CRIMP FREQUENCY OF A VISCOELASTIC FIBER IN A CRIMPING PROCESS

by

Hua HUANG [a], ***Cong-Tao ZHAO*** [b], ***Min-Feng SONG*** [b], ***Peng LIU*** [c], ***Hai-Yan KONG*** [c], ***and Ji-Huan HE*** [c*]

[a] Tongji University, Shanghai, China
[b] Nantong Cellulose Fibers Company Ltd., Nantong, China
[c] National Engineering Laboratory for Modern Silk, College of Textile and Engineering, Soochow University, Suzhou, China

Original scientific paper
https://doi.org/10.2298/TSCI160101077H

Crimp frequency during the stuffer box crimping process is of great importance for controlling morphology of crimped fibers. A fiber is considered as a viscoelastic rheological fluid, and an approximate formulation for crimp frequency is obtained revealing the main factors affecting the crimping process.

Key words: stuffer box crimping, crimped fiber, viscoelastic rheological model, crimped frequency

Introduction

Stuffer box crimping [1-4] is widely used for fabrication of microscale crimped fibers, and its mechanism is also used for fabrication of nanoscale crimped fibers [4] by bubble electrospinning [5, 6] or bubbfil spinning [7-10]. The governing equations were established for the transverse vibration of an axially moving fluid arising in the stuffer box crimping by the Hamilton's principle [1]. The viscoelastic fluid is subject to a complex vibration, and it will be crimped after solvent evaporation. The crimp frequency will greatly affect the morphology and properties of the crimped fibers. This paper search for an approximate frequency formulation to elucidate the main factors affecting the crimping process, so that the process can be optimized and controlled.

Governing equation

Governing differential equations for an axially moving slender fiber of viscoelastic fluid were obtained in [1]. The previous study revealed that the velocity of the axial motion affects greatly the natural frequencies of transverse vibration. If transverse vibration occurs just before fiber solidification, a crimped fiber can be obtained, and the crimped frequency can be adjusted by the velocity of axial motion [1]. This paper focuses on the effect of fiber viscoelastic rheological property on the crimping process. The viscoelastic rheological model for a moving fiber with axial stiffness *EA* and flexural rigidity *EI* is adopted in the form:

* Corresponding author, e-mail: hejihuan@suda.edu.cn

$$\sigma_L = E\varepsilon_L + \eta \frac{\mathrm{D}\varepsilon_L}{\mathrm{D}t} = E\varepsilon_L + \eta\left(\frac{\partial \varepsilon_L}{\partial t} + u\frac{\partial \varepsilon_L}{\partial x}\right) \tag{1}$$

where σ_L is the axial stress, η – the parameter for viscoelastic rheological property, ε_L – the non-linear strain, D/Dt – the material derivative of the transverse displacement, defined:

$$\frac{\mathrm{D}}{\mathrm{D}t} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} \tag{2}$$

where u is the velocity of the moving fiber. The non-linear strain is defined:

$$\varepsilon_L = \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{3}$$

where w is the transverse displacement.

The governing equation was obtained by Hamilton principle [1], which is:

$$\rho A\frac{\partial^2 w}{\partial t^2} + 2u\rho A\frac{\partial^2 w}{\partial x\partial t} + \rho A u^2\frac{\partial^2 w}{\partial x^2} - N\frac{\partial^2 w}{\partial x^2} -$$
$$-\frac{3}{2}AE\frac{\partial}{\partial x}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2} - \eta A\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)\frac{\partial^2 w}{\partial t\partial x}\frac{\partial^2 w}{\partial x^2} - \frac{1}{2}\eta A\left(\frac{\mathrm{d}w}{\mathrm{d}x}\right)^2\frac{\partial^3 w}{\partial t\partial^2 x} -$$
$$-\eta Au\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial^2 w}{\partial^2 x}\right)^2 - \frac{1}{2}\eta Au\left(\frac{\partial w}{\partial x}\right)^2\left(\frac{\partial^3 w}{\partial^3 x}\right) + EI\frac{\partial^4 w}{\partial x^4} = 0 \tag{4}$$

where w is the transverse displacement, A – the section area, u – the velocity of the moving fiber, F – the liquid fiber tension per area, P – the fluid pressure, and ρ – the density.

