

## A MATHEMATICAL MODEL OF META-ARAMID FIBRID FORMATION

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*Meta-aramid fibril is a key material for aramid paper preparation. Its formation mechanism is elucidated and a mathematical model is established to reveal the main effects of coagulation properties and shear stress on the properties of aramid paper.*

Key words: *meta-aramid fibril, coagulation, formation, mathematical model*

### Introduction

Various kinds of fibrils are produced through two convenient methods, including beating. In beating, the liquid suspension of a shaped structure formed through an interfacial-forming procedure is beaten. In the interface of a two-liquid phase, a rapid reaction occurs between a polymer solution and a coagulant, and solidified intermediates are formed within a short period, which affects the shape and size of the prepared fibrils. This process was first described in the U. S. Patent 2708617 [1].

The properties of aramid fibers or aramid composites have been extensively investigated [2-7]. However, the preparation [8, 9] and formation mechanism of fibrils have been rarely explored. Meta-aramid fibril and its properties have also been examined [10, 11]. In the current study, the formation of meta-aramid fibril under turbulent conditions is described by using our homemade equipment.

### Meta-aramid fibril preparation

A coagulation bath composed of a H<sub>2</sub>O/DMAc mixture was placed in a closed vessel. A needle was injected into the gap between a rotator and a stator from the side of the vessel. Rotating speed was set, and a polymer solution was fed through a feeding pump. A needle with an inner diameter of 0.5 mm was then used when the display speed reached the set value. Feeding speed was set to 5 ml per minute, and approximately 5-10 ml feed was generated at a time. After precipitation was completed, the fibril suspension was filtered through an 80-mesh web, and wet fibrils were washed with tap water five times and dispersed in water until use. The fibril suspension was then dropped onto the slide glass. Twenty samples were randomly examined, and their mean was calculated. The mean length of <1 mm fibrils was measured under a microscope.

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## Discussion

In a high shearing fluid flow, a viscous droplet is elongated at  $x_1$  direction by the shearing force and the thickness of the droplet is decreased at  $x_2$  direction. The droplet solidifies when it is penetrated by the coagulant. Thus, deformation and coagulation simultaneously occur. Deformation stops when the coagulant concentration in the droplet reaches the critical value, and the fibril is formed.

In fibril formation, fig. 1, the spherical droplet in the coagulant at  $t_0$  is elongated by shearing force, and deformation stops at  $t_2$ . The coagulant does not penetrate the center of the droplet when the fibril is deformed. This phenomenon occurs because the droplet solidifies and deformation stops at a critical time.

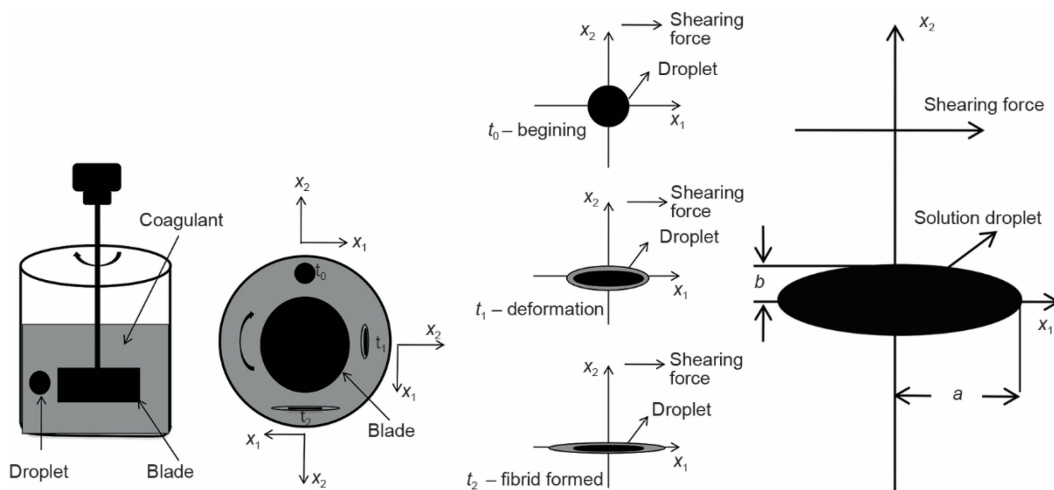
Considering that a drop of aramid solution is suspended in the coagulant, fig. 2, we use a 2-D hyperbolic flow field to describe the drop deformation in a flow field far from the drop:

$$\mu_1 = kx_1 \quad (1)$$

$$\mu_2 = kx_2 \quad (2)$$

$$\mu_3 = kx_3 \quad (3)$$

$$\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1 \quad (4)$$



**Figure 1. Deformation of meta-aramid droplet under shearing coagulant**

**Figure 2. Fibril in the  $x_1$ - $x_2$  plane**

where  $\mu_1$ ,  $\mu_2$ , and  $\mu_3$  are the velocity components in the  $x_1$ ,  $x_2$ , and  $x_3$  directions, respectively. Assuming that the fluids inside and outside the drop exhibit a Newtonian behavior, we demonstrate that an initial spherical drop is deformed into an ellipsoidal drop with semi-axes  $a$ ,  $b$ , and  $c$  in the  $x_1$ ,  $x_2$ , and  $x_3$  directions, respectively. The distance of semi-axes  $c$  is also approximately constant during this deformation. The interfacial tension between the drop and the suspending medium is set to 0. This assumption corresponds to the practical condition observed in fibril formation because the polymer solvent and the coagulant are miscible. An ini-

tial spherical drop is deformed into an ellipsoidal drop with semi-axes  $a$ ,  $b$ , and  $c$  in the  $x_1$ ,  $x_2$ , and  $x_3$  directions, respectively. The diffusion coefficient of a viscous droplet is assumed negligible relative to that of the coagulant:

