HALL CURRENT AND JOULE HEATING EFFECTS ON FREE CONVECTION FLOW OF A NANOFLUID OVER A VERTICAL CONE IN PRESENCE OF THERMAL RADIATION

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The effects of Hall current and Joule heating on flow and heat transfer of a nanofluid along a vertical cone in the presence of thermal radiation is considered. The flow is subjected to a uniform strong transverse magnetic field normal to the cone surface. Similarity transformations are used to convert the nonlinear boundary layer equations for momentum and energy equations to a system of nonlinear ordinary differential equations which are then solved numerically with appropriate boundary conditions. The solutions are presented in terms of local skin-friction, local Nusselt number, velocity and temperature profiles for values of magnetic parameter, Hall parameter, Eckert number, radiation parameter, and nanoparticle volume fraction. Comparison of the numerical results made with previously published results under the special cases, the results are found to be in an excellent agreement. It is also found that, nanoparticle volume fraction parameter and types of nanofluid play an important role to significantly determine the flow behavior.

key words: Heat transfer, Vertical cone, Hall current, Thermal radiation, Nanofluid.

Introduction

Laminar free convection flow and heat transfer over a heated vertical surface has many practical applications in industrial process and technology such as polymer extrusion processes where the object enters the fluid for cooling below a certain temperature, cooling of metallic surfaces in a bath and heat dissipation from electronic components, paper production, wire drawing, geophysical fluid dynamics, and condensation process of metallic plate in a cooling bath and glass. Mark and Prins[1], developed the general relations for similar solutions on isothermal axisymmetric forms and showed that a refinement of the Squire-Eckert approximation for the thermal convection in laminar boundary layers leads to better agreement. Chamkh [2], investigated the laminar free convection flow with considered a transverse magnetic field along a vertical cone and a wedge immersed in an electrically conducting fluid-saturated porous medium. Similarity solutions for free convection from the vertical cone have been exhausted by Hering and Grosh [3]. Alam et al. [4], studied the laminar free convection flow from a vertical permeable circular cone maintained at non-uniform surface temperature with pressure work. Paolucci and Zikoski [5], used Lie group theory to study the free convective flow along a heated vertical wall immersed in a thermally stratified. The effects of shear-dependent viscosity and aspect ratio on the laminar free convection heat transfer flow immersed in unbounded quiescent power-law fluids are investigated by Gupta et al. [6].

Nanofluid concept is utilized to describe a fluid in which nanometer-sized particles (nominally 1-100 nm in size) are suspended in conventional heat transfer basic fluids. Nanofluids, as coined by Choi [7] in 1995, he showed that the addition of a small amount of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the base fluid [8].

Since then, many research explained that nanofluids clearly exhibit enhanced thermal conductivity, which goes up with increasing volumetric fraction of nanoparticles $[9-14]$. Mahdy and Sameh [15] reported numerical analysis for laminar free convection over a vertical wavy surface embedded in a porous medium saturated with a nanofluid. Khan and Pop [16] investigated numerically the problem of laminar fluid flow resulting from the stretching of a flat surface in a nanofluid.

The study of Magnetohydrodynamic (MHD) flow and heat transfer has received considerable attention in the recent years because of their wide variety of many engineering and scientific applications such as MHD generator, cooling of nuclear reactor, MHD pump, and plasma physics, etc., $[17-18]$. Hassan [19], investigated the solution of the MHD boundary layer flow for an incompressible viscous fluid over a sheet stretching according to a power-law velocity. He applied the Lie-group theory to the equations of motion for determining symmetry reductions of partial differential equations. The MHD boundary layer flow by employing the modified Adomian decomposition method and the Pad approximation was investigated by Hayat et al. [20]. The Hall current effects are very important for the applications of MHD is towards a strong magnetic field, because for strong magnetic field electromagnetic force is prominent and it has a marked effect on the magnitude and direction of the current density and consequently on the magnetic force term. Also it is more realistic to include Ohmic heating to explore the effect of the magnetic field on thermal transport in the boundary layer $[21-22]$. Abo-Eldahab and El Aziz [23] analyzed the laminar mixed convection boundary layer flow on a constantly rotating cone of an electrically conducting micropolar fluid and the effects of Hall current and Ohmic heating were considered. The Hall effect and temperature dependent physical properties for the transient generalized Couette flow and heat transfer through a porous medium between two parallel plates was studied by.$[24-25]$. Zaman et al. [22], Used the homotopy analysis method (HAM) to study the effects of Hall current on MHD boundary layer flow induced by a Continuous Surface with heat transfer in a parallel free stream of a second-order viscoelastic fluid.

Thermal radiation is of major importance in many processes in engineering areas which occur when the temperature difference between the surface and ambient is large. The Rosseland approximation is used to describe the radiative heat flux in the energy equation [26]. Many research workers have paid their attention towards the effect of Thermal radiation in free convection flow in different category $[27-30]$.

Motivated by the works mentioned above, the aim of the present work is to provide a boundary layer analysis for the effect of Hall current and Joule heating on the flow and heat transfer of a nanofluid from a vertical circular cone in presence of thermal radiation. We study the effects of the magnetic field parameter, Hall parameter, Eckert number, radiation parameter, and nanoparticle volume fraction on the velocity and temperature profiles for three different types of nanoparticles.