Crimp frequency

We assume that the solution of eq. (4) can be expressed in the form:

$$w = w_0 + \eta w_1 \tag{5}$$

where w_0 and w_1 are solved, respectively, from the following equations:

$$\rho A\frac{\partial^2 w_0}{\partial t^2} + 2u\rho A\frac{\partial^2 w_0}{\partial x\partial t} + \rho A u^2\frac{\partial^2 w_0}{\partial x^2} - N\frac{\partial^2 w_0}{\partial x^2} - 3AE\frac{\partial w_0}{\partial x}\left(\frac{\partial^2 w_0}{\partial x^2}\right)^2 + EI\frac{\partial^4 w_0}{\partial x^4} = 0 \tag{6}$$

and

$$\rho A\frac{\partial^2 w_1}{\partial t^2} + 2u\rho A\frac{\partial^2 w_1}{\partial x\partial t} + \rho A u^2\frac{\partial^2 w_1}{\partial x^2} - N\frac{\partial^2 w_1}{\partial x^2} -$$
$$-\frac{3}{2}AE\frac{\partial}{\partial x}\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial^2 w_1}{\partial x^2} - \frac{3}{2}AE\frac{\partial}{\partial x}\left(\frac{\partial w_1}{\partial x}\right)^2\frac{\partial^2 w_0}{\partial x^2} - A\left(\frac{\mathrm{d}w_0}{\mathrm{d}x}\right)\frac{\partial^2 w_0}{\partial t\partial x}\frac{\partial^2 w_0}{\partial x^2} - \frac{1}{2}A\left(\frac{\mathrm{d}w_0}{\mathrm{d}x}\right)^2\frac{\partial^3 w_0}{\partial t\partial^2 x} -$$
$$-Au\left(\frac{\partial w_0}{\partial x}\right)\left(\frac{\partial^2 w_0}{\partial^2 x}\right)^2 - \frac{1}{2}Au\left(\frac{\partial w_0}{\partial x}\right)^2\left(\frac{\partial^3 w_0}{\partial^3 x}\right) + EI\frac{\partial^4 w_1}{\partial x^4} = 0 \tag{7}$$

The solution of eq. (6) can be presented in the form:

$$w_0(x,t) = W_0(x)\cos\omega t \tag{8}$$

where W_0 is the normal function, ω – the natural frequency, and $N = FA - PA$.

The differential equation for the mode shape of vibration, after submitting eq. (8) into eq. (6), is obtained:

$$-\omega^2 \rho A W_0 + (\rho A u^2 - N) W_0'' + EI W_0^{(4)} = 0 \tag{9}$$

The expression of the normal function is assumed to have the form:

$$W_0 = w_{\max} \sin\left(\frac{\pi}{L} x\right) \tag{10}$$

we obtain the following fundamental frequency of vibration:

$$\omega = \sqrt{-\frac{\pi^2}{\rho A L^2}(\rho A u^2 - N) + \frac{\pi^4}{\rho A L^4} EI} \tag{11}$$

The normal mode shape of vibration reads:

$$w_0(x,t) = w_{\max} \sin\left(\frac{\pi}{L} x\right) \cos \omega t \tag{12}$$

Now submitting eq. (12) into eq. (7) results in a differential equation for $w_1(x, t)$:

$$\rho A \frac{\partial^2 w_1}{\partial t^2} + 2u\rho A \frac{\partial^2 w_1}{\partial x \partial t} + \rho A u^2 \frac{\partial^2 w_1}{\partial x^2} - N \frac{\partial^2 w_1}{\partial x^2} + EI \frac{\partial^4 w_1}{\partial x^4} -$$
$$-\frac{3}{2} AE \frac{\partial}{\partial x}\left(\frac{\partial w_0}{\partial x}\right)^2 \frac{\partial^2 w_1}{\partial x^2} - \frac{3}{2} AE \frac{\partial}{\partial x}\left(\frac{\partial w_1}{\partial x}\right)^2 \frac{\partial^2 w_0}{\partial x^2} - A\left(\frac{\mathrm{d}w_0}{\mathrm{d}x}\right)\frac{\partial^2 w_0}{\partial t \partial x}\frac{\partial^2 w_0}{\partial x^2} - \frac{1}{2} A \left(\frac{\mathrm{d}w_0}{\mathrm{d}x}\right)^2 \frac{\partial^3 w_0}{\partial t \partial^2 x} -$$
$$-Au\left(\frac{\partial w_0}{\partial x}\right)\left(\frac{\partial^2 w_0}{\partial^2 x}\right)^2 - \frac{1}{2} Au \left(\frac{\partial w_0}{\partial x}\right)^2 \left(\frac{\partial^3 w_0}{\partial^3 x}\right) = 0 \tag{13}$$

