Numerical Study of Heat Transfer of a Micropolar Fluid through a Porous Medium with Radiation

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Abstract

An efficient Spectral Collocation method based on the shifted Legendre polynomials was applied to get solution of heat transfer of a micropolar fluid through a porous medium with radiation. A similarity transformation is applied to convert the governing equations to a system of nonlinear ordinary differential equations. Then, the shifted Legendre polynomials and their operational matrix of derivative are used for producing an aproximate solution for this system of nonlinear differential equations. The main advantage of the proposed method is that the need for guessing and correcting the initial values during the solution procedure is eliminated and a stable solution with good accuracy can be obtained by using the given boundary conditions in the problem. A very good agreement is observed between the obtained results by the proposed Spectral Collocation method and those of previously published ones.

Keywords: Micropolar fluid; Porous medium; Spectral Collocation method; Shifted Legendre polynomials

1 Introduction

Since the time of Fourier, orthogonal functions and polynomials have been used in the analytic study of differential equations and their applications for numerical solution of ordinary differential equations refer, at least, to the time of Lanczos (1938). It is well known that the eigenfunctions of certain singular Sturm-Liouville problems such as Legendre or Chebyshev orthogonal polynomials allow the approximation of functions $C^{\infty}[a,b]$ where truncation error approaches zero faster than any negative power of the number of basic

functions used in the approximation, as that number (order of truncation N) tends to infinity. This phenomenon is usually referred to as spectral accuracy [1–3]. The collocation approach appears to have been first used by Slater and by Kantorovic (1934) in specific applications. This approach is especially attractive whenever it applies to variable-coefficient and even nonlinear problems [4]. Some major advantages of the collocation methods are as follow:
(i) Since no integration is required, the construction of the final system of equations is very efficient
(ii) The functions must be evaluated only at the collocation nodes in contrast to other methods
(iii) Computational cost of calculating nonlinear terms is reasonably low with good numerical accuracy.

The spectral collocation method has been applied for numerical solution of different kind of differential and integral equations. For example, it has been used for deriving approximate solution of stochastic Burgers equation [4], Burgers-type equation [5], Navier-Stokes equations [6], two-point boundary value problem in modelling viscoelastic flows [7], Poisson equation in polar and cylindrical coordinates [8], Volterra integral equations [9,10], compressible flow, two-dimensional and axisymmetric boundary layer problems [11], hypersonic boundary layer stability [12], Helmholtz and variable coefficient equations in a disk [13] and Burgers-Huxley equation [14].

The Legendre polynomials [1] are well known family of orthogonal polynomials on the interval $[0, 1]$ of the real line. These polynomials present very good properties in the approximation of functions. Therefore, Legendre polynomials appear frequently in several fields of mathematics, physics and engineering. Spectral methods based on Legendre polynomials as basis functions for solving numerically differential equations have been used by many authors, (see for example [15–17]).

The problem of micropolar fluids past through a porous media has received much attention in several industrial and engineering processes such as porous rocks, foams and foamed solids, aerogels, alloys, polymer blends and microemulsions. The simultaneous effects of a fluid inertia force and boundary viscous resistance upon flow and heat transfer in a constant porosity porous medium were analyzed by Vafai and Tien [18]. Raptis [19] investigated boundary layer flow of a micropolar fluid through a porous medium. Abo-Eldahab and El Gendy [20] considered the convective heat transfer past a stretching surface embedded in nondarcian porous medium in the presence of magnetic field. Abo-Eldahab and Ghonaim [21] studied the radiation effect on heat transfer of a micropolar fluid past on unmoving horizontal plate through a porous medium. In [22] the DTM was applied successfully to find the analytical solution of heat transfer of a micropolar fluid through a porous medium with radiation. Rashidi et al. [23] presented complete analytic solution to heat transfer of a micropolar fluid through a porous medium with radiation.