Mathematical Model

Consider steady, two-dimensional laminar, free convection boundary-layer flow of a viscous incompressible electrically conducting fluid over a vertical cone in the presence of thermal radiation effects, as shown in fig.1. The fluid is a water-based nanofluid containing three different types of the nanoparticles namely, Copper Cu, Silver Ag, and Titania TiO_2. The thermo physical properties of the base fluid and nanoparticles are listed in Table 1 [31]. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium and no slip occurs between them.The rectangular curvilinear coordinate systems (x,y) have been employed and the origin of the coordinates is placed at the vertex of the cone, where x represents the coordinate along the surface of the cone and y represents the coordinate normal to the surface of the cone. The coordinate system and flow configuration are shown in fig 1. A uniform magnetic field of strength B_o is applied normal to the flow direction (y-direction). The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. The electron-atom collision frequency is assumed to be relatively high, so that the Hall effect cannot be neglected. The cone surface is maintained at a constant temperature T_w and the ambient temperature far away from the surface of the cone T_∞ is assumed to be uniform.

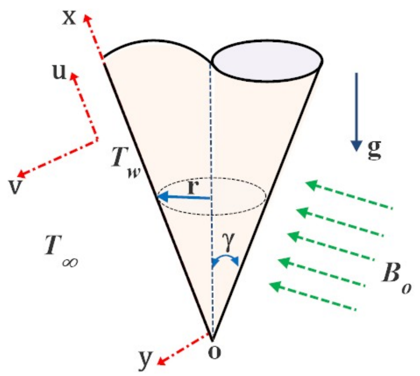

Figure 1: Physical model and coordinates system.

Under the Boussinesq approximation the governing equations the steady-state conservation of mass, momentum, and energy for the boundary layer can be written as:

Continuity equation:

$$\frac{\partial}{\partial x}(ur) + \frac{\partial}{\partial y}(vr) = 0 \tag{1}$$

Momentum equation:

$$\rho_{nf}(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = \mu_{nf}\frac{\partial^2 u}{\partial y^2} + g(\rho\beta)_{nf}\cos\gamma(T - T_\infty) - \frac{\sigma B_o^2}{1+m^2}u \tag{2}$$

Energy equation:

$$(\rho C_p)_{nf}(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = k_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_o^2}{1+m^2}u^2 - \frac{\partial q_r}{\partial y} \tag{3}$$

Where,u,and $v$are the fluid velocity components in x and y directions respectively,r is the local radius of the cone, μ_{nf} is the nanofluid dynamic viscosity, g is the acceleration due to gravity, β_{nf} is the nanofluid coefficient of volume expansion, B_o is the magnetic induction, σ is the electrical conductivity, m is the Hall parameter, k_{nf} is the thermal conductivity, γ is the cone apex half angle, ρ_{nf} is the nanofluid density, $(Cp)_{nf}$ is the specific heat of the nanofluid at constant pressure, and q_r is the radiative heat flux.

In addition, according to the Rosseland approximation the radiative heat flux is described as:

$$q_r = -\frac{4\sigma^*}{3\alpha^*}\frac{\partial T^4}{\partial y} \tag{4}$$

Where σ^* and α^* are the Stefan-Boltzman constant and mean absorption coefficient respectively. We assume that the temperature differences within the flow is such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms we get:

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{5}$$

Using equations (4) and (5), the energy equation (3) becomes:

$$(\rho C_p)_{nf}(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}) = k_{nf}\frac{\partial^2 T}{\partial y^2} + \frac{\sigma B_o^2}{1+m^2}u^2 - \frac{16\sigma^* T_\infty^3}{3\alpha^*}\frac{\partial^2 T}{\partial y^2} \quad (6)$$

The initial and boundary conditions are:

$$\begin{aligned} u &= 0, \quad v = 0, \quad T = T_w \quad at \;\; y = 0 \\ u &= 0, \quad T = T_\infty \quad at \;\; y \to \infty \end{aligned} \quad (7)$$

The thermo-physical properties of the nanofluid were determined In the current study using the following relations [32]:

$$\begin{aligned} &\frac{\mu_{nf}}{\mu_f} = (1-\phi)^{2.5}, \quad \frac{\rho_{nf}}{\rho_f} = (1-\phi) + \phi\frac{\rho_s}{\rho_f}, \quad \frac{(\rho\beta)_{nf}}{(\rho\beta)_f} = (1-\phi) + \phi\frac{(\rho\beta)_s}{(\rho\beta)_f} \\ &\frac{(\rho Cp)_{nf}}{(\rho Cp)_f} = (1-\phi) + \phi\frac{(\rho Cp)_s}{(\rho Cp)_f}, \quad \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)} \end{aligned} \quad (8)$$

Where, ϕ is nanoparticle volume fraction, it is worth mentioning that the study reduces to those of a viscous or regular fluid when (ϕ=0).

Table 1: Thermo-physical properties of water and nanoparticles [31].