$$\frac{\partial C}{\partial t} + \mu_1 \frac{\partial C}{\partial x_1} + \mu_2 \frac{\partial C}{\partial x_2} + \mu_3 \frac{\partial C}{\partial x_3} = D' \left( \frac{\partial^2 C}{\partial x_1^2} + \frac{\partial^2 C}{\partial x_2^2} + \frac{\partial^2 C}{\partial x_3^2} \right) \quad (5)$$

where  $t$  and  $C$  express the time and mass concentration of the coagulant, respectively. A smaller drop of dimensions in the  $x_1$  and  $x_3$  directions is formed because the effect of diffusion outside the drop is restricted to a thick boundary layer. Flow field is assumed the same as that inside the drop throughout this diffusion boundary layer because the boundary layer thickness is lower than any typical length scale of the low-Reynolds-number flow outside the drop:

$$\mu_1 = kx_1 \quad (6)$$

$$\mu_2 = -kx_2 \quad (7)$$

$$\mu_3 = 0 \quad (8)$$

where  $k$  changes over time,  $t$ . The velocity field inside the drop generates a flow outside the drop. The velocity field is far from the drop, whereas the velocity is continuous on the drop surface. Thus, eq. (5) governing coagulant concentration is expressed:

$$\frac{\partial C}{\partial t} + kx_1 \frac{\partial C}{\partial x_1} - kx_2 \frac{\partial C}{\partial x_2} = D' \left( \frac{\partial^2 C}{\partial x_1^2} + \frac{\partial^2 C}{\partial x_2^2} \right) \quad (9)$$

The diffusion constant,  $D'$ , is the same inside and outside the drop, and no variation is observed in the concentration profile in the  $x_3$  direction. The latter assumption is only valid when the drop is not too far from the central plane  $x_3 = 0$ , where the drop thickness is approximately constant and equal to  $b$ . According to Kalb's theory [12], the following equation is obtained when  $i \geq 3$ :

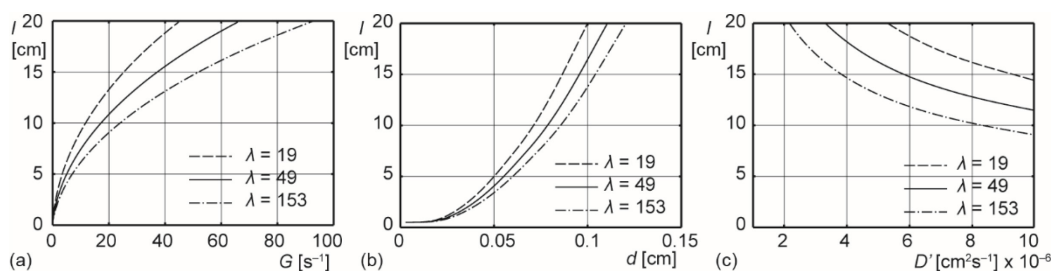
$$T(t) - T(t') = \frac{2D'}{Gd^2} \int_3^i \left[ \frac{4\lambda}{i^{*3}} \left( \ln \frac{4i^{*2}}{i^*} - \frac{3}{2} \right) + 1 \right] i^* di^* \quad (10)$$

where  $T(t)$  is the time function  $T(t) - T(t') \approx T(t)$ , which can be calculated for different polymer solutions,  $G$  – the shear rate,  $d$  – the original diameter of the droplet,  $D'$  – the diffusion coefficient of coagulant in the polymer solution,  $\lambda$  – the viscosity ratio of the polymer solution and the coagulation bath, and  $i$  – the dimensionless length of fibril, which is the ratio of the length after deformation to the original diameter of droplet, and  $i^*$  – the dimensionless length of fibril, which is the ratio of the length after deformation to diameter of droplet at deformation time. For example,  $T(t) \approx 1.085$  for meta-aramid, the following expression is obtained to predict the length of meta-aramid fibril:

$$\frac{i^2}{8\lambda} - \frac{1.89}{i} - \frac{1.125}{\lambda} + 0.51 \approx \frac{0.034Gd^2}{D'\lambda} \quad (11)$$

Equation (11) can be used to predict the effect of  $G$ ,  $\lambda$ ,  $D'$ , and  $d$  on the final length of the fibril. The influence of various parameters on the fibril length is shown in fig. 3. Theo-

retical data show that fibril length is directly related to  $G$  and the radius of the initial droplet  $d$ . By comparison, fibril length is inversely related to  $D'$  and  $\lambda$ .



**Figure 3.** Relationship between (a) fibril length and shear rate,  $G$ , (b) diameter of droplet,  $d$ , and (c) diffusion coefficient,  $D'$

It was assumed that the solution droplet does not break during elongation in the shearing coagulation bath. However, the droplet may break when the shearing rate is sufficiently high. As such, a new fibril model with a high-shearing coagulant should be established to describe fibril formation accurately.

## Conclusions

A mathematical model is used to describe the formation of meta-aramid fibril at a low shear rate. The formation of meta-aramid fibril usually involves a two-step process. A viscous polymer solution is initially dispersed as a droplet in the shearing coagulant. Droplet deformation occurs in the turbulent flow, where a polymer droplet is elongated by shearing force. Coagulation subsequently occurs when a sufficient amount of coagulant is diffused across the boundary between a deformed droplet and a coagulation medium. Thus, a solid fiber-like particle is fabricated. The length and morphological characteristics of fibrils are determined on the basis of shearing coagulants and polymer solutions.

Theoretical data reveal that fibril length is directly related to  $G$  and the radius of the initial droplet  $d$ . However, fibril length is inversely related to  $D'$  and  $\lambda$ .

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