2 Flow Analysis and Mathematical Formulation

Consider a steady two-dimensional flow of an incompressible micropolar fluid through a porous medium past a continuously semi-infinite horizontal plate (Fig. ??). The governing equations of boundary layer to micropolar fluid through a porous medium are given as follows [21–23]

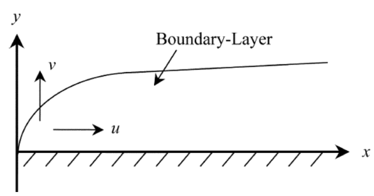


Figure 1: Geometry of the problem and coordinate system.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + k_1\frac{\partial \sigma}{\partial y} + \frac{\nu\varphi}{K}(U-u) + C\varphi\left(U^2 - u^2\right), \tag{2}$$

$$G_1\frac{\partial^2 \sigma}{\partial y^2} - 2\sigma - \frac{\partial u}{\partial y} = 0, \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial u}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \tag{4}$$

subject to the following boundary conditions

$$\begin{aligned} &u = 0, v = 0, \sigma = 0, T = T_w \quad as \quad y = 0, \\ &u = U_0, \sigma = 0, T = T_\infty \quad as \quad y \to \infty, \end{aligned} \tag{5}$$

where $k_1 = \rho S$ and $\nu = \frac{\mu + S}{\rho}$. By using the Rosselant approximation, we have

$$q_r = \frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \tag{6}$$

where σ^* the Stefan-Boltzman constant and k^* the mean absorption coefficient. If the flux is sufficiently small, T^4 can be expanded as a Taylor series about T. By neglecting higher order terms of the Taylor series, we have

$$T^4 \cong 4T^3 T_\infty - 3T_\infty^4. \tag{7}$$

By substitution from equations (6) and (7) in equation (4) we have

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^*}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2}. \tag{8}$$

By introducing the following similarity transforms [21, 24, 25]

$$\begin{aligned} &\psi(x,y) = (2\nu U_0 x)^{\frac{1}{2}} f(\eta), \quad \sigma = \left(\tfrac{U_0}{2\nu x}\right)^{\frac{1}{2}} U_0 g(\eta), \\ &\eta = \left(\tfrac{U_0}{2\nu x}\right)^{\frac{1}{2}} y, \qquad \theta = \tfrac{T - T_\infty}{T_w - T_\infty}, \end{aligned} \tag{9}$$

where ψ is the stream function and is defined as $u = \frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$, the governing equations (1)-(4) are reduced to the following system of ordinary differential equations

$$f''' + ff'' + \Delta g' + \frac{1}{M}\left(1 - f'\right) + N\left(1 - f'^2\right) = 0, \tag{10}$$

$$Gg'' - 2\left(2g + f''\right) = 0, \tag{11}$$

$$(3R + 4)\,\theta'' + 3R\,\mathrm{Pr}\, f\theta' = 0, \tag{12}$$

subject to the boundary conditions

$$\begin{aligned} &f(0) = 0, \quad f'(0) = 0, \quad g(0) = 0, \quad \theta(0) = 1, \\ &f'(\infty) = 1, \quad g(\infty) = 0, \quad \theta(\infty) = 0, \end{aligned} \tag{13}$$

where

$$\begin{aligned} &Pr = \tfrac{\rho\nu c_p}{k} \quad \text{(Prandtl number)}, \\ &\Delta = \tfrac{k_1}{\nu} \quad \text{(Coupling constant)}, \\ &G = \tfrac{G_1 U_0}{\nu x} \quad \text{(Microrotation parameter)}, \\ &R = \tfrac{k^* k}{4\sigma^* T_\infty^3} \quad \text{(Radiation parameter)}, \\ &M = \tfrac{K U_0}{2\varphi\nu x} \quad \text{(Permeability parameter)}, \\ &N = 2C\varphi\nu x \quad \text{(Inertia coefficient parameter)}. \end{aligned} \tag{14}$$

3 Shifted Legendre Polynomials and their Properties

The well known Legendre polynomials are defined on the interval and can be determined with the aid of the following recurrence formulae [1, 26–28]

$$L_{m+1}(t) = \frac{2m+1}{m+1}\, t\, L_m(t) - \frac{m}{m+1}\, L_{m-1}(t), \qquad m = 1,\ 2,\ 3,\ ..., \tag{15}$$

where $L_0(t) = 1,\ \ L_1(t) = t$. In order to use Legendre polynomials on the interval $[0, 1]$ we define the so-called shifted Legendre polynomials by introducing the change of variable $t = 2x - 1$. The orthogonality condition for these polynomials is

$$\int_0^1 P_m(x) P_n(x)\, dx = \begin{cases} \frac{1}{2m+1} & for\ m = n, \\ 0 & for\ m \neq n. \end{cases} \tag{16}$$