	$\rho(kg/m^3)$	$C_p(J/kgK)$	$k(W/mK)$	$\beta \times 10^5(1/K)$
H_20	997.1	4179	0.6130	21.0
Cu	8933	385.0	401.00	1.67
Ag	10500	235.0	429.00	1.89
TiO_2	4250	686.2	8.9538	0.90

We introduce the following non-dimensional variables to transform the above governing equations to dimensionless form:

$$\begin{aligned} &\eta = \frac{y}{x}(Gr_x)^{1/4}, \quad \psi = \nu_f r(Gr_x)^{1/4} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad M^2 = \frac{\sigma B_o^2 x^2}{\mu_f (Gr_x)^{1/2}}, \\ &Gr_x = \frac{g\beta_f x^3 \cos\gamma(T_w - T_\infty)}{\nu_f^2}, \quad Pr = \frac{\nu_f}{\alpha_f}, \quad Ec = \frac{\nu_f Gr_x}{(Cp)_f x^2 (T_w - T_\infty)}, \quad N_R = \frac{16\sigma^* T_\infty^3}{3\alpha^8 k_f} \end{aligned} \quad (9)$$

Where, Gr_x is the local Grashof number, η is the pseudo-similarity variable, v_f is the kinamatic viscosity of the base fluid (water), M is the Hartmann number, m is the Hall parameter, Pr is Prandlt number, N_R is the radiation parameter, Ec is the Eckert number, and $r = x\sin\gamma$. The functions$f(\eta)$ and $\theta(\eta)$ are the dimensionless stream and temperature function of the nanofluid, respectively in the boundary layer region, and ψ is the stream function which is defined as:

$$ru = \frac{\partial\psi}{\partial y}, \quad and \quad rv = -\frac{\partial\psi}{\partial x}$$

With above transformations, Continuity equation (1) is identically satisfied and equations (2), and (6) reduce to the following non-dimensional system of ordinary differential equations as:

$$\frac{(1-\phi)^{2.5}}{(1-\phi)+\phi\frac{\rho_s}{\rho_f}}f''' + \frac{7}{4}ff'' - \frac{1}{2}f'^2 + \frac{(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}}{(1-\phi)+\phi\frac{\rho_s}{\rho_f}}\theta - \frac{M^2}{(1+m^2)((1-\phi)+\phi\frac{\rho_s}{\rho_f}}f' = 0 \quad (10)$$

$$\frac{1}{Pr}\left(\frac{\frac{(k_s+2k_f-2\phi(k_f-k_s))}{(k_s+2k_f+\phi(k_f-k_s))}+N_R}{(1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}}\right)\theta'' + \frac{7}{4}f\theta' + \frac{M^2 Ec}{(1+m^2)((1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}}f' = 0 \quad (11)$$

The corresponding non-dimensional boundary conditions are:

$$f = 0, \quad f' = 0, \quad \theta = 1 \quad at \ \eta = 0$$
$$f' = 0, \quad \theta = 0 \quad at \ \eta \to \infty \quad (12)$$

The physical quantities of interest to study the local as well as the skin friction coefficient, and Nusselt number which are indicate physically to surface shear stress, and rate of heat transfer respectively. This characteristics affect directly on the mechanical properties of the surface during heat treatment process, such that increasing the rate of heat transfer (heat flux) from the surface accelerates the cooling of the surface which improve the hardness and shear strength of the surface but on the other hand decrease the ductility of the surface and increase surface cracking. These physical parameters are defined as:

Surface shear stress:

$$\tau_w = \mu_{nf}(\frac{\partial u}{\partial y})_{y=0} = \mu_{nf}[\frac{\nu_f(Gr_x)^{3/4}}{x^2}f''(0)]$$

Skin friction coefficient:

$$C_f = \frac{2\tau_w}{\rho_f U^2}$$

Where, $U = \frac{\nu_f(Gr_x)^{1/2}}{x}$ is the reference velocity.

Surface heat flux:

$$q_w = -k_{nf}(\frac{\partial T}{\partial y})_{y=0} = -k_{nf}[\frac{(T_w - T_\infty)(Gr_x)^{1/4}}{x}\theta'(0)]$$

Nusselt number:

$$Nu_x = \frac{xq_w}{k_f(T_w - T_\infty)}$$

Numerical Procedure

The set of ordinary nonlinear homogeneous differential equations (10) and (11) subject to the boundary condition (12) are converted into the following simultaneous system of linear differential equations of first order by the substitution as:

$$\{f, f', f'', \theta, \theta''\} = \{z_1, z_2, z_3, z_4, z_5\} \quad (13)$$

Using equation (13), the governing equations (10) and (11) becomes:

$$z_1' = z_2, \quad z_2' = z_3, \quad z_4' = z_5,$$

$$z_3^{'} = \frac{(1-\phi)+\phi\frac{\rho_s}{\rho_f}}{2(1-\phi)^{2.5}}z_2^2 - \frac{7((1-\phi)+\phi\frac{\rho_s}{\rho_f})}{4(1-\phi)^{2.5}}z_1z_3 - \frac{(1-\phi)+\phi\frac{(\rho\beta)_s}{(\rho\beta)_f}}{(1-\phi)^{2.5}}z_4 + \frac{M^2}{(1+m^2)((1-\phi)^{2.5})}z_2, \tag{14}$$

$$z_5^{'} = -Pr\frac{(1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}}{\frac{(k_s+2k_f-2\phi(k_f-k_s))}{(k_s+2k_f+\phi(k_f-k_s)}+N_R}[\frac{7}{4}z_1z_5 + \frac{M^2Ec}{(1+m^2)(1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}}z_2^2]$$

Subjected to the initial conditions:

$$z_1(0)=0, \quad z_2(0)=0, \quad z_3(0)=s_1, \quad z_4(0)=1, \quad z_5(0)=s_2, \tag{15}$$

The coupled ordinary differential equation (14) subject to the boundary conditions (15) is solved numerically using Fourth order Runge-Kutta method with shooting technique. The unspecified initial conditions s_1 and s_2 are determined upon solving the equations $z_2(\eta_{max}) = 0$, and $z_4(\eta_{max}) = 0$ to get the solution satisfy the equations The process is repeated until the solution at η_{max} shoots to the boundary conditions at that point. Once s_1 and s_2 are determined the equation system (14) and (15) is closed and can be solved numerically using Runge-Kutta scheme.

