

A DELAYED FRACTIONAL MODEL FOR COCOON HEAT-PROOF PROPERTY

by

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Silkworm cocoon is extremely insensitive to environment change, and a pupa can be survived in a harsh environment. This paper gives a mathematical explanation to this superior survival ability and an experiment is carefully carried out to verify the mechanism. The results are of great importance to design functional clothing for harsh environment, e. g., a moon suit.

Key words: *fractional model, heat-proof property, silkworm cocoon*

Introduction

Silkworm cocoons have superior ability to survive in a harsh environment. It has already been revealed that their hierarchical structure plays an important role in heat-proof property [1, 2] and highly selective permeability for oxygen [3, 4]. Such natural phenomena have been caught much attention. In this paper, we will establish a thermal model to explain cocoon thermal regulation for pupa.

Fractional Fourier's law

The cocoon is proved to be a fractal porous hierarchy, the fractional partner of Fourier's law for fractal porous media can be written in the forms:

$$q = -D \frac{\partial^\alpha T}{\partial x^\alpha} \quad (1)$$

and

$$\frac{\partial T}{\partial t} = -\frac{\partial^\alpha q}{\partial x^\alpha} \quad (2)$$

Combining eqs. (1) and (2), we have a fractional differential equation for heat transfer in cocoon hierarchy:

$$\frac{\partial T}{\partial t} + \frac{\partial^\alpha}{\partial x^\alpha} \left(D \frac{\partial^\alpha T}{\partial x^\alpha} \right) = 0 \quad (3)$$

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with boundary conditions:

$$T(0, t) = T_0, \quad T(L, t) = T_L \quad (4)$$

and initial condition:

$$T(x, 0) = g(x) \quad (5)$$

where T is the temperature, D – the thermal conductivity of heat flux in the fractal medium, $g(x)$ – the initial temperature distribution, α – the fractal dimensions of the fractal medium, and $\partial^\alpha / \partial x^\alpha$ – the fractional derivative defined [5, 6]:

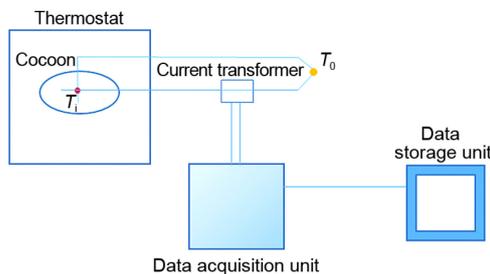
$$\frac{\partial^\alpha T}{\partial x^\alpha} = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_{t_0}^t (s-x)^{n-\alpha-1} [T_0(s) - T(s)] ds \quad (6)$$

where $T_0(x)$ can be the solution of its continuous partner of the problem with the same boundary/initial conditions of the fractal partner.

Experimental

The experiment set-up to measure for temperature change in the inner cocoon is shown in fig. 1. The T_0 is the external temperature of the thermostat, and T_i represents the real-time temperature inside the cocoon. Generally, T_0 is the room temperature. In the experiment, the cocoon with temperature of T_0 was put into the thermostat with a certain temperature. The data of real-time temperature began to be collected by data acquisition unit. The time interval was 4 seconds. Finally, the complete data were obtained in data storage unit.

Figure 1. The experiment set-up for measuring temperature change in the inner cocoon



Due to the cocoon wall, the inner temperature will not change immediately, there is a delayed time Δt , eq. (3) for our experimental study is modified:

$$\frac{\partial T(x, t + \Delta t)}{\partial t} + \frac{\partial^\alpha}{\partial x^\alpha} \left[D \frac{\partial^\alpha T(x, t)}{\partial x^\alpha} \right] = 0 \quad (7)$$

and the initial condition becomes:

$$T(x, t) = g(x), \quad t \leq \Delta t \quad (8)$$

where Δt is the delay time.

Fractional complex transform

The fractional complex transform [7-10] is to convert a fractional differential equation to a partial differential equation. Introducing a fractional complex transform [11-13]:

$$s = \frac{x^\alpha}{\Gamma(1+\alpha)} \quad (9)$$

We can convert eq. (7) to a partial differential equation, which is:

$$\frac{\partial T(x, t + \Delta t)}{\partial t} + \frac{\partial}{\partial s} \left[D \frac{\partial T(x, t)}{\partial s} \right] = 0 \quad (10)$$

Considering its steady-case, eq. (10) becomes:

$$\frac{\partial}{\partial s} \left(k \frac{\partial T}{\partial s} \right) = 0 \quad (11)$$

Equation (11) has the solution:

$$T = a + bs = a + \frac{bx^\alpha}{\Gamma(1 + \alpha)} \quad (12)$$

After incorporating the boundary conditions, we have:

$$T = T_0 + \frac{(T_L - T_0)}{L^\alpha} x^\alpha \quad (13)$$

It is obvious that the solution has the following remarkable property:

$$\frac{dT}{dx}(x=0) = \begin{cases} 0, & \alpha > 1 \\ \frac{(T_L - T_0)}{L}, & \alpha = 1 \\ \infty, & \alpha < 1 \end{cases} \quad (14)$$

In order to elucidate the temperature change in inner cocoon, we write eq. (7):

$$\frac{\partial T(x, t + \Delta t)}{\partial t} + \frac{\partial^\alpha q(x, t)}{\partial x^\alpha} = 0 \quad (15)$$