A function $f(t)$ definedover $[0, 1]$ may be expanded in the terms of shifted Legendre polynomials as

$$f(t) = \sum_{k=0}^{\infty} c_k P_k(t), \tag{17}$$

where $c_k = (f(t), P_k(t))$, in which $(.,.)$ denotes the inner product. If theinfinite series in Eq. (17) is truncated, then it can be written as

$$f(t) = \sum_{k=0}^{N} c_k P_k(t) = C^T \Phi(t), \tag{18}$$

where C and $\Phi(t)$ are $(N + 1)$ vectors given by

$$C^T = [c_1, c_2, ..., c_N]\,, \qquad \Phi(t) = [P_1(t), P_2(t), ..., P_N(t)]\,. \tag{19}$$

In the next theorem we derived a relation between shifted Legendre polynomials and their derivatives that is very important for deriving the operational matrix of derivative for shifted Legendre polynomials.

Theorem 1. *[26–28] Let $\Psi(t)$ be the Legendre polynomial vector defined as*

$$\Phi(t) = [P_0(t), P_1(t), ..., P_N(t)], \tag{20}$$

the derivative of this vector can be expressed by

$$\frac{d\Phi(t)}{dt} = D\Phi(t), \tag{21}$$

which D is $(N + 1) \times (N + 1)$ matrix and its (i, j)-th element is defined as below

$$D_{i,j} = \begin{cases} 2(2j-1) & j = 1, ..., i-1\ and\, (i+j)\, odd, \\ 0 & o.w. \end{cases} \tag{22}$$

4 Method of Solution

Consider the coupled nonlinear differential equations (7)-(8) subject to boundary conditions (10). By using change of variable

$$t = \frac{\eta}{\eta_\infty}, \quad F(t) = f(t\eta_\infty), \quad H(t) = g(t\eta_\infty), \quad \vartheta(t) = \theta(t\eta_\infty), \tag{23}$$

we have the following nonlinear differential systems in the interval $[0, 1]$,

$$\frac{d^3F}{dt^3} + \eta_\infty F\frac{d^2F}{dt^2} + \eta_\infty^2 \Delta \frac{dH}{dt} + \frac{1}{M}\left(\eta_\infty^3 - \eta_\infty^2 \frac{dF}{dt}\right) + N\left(\eta_\infty^3 - \eta_\infty\left(\frac{dF}{dt}\right)^2\right) = 0, \tag{24}$$

$$G\frac{d^2H}{dt^2} - 2\eta_\infty^2\left(2H + \frac{d^2F}{dt^2}\right) = 0, \tag{25}$$

$$(3R + 4)\,\frac{d^2\vartheta}{dt^2} + 3\eta_\infty R \,\mathrm{Pr}\, F\frac{d\vartheta}{dt} = 0, \tag{26}$$

the boundary conditions become

$$\begin{aligned} &F(0) = 0, \quad F'(0) = 0, \quad H(0) = 0, \quad \vartheta(0) = 1, \\ &F'(1) = \eta_\infty, \quad H(1) = 0, \quad \vartheta(1) = 0. \end{aligned} \tag{27}$$

Now we expand the unknown function $F(t), H(t)$ and $\vartheta(t)$ by the shifted Legendre polynomial into interval $[0, 1]$ as

$$F(t) \simeq C_1^T \Phi(t), \qquad H(t) \simeq C_2^T \Phi(t), \qquad , \vartheta(t) \simeq C_2^T \Phi(t), \tag{28}$$

where C_1, C_2 and C_3 are the unknown shifted Legendre polynomial coefficient vectors defined in (19). By using the operational matrix derived in (21) we get

$$\frac{dF}{dt} \simeq C_1^T D\Phi(t), \qquad \frac{d^2F}{dt^2} \simeq C_1^T D^2\Phi(t), \qquad \frac{d^3F}{dt^3} \simeq C_1^T D^3\Phi(t), \tag{29}$$

$$\frac{dH}{dt} \simeq C_2^T D\Phi(t), \qquad \frac{d^2H}{dt^2} \simeq C_2^T D^2\Phi(t), \tag{30}$$