To validation the accuracy of our numerical results, the present results are compared with the previously published work in the literatures. The present numerical for $f^{''}(0)$ and $-\theta^{'}(0)$ with ($M = m = Ec = N_R = \phi = 0$) and for different values of Pr are compared with those Hering and Grosh [3] and Roy [31]. The quantitative comparison is shown in table 2 and it is found to be in a very good agreement.

Table 2: Comparison of values of $f^{''}(0)$ and $-\theta^{'}(0)$ for various values of Prandtl number (Pr =0.1, 1, 10) at ($M = m = Ec = N_R = \phi = 0$).

	$f^{''}(0)$			$-\theta^{'}(0)$		
Pr	Hering and Grosh [3]	Roy [31]	Present results	Hering and Grosh [3]	Roy [31]	Present results
0.1	1.0960	-	1.0959	0.2113	-	0.2113
1	0.7694	0.8600	0.7694	0.5104	0.5275	0.5104
10	-	0.4899	0.4877	-	1.0354	1.0340

Results and Discussions

In the present study, three different types of nanoparticles, Cu, Ag and are considered with water as the base fluid. The influence of manetic parameter M, Hall parameter m, Eckert number Ec, radiation parameter N_R, nanoparticle volume fraction ϕ and nanoparticles type on the velocity and temperature within the boundary layer, are shown in figs. 2-13. The prandtl number of the base fluid (water) is kept constant at 7.

Figs., 2 and 3,present the effect of magnetic parameter M on the velocity and temperature within the boundary layer of (Cu -Water nanofluid). It is observed that,increase in the magnetic field parameter decreases the velocity but increases the temperature of the fluid. This can be attributed to the existence of magnetic field within the flow region which causes a force called Lorentz force. This force works opposite to the flow direction and it resists the flow and increases the boundary layer thickness.

The effects of Hall parameter m, and Eckert number Ec on the velocity and temperature within the boundary layer of Cu-Water nanofluid are shown in figs. 4-7. it is observed that, the increase of Hall parameter m increases the velocity and decreases the temperature within the boundary layer, as shown in figs. 4 and 5. Also, it is found that, with increases of Eckert number Ec both the velocity and temperature increases within the boundary layer, as shown in figs. 6 and 7.

Figs., 8 and 9, present the effect of radiation parameter N_R on the temperature and velocity of Cu-Water nanofluid. Figs. shows that, both the velocity and temperature increases with increases of radiation parameter. On other hand, the effect of nanoparticles concentration (volume fraction)ϕ on the velocity and temperature of (Cu-Water nanofluid) are shown in figs. 10 and 11. It is observed that, with increase of nanoparticle volume fraction decrease the velocity but increase the temperature within the boundary layer. In fact, high values of ϕ causes the fluid becomes more viscous. As results, the natural convection is reduced which causes the fluid flows is very slowly. This velocity reduction causes an increase in thermal boundary layer thickness, which in turn, increases the temperature within the boundary layer.

Figs., 12 and 13, illustrate the profiles of fluid velocity and fluid temperature for Cu-Water, Ag-Water and Tio_2-Water nanofluid. It is observed that, the fluid motion becomes faster by adding Ag nanoparticles and the Tio_2 nanoparticles give slower motion for nanofluid than other nanoparticles near the surface to a certain point within the boundary layer, then the fluid motion becomes faster by adding Tio_2 nanoparticles and the Ag nanoparticles give slower motion for nanofluid than other nanoparticles before decaying the velocity to zero. In other words, there is a location within the boundary layer not affected by changes in nanoparticles type. On the other hand, the high value of thermal conductivity of Ag causes to increase the fluid temperature whereas, the Tio_2 nanoparticles leads to decrease the fluid temperature.

Table 3 show, the values of velocity gradient and temperature gradient at the surface and the corresponding values of skin friction and Nusselt number for different values of embedded parameters at $Gr_x = 10^9$ and $Pr = 7$. The effect of nanoparticles type, nanoparticle concentration, magnetic field, Hall current, Eckert number and thermal radiation on the surface shear stress, surface heat flux and the mechanical properties (hardness, stiffness, strength, surface cracking, etc.) are discussed below:

• **Type of nanoparticles:** It is clear from, that the values of velocity gradient at the surface increased gradually by changing the nanoparticle from Tio_2 to Cu to Ag but the opposite occurs on temperature gradient. On the other hand, the skin friction and surface shear stress are higher in the case of Ag-nanofluid than that in Cu and Tio_2-nanofluid. Also,the Nusselt number and rate of heat transfer from the surface are higher in the case of Cu-nanofluid than that in Tio_2 and Ag-nanofluid, which means that using Cu-nanofluid as a cooling medium is more useful for the surface hardness and strength.

• **Concentration of nanoparticles:** it is observed that,the values of velocity and temperature gradient at the surface deccreases gradually by increase the particle volume fraction $(5-10\%)$in the case of Cu-nanoparticle. On the other hand the skin friction and rate of heat transfer both increase with increase of the concentration of nanoparticle within the base fluid. So one can say that the nanofluid with 10% nanoparticle is more affect on the mechanical properties than that with 5% nanoparticle. In general using a nanofluid in the cooling process is more active to improve the mechanical properties of the surface, such that using nanofluid increase the rate of heat transfer by $(10 - 30\%)$ more than in the case of pure water that leads to accelerate the cooling of the surface which increases the surface hardness and strength.