We assume that the second mode shape of vibration can be expressed in the form:

$$w_1(x,t) = \varepsilon \sin\left(\frac{\pi}{2L} x\right) \cos \omega t \tag{14}$$

Submitting eq. (14) into eq. (13) yields a residual equation, setting the residual equation be zero at $x = L$, $t = 0$, we obtain:

$$\varepsilon \omega^2 \rho A + \frac{\pi^2(\rho A u^2 - N)}{4L^2} + \frac{\pi^4 EI}{16 L^4} = 0 \tag{15}$$

From eq. (14), ε can be identified, which is:

$$\varepsilon = -\frac{1}{\omega^2 \rho A}\left[\frac{\pi^2(\rho A u^2 - N)}{4L^2} + \frac{\pi^4 EI}{16 L^4}\right] \tag{16}$$

Therefore, we obtain the mode shape of vibration of the fiber:

$$w(x,t) = w_{\max} \sin\left(\frac{\pi}{L} x\right) \cos \omega t + \varepsilon \eta \sin\left(\frac{\pi}{2L} x\right) \cos \omega t \tag{17}$$

where ω is defined in eq. (11), and ε is defined in eq. (16).

Discussion and conclusion

The transverse vibration of an axially moving slender fiber of viscoelastic fluid can be approximately expressed by eq. (17) for normal mode shape of vibration and second mode shape of vibration. The viscoelastic property, η, mainly affects the second mode shape of vibration, and crimp frequency, ω, mainly depends upon the axially moving speed, fiber mechanical property, and geometrical property of the stuffer box. The obtained approximate crimp frequency given in eq. (11) can be used for optimization of the crimping process for practical applications.

Acknowledgment

The work is supported by PAPD (A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions).

Nomenclature

A – section area
EA – fiber axial stiffness
EI – fiber flexural rigidity
F – liquid fiber tension per area
L – effective crimp length
$N = FA – PA$
P – fluid pressure
u – velocity of the moving fiber
w – fiber transverse displacement
$w_{\max}$ – maximal transverse displacement

Greek symbols

ε – small transverse displacement defined in eq. (16)
ε_L – non-linear strain
η – viscoelastic parameter
ρ – the density
σ_L – axial stress
ω – fundamental frequency defined in eq. (11)

References

[1] Huang, H., *et al.*, A Mathematical Model for an Axially Moving Slender Fiber of Viscoelastic Fluid: Part 1 Fabrication of Crimped Fiber, *SYLWAN*, *158* (2015), 5, pp. 285-293
[2] Huang, J. X., *et al.,* Transverse Vibration of an Axially Moving Slender Fiber of Viscoelastic Fluid in Bubbfil Spinning and Stuffer Box Crimping, *Thermal Science*, *19* (2013), 4, pp. 1437-1441
[3] Huang, J. X., *et al.*, Effect of Temperature on Nonlinear Dynamical Property of Stuffer Box Crimping and Bubble Electrospinning, *Thermal Science, 18* (2014), 3, pp. 1049-1053
[4] Wang, P., *et al.*, Nonlinear Vibration Mechanism for Fabrication of Crimped Nanofibers with Bubble Electrospinning and Stuffer Box Crimping Method, *Textile Research Journal*, On-line first, https: //doi.org.10.1177/0040517516658513
[5] Chen, R. X., *et al.,* Mechanism of Nanofiber Crimp, *Thermal Science, 17* (2013), 5, pp. 1473-1477
[6] Liu, P., *et al.*, Facile Preparation of Alpha-Fe_2O_3 Nanobulk via Bubble Electrospinning and Thermal Treatment, *Thermal Science, 20* (2016), 3, pp. 967-972
[7] Shen, J., *et al.*, Primary Study of Ethyl Cellulose Nanofiber for Oxygen-Enrichment Membrane, *Thermal Science, 20* (2016), 3, pp. 1008-1009
[8] He, C. H., *et al.*, Bubbfil Spinning for Fabrication of PVA Nanofibers, *Thermal Science, 19* (2015), 2, pp. 743-746
[9] Liu, P., *et al.*, Micro-Nanofibers with Hierarchical Structure by Bubbfil-Spinning, *Thermal Science, 19* (2015), 4, pp. 1455-1456
[10] Li, Y., *et al.* Bubbfil Electrospinning of PA66/Cu Nanofibers, *Thermal Science, 20* (2016), 3, pp. 993-998

Paper submitted: January 1, 2016
Paper revised: September 5, 2016
Paper accepted: September 6, 2016