$$\frac{d\vartheta}{dt} \simeq C_3^T D\Phi(t), \qquad \frac{d^2\vartheta}{dt^2} \simeq C_3^T D^2\Phi(t), \tag{31}$$

substituting Eqs. (29)-(31) in (24)-(26), we have

$$C_1^T D^3\Phi(t) + \eta_\infty(C_1^T D\Phi(t))(C_1^T D^2\Phi(t)) + \eta_\infty^2 \Delta C_2^T D\Phi(t)$$

$$+\frac{1}{M}\left(\eta_\infty^3-\eta_\infty^2 C_1^T D\Phi(t)\right)+N\left(\eta_\infty^3-\eta_\infty\left(C_1^T D\Phi(t)\right)^2\right)=0, \tag{32}$$

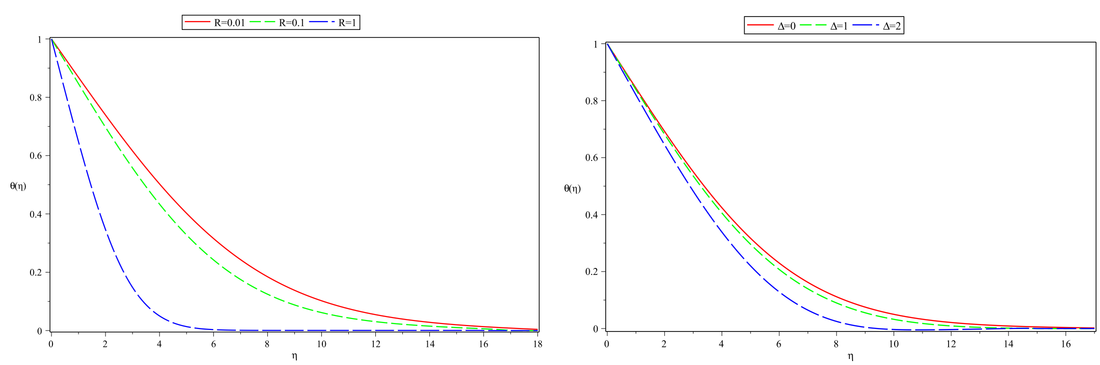
$$GC_2^T D^2\Phi(t)-2\eta_\infty^2\left(2C_2^T\Phi(t)+C_1^T D^2\Phi(t)\right)=0, \tag{33}$$

$$(3R+4)\, C_3^T D^2\Phi(t)+3\eta_\infty R\,\mathrm{Pr}\, C_1^T\Phi(t)C_3^T D\Phi(t)=0. \tag{34}$$

Moreover, boundary conditions (27) result

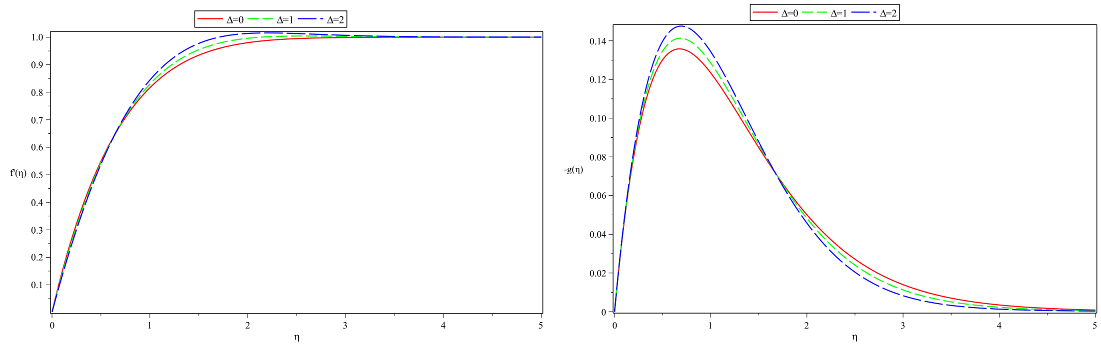
$$\begin{array}{l} C_1^T\Phi(0)=0, \quad C_1^T D\Phi(0)=0, \quad C_2^T\Phi(0)=0, \quad C_3^T\Phi(0)=1, \\ C_1^T D\Phi(1)=\eta_\infty, \quad C_2^T\Phi(1)=0, \quad C_3^T\Phi(1)=0. \end{array} \tag{35}$$