• **Magnetic field:** the values of both the skin friction (surface shear stress) and Nusselt number (rate of heat transfer) decrease with increasing the magnetic field in the case of Cu-nanoparticle that means the hardness and the strength of the surface will be poor in the presence of magnetic field.

• **Hall ccrrent:** It is observe that, the values of both the skin friction and Nusselt number increasing with increases the Hall current in the case of Cu-nanoparticle that means the hardness and the strength of the surface will be good in the presence of Hall current.

• **Eckert number:** It is clear, with increasing the Eckert number increase the values of the skin friction in the case of Cu-nanoparticle, but on the other hand this increasing in the Eckert number decrease Nusselt number. That means the hardness and the strength of the surface will be poor with the increasing of Eckert number.

• **Thermal radiation:** It is observed that, increasing the thermal radiation increase the values of the skin friction and decrease the values of Nusselt number in the case of Cu-nanoparticle.That means the hardness and the strength of the surface will be decrease in the presence thermal radiation.

Conclusion

We present in this study a mathematical model of free convection along a vertical cone embedded into a nanofluid and heat transfer is considered in the presence of thermal radiation. The flow was subjected to a uniform strong transverse magnetic field normal to the cone surface. The study based on three different types of nanoparticles Cu, Ag and Tio_2 with water as the base fluid. The governing boundary layer equations have been non-dimensionalised and solved numerically with appropriate boundary conditions. The mechanical properties and heat transfer characteristics of the surface were our goal in this study and the following results are obtained:

- The velocity within the boundary layer increases with increase of Hall parameter m, radiation parameter N_R and Eckert number Ec and decreases with increases of magnetic parameter M and nanoparticles concentration ϕ.
- The temperature increases with increase of magnetic parameter, radiation parameter, nanoparticles concentration and Eckert number and decrease with increase of Hall parameter.
- Near the surface the fluid motion becomes slowly by adding Tio_2 nanoparticles. However, the Ag nanoparticles give faster motion for nanofluid than other nanoparticles but the opposite occurs at the end of boundary layer. On the other hand, the high value of thermal conductivity of Ag causes to increase the fluid temperature whereas, the Tio_2 nanoparticles leads to decrease the fluid temperature.
- Using nanofluid as a cooling medium is useful to improve the mechanical properties (hardness and strength) according to the type and concentration of nanoparticles used.
- It is found that, Cu-nanofluid is the best type used to improve the mechanical properties of the surface (increase the heat flux) and the best type used to decrease the surface shear stress is Tio_2-nanofluid
- The presence of magnetic field, thermal radiation and Ohmic heating in the cooling process of the cone surface have a negative effect on the mechanical properties of the surface. On the other hand presence of Hall current has a positive effect on the mechanical properties of the surface.

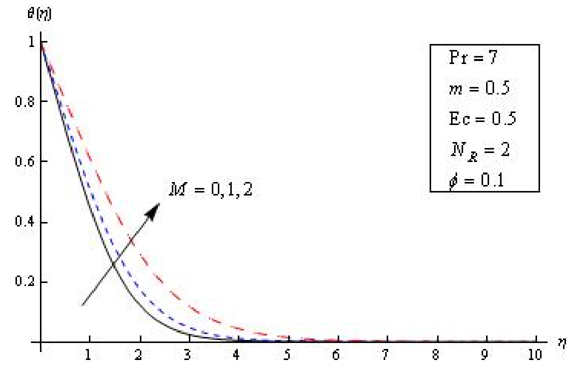

Figure 2: The temperature profiles $\theta(\eta)$ for Cu-Water with increasing of magnetic parameter M.

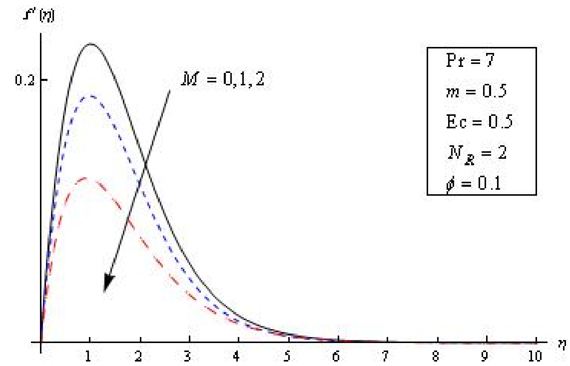

Figure 3: The velocity profiles $f^{'}(\eta)$ for Cu-Water with increasing of magnetic parameter M.

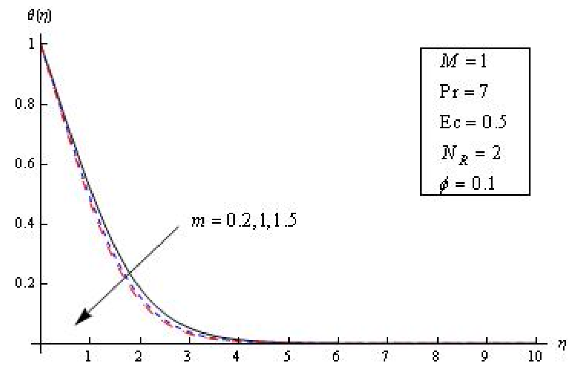

Figure 4: The temperature profiles $\theta(\eta)$ for Cu-Water with increasing of Hall parameter m.

