Silkworm Cocoon Hierarchy, *Thermal Science*, *19* (2015), 4, pp. 1155-1159

[3] Fan, J., He, J. H., Biomimic Design of Multi-scale Fabric with Efficient Heat Transfer Property, *Thermal Science*, *16* (2012), 5, pp. 1349 – 1352
[4] Fan, J., He, J. H., Fractal Derivative Model for Air Permeability in Hierarchic Porous Media, *Abstract and Applied Analysis*, 2012, Article ID 354701
[5] He, J. H., A Tutorial Review on Fractal Spacetime and Fractional Calculus, *Journal of Theoretical Physics*, *53* (2014), 11, pp. 3698-3718
[6] He, J. H., A New Fractal Derivation, *Thermal Science*, *15* (2011), S145-S147
[7] Li, Z. B., He, J. H., Fractional Complex Transform for Fractional Differential Equations, *Mathematical & Computational Applications*, *15* (2010), 5, pp. 970-973
[8] He, J. H., Li, Z. B., Converting Fractional Differential Equations into Partical Differential Equations, *Thermal Science*, *16* (2012), 2, pp. 331-334
[9] He, J. H., Asymptotic Methods for Solitary Solutions and Compactons, *Abstract and Applied Analysis*, 2012, Article ID 916793
[10] Li, Z.B., Zhu, W.H., Fractional Series Expansion Method for Fractional Differential Equations, *International Journal of Numerical Methods for Heat & Fuid Flow*, *25* (2015), 7, pp. 1525-1530
[11] He, J. H., Some Asymptotic Methods for Strongly Nonlinear Equations, *International Journal of Modern Physics B*, *20* (2006), 10, pp. 1141-1199
[12] He, J. H., Elagan, S. K., Li, Z. B., Geometrical Explanation of the Fractional Complex Transform and Derivative Chain Rule for Fractional Calculus, *Physics Letters A* , 376 (2012), 4, pp. 257-259
[13] Zhou, J., Liu, F. J., He, J. H., On Richards' Equation for Water Transport in Unsaturated Soils and Porous Fabrics, *Computers and Geotechnics*, *54* (2013), pp. 69–71

Paper submitted: April 15, 2016
Paper revised: May 10, 2016
Paper accepted: July 20, 2016