to find the approximate solution of the nonlinear system (24)-(26), we use the typical collocation method and collocate Eqs. (33) and (34) at $(M-1)$ different points and Eq. (32) at $(M-2)$ different points in the interval $[0,1]$. For choosing suitable collocation points, we use the first roots of shifted Legendre $P_{M+1}(t)$. These equations together with equations in (35) generate $3(M+1)$ nonlinear equations. The well-known Newton-Raphson have been used for approximate solution of derived nonlinear systems. After finding the solution of this nonlinear systems we obtain unknown vectors C_1, C_2 and $C3$. By substituting these vectors in Eq. (28) the solution functions $F(t), H(t)$ and $\vartheta(t)$ can be approximated. Now, the change of variable in Eq. (23) resuls approximation of functions $f(\eta), g(\eta)$ and $\theta(\eta)$.



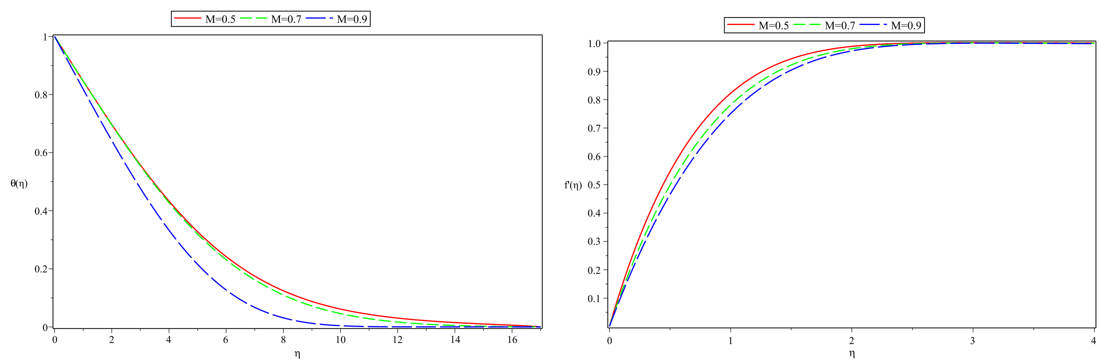
(a) (b)

Figure 2: Variation of the dimensionless temperature: (a) $M=\Delta=0.5, N=0.1, G=2, Pr=0.7$ and different values of R. (b) $M=0.5, R=N=0.1, G=2, Pr=0.7$ and different values of Δ.



(a) (b)

Figure 3: Variation of the dimensionless velocity: (a) $M = 0.5, R = N = 0.1, G = 2, Pr = 0.7$ and different values of Δ. (b) $M = 0.5, R = N = 0.1, G = 2, Pr = 0.7$ and different values of Δ.



(a) (b)

Figure 4: (a) Variation of the dimensionless temperature $\Delta = 0.5, R = N = 0.1, G = 2, Pr = 0.7$ and different values of M. (b) Variation of the dimensionless velocity for $\Delta = 0.5, R = N = 0.1, G = 2, Pr = 0.7$ and different values of M.

5 Numerical results

In this section, the Legendre Collocation method presented in Section 4 was applied to approximate solution of the nonlinear differential Eqs. (10)-(12) subject to the boundary conditions (13). In order to verify the results of this study, the results have been compared with previously published results from the literature. Table 1 shows the values of $f''(0), -g'(0)$ and $-\theta'(0)$ for different values of permeability parameter, M, coupling constant, Δ, radiation parameter, R and $G = 2, N = 0.1, Pr = 0.7$. The results reveal that as the permeability parameter increase the wall temperature gradient $\theta'(0)$ and rate of change $g'(0)$ are increase while the shear stress decrease. Moreover, as the radiation parameter decrease, the wall temperature gradient $\theta'(0)$ increases and shear stress $f''(0)$