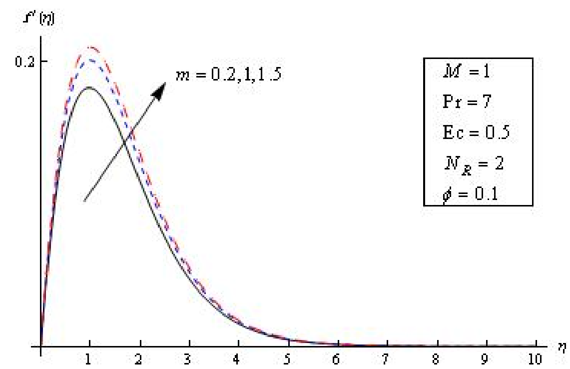

Figure 5: The velocity profiles $f^{'}(\eta)$ for Cu-Water with increasing of Hall parameter m.

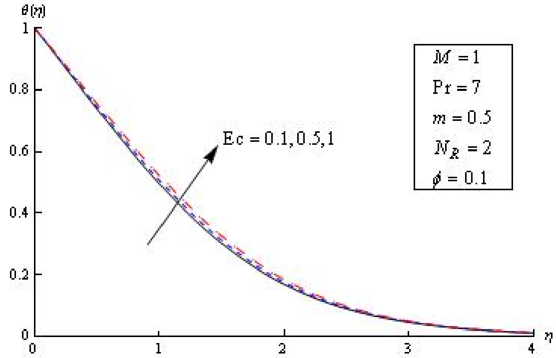

Figure 6: The temperature profiles $\theta(\eta)$ for Cu-Water with increasing of Eckert number Ec.

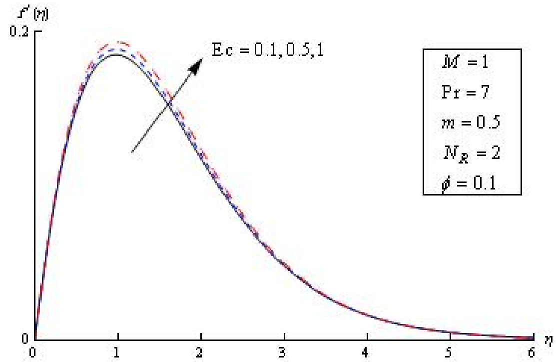

Figure 7: The velocity profiles $f^{'}(\eta)$ for Cu-Water with increasing of Eckert number Ec.

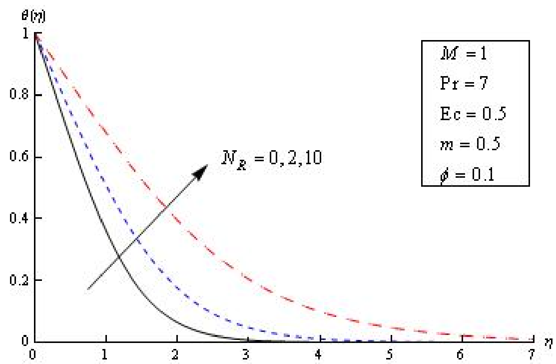

Figure 8: The temperature profiles $\theta(\eta)$ for Cu-Water with increasing of radiation parameter N_R.

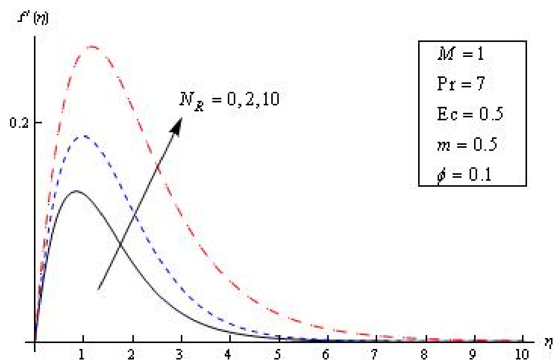

Figure 9: The velocity profiles $f^{'}(\eta)$ for Cu-Water with increasing of radiation parameter N_R.

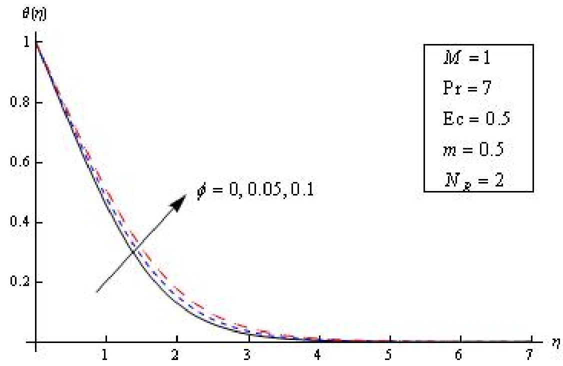

Figure 10: The temperature profiles $\theta(\eta)$ for Cu-Water) with increasing of nanoparticle volume fraction ϕ.

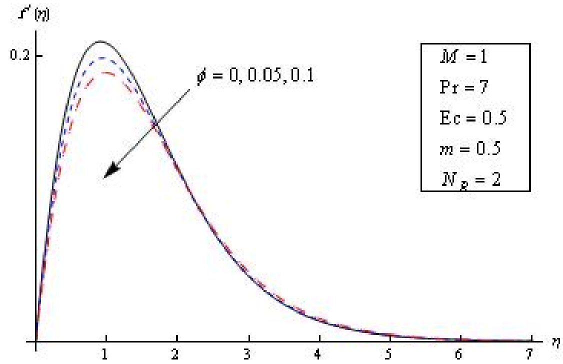

Figure 11: The velocity profiles $f^{'}(\eta)$ for Cu-Water with increasing of nanoparticle volume fraction ϕ.