Our aim focuses on the temperature change at $x = 0$, and we assume that:

$$\frac{\partial T(0, t + \Delta t)}{\partial t} = Q \quad (16)$$

where Q is assumed to be an approximate constant: $Q = -[\partial^\alpha q(0, t)]/\partial x^\alpha$. Equation (16) can be approximately written in the form:

$$\frac{\partial T}{\partial t}(0, t) + \Delta t \frac{\partial^2 T}{\partial t^2}(0, t) = Q \quad (17)$$

Its solution reads:

$$T(0, t) = T_0 + Qt + \frac{Q}{\Delta t} \exp\left(-\frac{t}{\Delta t}\right) \quad (18)$$

Note that eq. (18) is valid for smaller time. It is obvious that:

$$\frac{\partial T}{\partial t}(0, 0) = 0 \quad (19)$$

Equation (19) implies unchanged temperature of inner cocoon at initial stage, this agrees well with the experiment data shown in figs. 2 and 3.

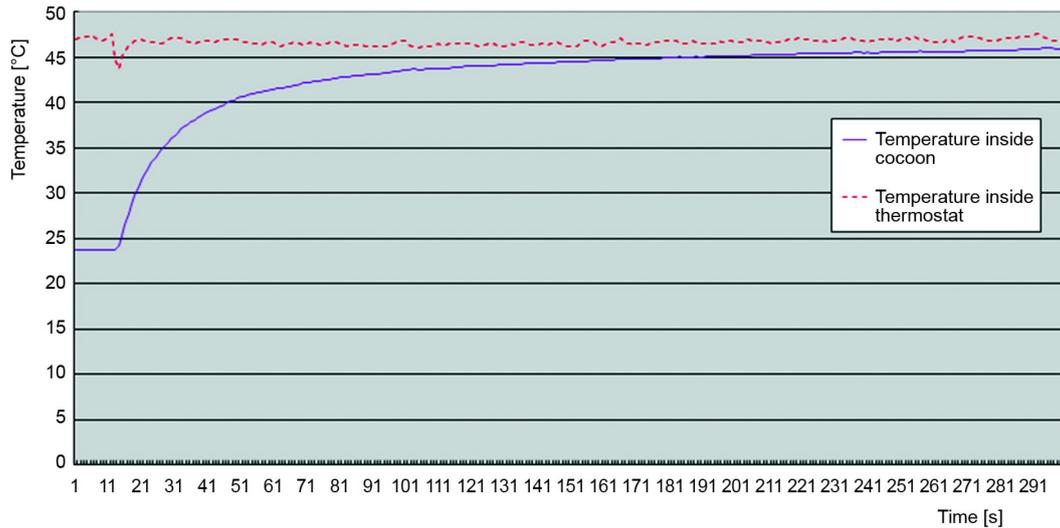


Figure 2. The real-time temperature change of inner cocoon in a high temperature environment

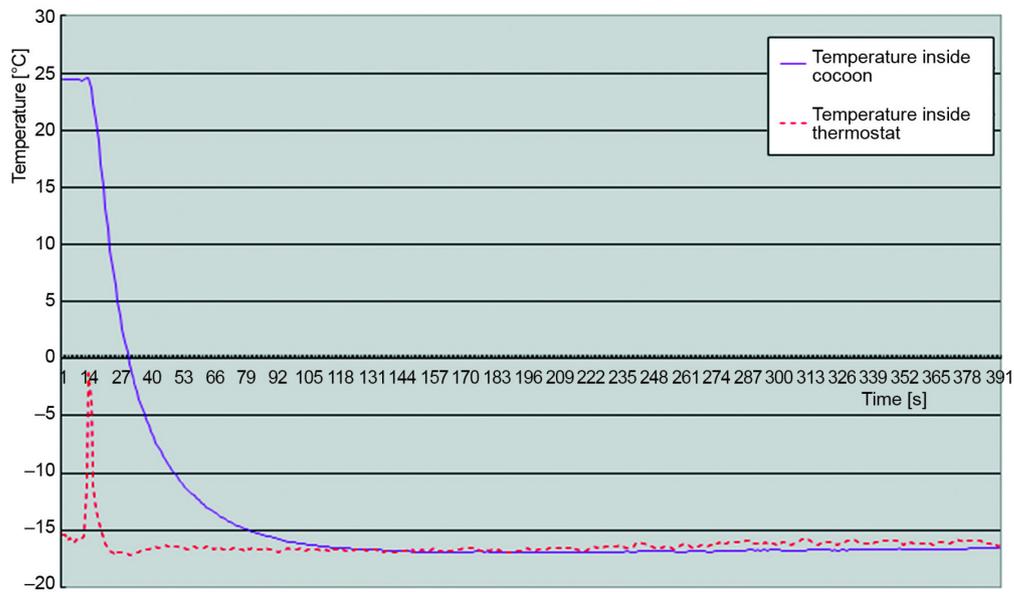


Figure 3. The real-time temperature change of inner cocoon in low temperature environment

Conclusion

This paper gives a theoretical explanation of silkworm cocoon thermal property for a harsh environment using a fractional model. The obtained result can be used for biomimetic

design of advanced clothing for harsh environment, *e. g.*, a moon suit, where a hierarchically porous structure is of great importance for air permeability and thermal protection.

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