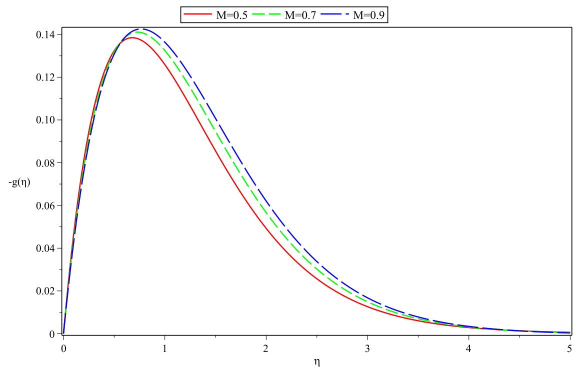


Figure 5: Variation of the dimensionless angular velocity g for $\Delta = 0.5, R = N = 0.1, G = 2, Pr = 0.7$ and different values of M.

Table 1: Variation of $f''(0), -g'(0)$ and $-\theta'(0)$ for different values of M, Δ, R and $G = 2, N = 0.1, Pr = 0.7$.

			$f''(0)$		$-g'(0)$		$-\theta'(0)$	
M	Δ	R	Present	Ref. [21]	Present	Ref. [21]	Present	Ref. [21]
0.5	0	0.1	1.519462	1.519402	0.533807	0.533808	0.203431	0.203438
0.5	0.5	0.01	1.473758	1.473730	0.534186	0.534188	0.170620	0.170621
0.5	0.5	0.1	1.473758	1.473730	0.534186	0.534188	0.203579	0.203581
0.5	0.5	1	1.473758	1.473730	0.534186	0.534188	0.359838	0.359834
0.5	0.5	2	1.473758	1.473730	0.534186	0.534188	0.413383	0.413389
0.5	0.5	4	1.473758	1.473730	0.534186	0.534188	0.453060	0.453065
0.5	0.5	10	1.473758	1.473730	0.534186	0.534188	0.484063	0.484069
0.6	0.5	0.1	1.360425	1.360340	0.516888	0.516889	0.202930	0.202933
0.7	0.5	0.1	1.273568	1.273590	0.502783	0.502788	0.202394	0.202391

and rate of change $g'(0)$ have no changes. Figs. 2a-4a show numerical solution derived by the Legendre Collocation method for various values of permeability, vortex-viscosity and radiation parameters. The effect of radiation on the dimensionless temperature θ profile are presented in Fig. 2a. Figs. 2b-3b show the effects of the vortex-viscosity parameter Δ on the velocity of the fluid, dimensionless temperature and angular velocity of the microstructures. It is easily concluded from this Figs. that as Δ increase, the fluid velocity and temperature of the microstructure increase. The variation of permeability parameter M and its effects on the velocity of fluid, temperature distribution and angular velocity of microstructures are plotted in Figs. 4a-5.

6 Conclusion

In this study, we have introduced an efficient Spectral Collocation method based on the shifted Legendre polynomials to get solution of heat transfer of a micropolar fluid through a porous medium with radiation. This proposed approach is simple in applicability, as it does not require initial values during the solution procedure. Moreover, by using the given boundary conditions in the problem, a stable solution with very good results can be obtained. The effects of permeability, vortex-viscosity and radiation parameters are examined on the velocity of fluid, temperature distribution and angular velocity of microstructures. A very good agreement is observed between the obtained results of by the proposed Spectral Collocation method and those of previously published ones.

Nomenclature

f	Dimensionless velocity functions	θ	Dimensionless temperature
g	Dimensionless microrotation angular velocity	ν	Kinematics viscosity
G_1	Microrotation constant	c_p	Specific heat
σ	Angular velocity	q_r	The radiate heat flux
σ^*	Stefan-Boltzman constant	U_0	Uniform stream velocity
Pr	Prandtl number	φ	Porosity
T	Temperature distribution	k^*	Mean absorption coefficient
μ	Dynamical viscosity	K	Permeability of the porous medium
S	Constant characteristic to the fluid	ψ	Stream function
u	Velocity in x-direction	L_m	Legendre polynomial
v	Velocity in y-direction	P_m	Shifted Legendre polynomial
x	Distance along the surface	Ψ	Shifted Legendre polynomial vector
y	Distance normal to the surface x and y axes	C_1, C_2, C_3	Shifted Legendre polynomial coeffcient
ρ	Density of fluid	D	Operational matrix of derivative

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