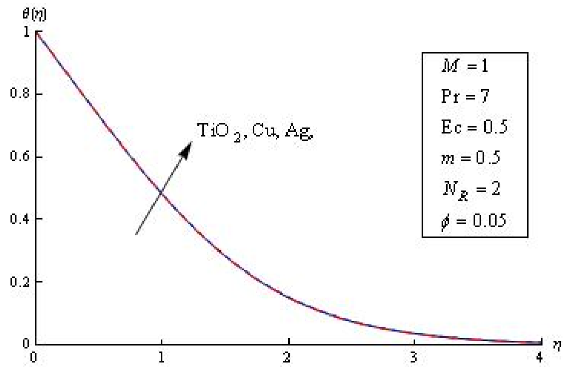

Figure 12: The temperature profiles $\theta(\eta)$ for different nanparticles types.

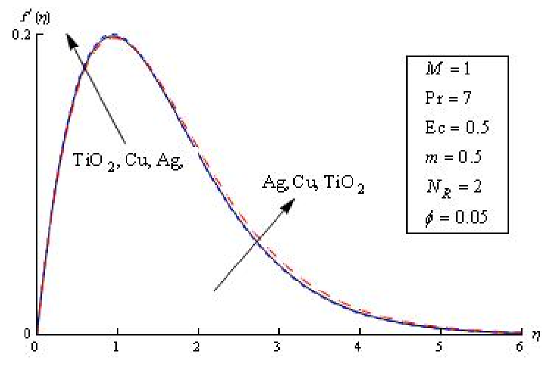

Figure 13: The velocity profiles $f^{'}(\eta)$ for different nanparticles types.

Table 3: Numerical values of velocity $f^{''}(0)$ and temperature $-\theta^{'}(0)$ gradient at the surface

Nanofluid Type	M	m	Ec	N_R	ϕ	$f^{''}(0)$	$-\theta^{'}(0)$	C_f	Nu_x
Cu-Water	0	0.5	0.5	2	0.1	0.52935	0.58791	0.00775	139.22
Cu-Water	1	0.5	0.5	2	0.1	0.46640	0.51395	0.00683	121.05
Cu-Water	2	0.5	0.5	2	0.1	0.35937	0.39294	0.00526	93.05
Cu-Water	1	0.2	0.5	2	0.1	0.45616	0.50214	0.00668	118.91
Cu-Water	1	1.0	0.5	2	0.1	0.48734	0.53828	0.00713	127.47
Cu-Water	1	1.5	0.5	2	0.1	0.50231	0.55583	0.00735	131.62
Cu-Water	1	0.5	0.1	2	0.1	0.46136	0.53028	0.00675	125.57
Cu-Water	1	0.5	0.5	2	0.1	0.46640	0.51395	0.00683	121.70
Cu-Water	1	0.5	1.0	2	0.1	0.47318	0.49183	0.00693	116.47
Cu-Water	1	0.5	0.5	0	0.1	0.39779	0.69791	0.00582	165.27
Cu-Water	1	0.5	0.5	2	0.1	0.46640	0.51395	0.00683	121.70
Cu-Water	1	0.5	0.5	10	0.1	0.55986	0.32726	0.00810	77.50
Cu-Water	1	0.5	0.5	2	0.0	0.56601	0.57383	0.00637	102.04
Cu-Water	1	0.5	0.5	2	0.05	0.51465	0.54338	0.00658	111.81
Cu-Water	1	0.5	0.5	2	0.1	0.46640	0.51395	0.00683	121.70
Cu-Water	1	0.5	0.5	2	0.05	0.51465	0.54338	0.00658	111.81
Ag-Water	1	0.5	0.5	2	0.05	0.52006	0.54161	0.00665	111.45
Tio_2-Water	1	0.5	0.5	2	0.05	0.50483	0.54369	0.00645	107.55

References

[1] Merk, E. J.,Prinss, J. A., Thermal convection in laminar boundary II. *App. Sci. Res.*, 4A(1954), pp. 195-206.

[2] Chamkha, A. J., Non-Darcy hydromagnetic free convection from a cone and a wedge in porous media. *Int. Commun. Heat Mass Transf.*, 23(1996), pp. 875-887.

[3] Hering, R. G., Grosh, R. J., Laminar free convection from a non-isothermal cone. *Z. Heat and Mass Transfer*, 5(1962), pp. 1059-1068.

[4] Alam, M. M., et al., Free convection from a vertical permeable circular cone with pressure work and non-uniform surface temperature. *Nonlinear Analysis Modeling and Control*, 261(2007), pp. 21-32.

[5] Paolucci, S.,Zikoski, Z. J., Free convective flow from a heated vertical wall immersed in a thermally stratified environment. *Int. J. Heat Mass Transf.*, 76(2013), pp. 1062-1071.

[6] Gupta, A. K., et al., Laminar and steady free convection in power-law fluids from a heated spheroidal particle: A numerical study. *Int. J. Heat Mass Transf.*, 75(2014), pp. 592-609.

[7] Choi, S. U. S., Enhancing thermal conductivity of fluids with nanoparticles in developments and applications of non-Newtonian flows. *NSiginer HP Wang Eds ASME*, 66(1995), pp. 99-105.

[8] Choi, S., et al., Anomalously Thermal Conductivity Enhancement in Nanotube Suspensions. *Applied Physics Letter*, 79(2001), pp. 2252-2254.

[9] Xuan, Y. Li, O., Heat Transfer Enhancement of Nanofluids. *International Journal of Heat and Fluid Flow*, 21(2000), pp. 58-64.

[10] Sheikholeslami, M., et al., Numerical investigation of MHD effects on Al2O 3water nanofluid flow and heat transfer in a semi-annulus enclosure using LBM. *Energy*, 60(2013), pp. 501-510.

[11] Wen, D., Ding, Y., Experimental Investigation into Convective Heat Transfer of Nanofluids at the Entrance Region under Laminar Flow Conditions. *International Journal Heat and Mass Transfer*, 47(2004), pp. 5181-5188.

[12] Bhattacharya, P., et al., Brownian Dynamics Simulation to Determine the Effect Thermal Conductivity of Nanofluids. *Journal of Applied Physics*, 95(2004), pp. 6492-6494.

[13] Mokmeli, A.,Saffar-Avval, M., Prediction of Nanofluid Convective Heat Transfer Using the Dispersion Model. *Nonlinear Analysis Modeling and Control*, 49(2010), pp. 471-478.

[14] Mansour, M., et al., Numerical Simulation of Mixed Convection Flows in a Square Lid-Driven Cavity Partially Heated from Below Using Nanofluid. *International Communications in Heat and Mass Transfer*, 37(2010), pp. 1504-1512.

[15] Mahdy, A., Ahmed, S. E, Laminar Free Convection over a Vertical Wavy Surface Embedded in a Porous Medium Saturated with a Nanofluid. *Transport in Porous Media*, 91(2012), pp. 423-435.

[16] Khan, W., Pop, I., Boundary-Layer Flow of a Nanofluid Past a Stretching Sheet. *International Journal of Heat and Mass Transfer*, 53(2010), pp. 2477-2483.

[17] Attia, H. A., et al., Effect of porosity on the flow and heat transfer between two parallel porous plates with the Hall effect and variable properties under constant pressure gradient. *Blug. Chem. Commun.*, 46(2014), pp. 535-544.

[18] Attia, H. A., et al., Effect of porosity on the flow of a dusty fluid between parallel plates with heat transfer and uniform suction and injection. *European Journal of Environmental and Civil Engineering*, 18(2014), pp. 241-251.

[19] Hassan, H. S., Symmetry Analysis for MHD Viscous Flow and Heat Transfer over a Stretching Sheet. *Appl. Math.*, 6(2015), pp. 78-94.

[20] Hayat, T., et al., The Modified Decomposition Method and Pad Approximants for the MHD Flow over a Non-Linear Stretching Sheet. *Nonlinear Analysis: Real World Applications*, 10(2009), pp. 966-973.

[21] Attia, H. A. , et al., Ion slip effect on unsteady Couette flow of a dusty fluid in the presence of uniform suction and injection with heat transfer. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, article in press, Available online 11, (2015) February.

[22] Zaman, H., et al., Effects of Hall Current on MHD Boundary Layer Second-Order Viscoelastic Fluid Flow Induced by a Continuous Surface with Heat Transfer. *Am. J. Comput. Math.*, 4(2014), pp. 143-152.

[23] Abo-Eldahab, E. M., El Aziz, M. A., Hall current and ohmic heating effects on mixed convection boundary layer flow of a micropolar fluid from a rotating cone with power-law variation in surface temperature. *Int. Commun. Heat Mass Transf.*, 31(2004), pp. 751-762.

[24] Abdeen, M. A. M, et al., Effectiveness of porosity on transient generalized Couette flow with Hall effect and variable properties under exponential decaying pressure gradient. *Int. Commun. Heat Mass Transf.*, 87(2013), pp. 767-775.

[25] Attia, H. A., et al., Heat transfer between two parallel porous plates for Couette ow under pressure gradient and Hall current. *Sadhana*, 40(2015), pp. 183-197.

[26] Reddy, M. G., et al., Effects of viscous dissipation and heat source on unsteady MHD flow over a stretching sheet. *Ain Shams Eng. J.*, 6(2015), pp. 1195-1201.

[27] Shakhaoath, Kh. Md., et al., Non-Newtonian MHD Mixed Convective Power-Law Fluid Flow over a Vertical Stretching Sheet with Thermal Radiation, Heat Generation and Chemical Reaction Effects. *Academic Research International*, 3(2012), pp. 80-92.

[28] Gerdroodbary, M. B., et al., Investigation of thermal radiation on traditional Jeffery-Hamel flow to stretchable convergent/divergent channels. *Case Stud. Therm. Eng.*, 6(2015), pp. 28-39.

[29] Hayat, T., et al., Magnetohydrodynamic three-dimensional flow of viscoelastic nanofluid in the presence of nonlinear thermal radiation. *J. Magn. Magn. Mater.*, 385(2015), pp. 222-229.

[30] Roy, S., Free convection over a slender vertical cone at high Prandtl numbers. *ASME J. Heat Mass Transfer*, 101(1974), pp. 174-176.

[31] Alloui, Z., et al., Natural Convection of Nanofluids in a Shallow Cavity Heated from Below. *International Journal of Thermal Sciences*, 50(2011), pp. 385-393.

[32] Kakac, S., Pramuanjaroenkij, A., Review of convective heat transfer enhancement with nanofluids, *International Journal of Heat and Mass Transfer*, 52(2009), pp. 3187-3196.