

PROGNOSTIC POTENTIAL OF FREE CONVECTION MODELS FOR ANALYSIS OF THERMAL CONDITIONS OF HEAT SUPPLY OBJECTS

by

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The article shows the results of mathematical simulation of convective turbulent heat transfer in the closed domain with heat-conducting walls and the source of heat emission. The system of equations in the model of thermal conductivity for the solid walls and Navier-Stokes equations for gas are solved. The article examines the possible versions of the calculation of the turbulent regime in the geometrically simple air region by means of conducting the simulation within the framework of algebraic models (Van Driest and Prandtl-Reichardt), and $k-\varepsilon$ model. On the basis of the obtained results the authors made a conclusion about the possibility of applying the algebraic model of Prandtl to describe the integral characteristics of turbulent flows in the conditions of natural convection in a geometrically simple area when the air heated by the heat source is moved by the lifting force. Besides, the temperature fields for a typical real object of heat supply are simulated in the article. The values of the dimensionless heat exchange coefficient at the air-wall interface are determined. The comparative analysis of two quite significantly different approaches to determine the average temperature in the heated room, i. e. the traditional balance approach and the approach based on the considered system of partial differential equations is executed. It is concluded that the balance models of the calculation of the temperature regime can adequately describe real temperatures of heat supply objects only for very large values of the characteristic times of the processes in question.

Key words: thermal conditions, heat supply object, free convection, mathematical modeling, turbulent regime

Introduction

The problem of secure energy supply has surely become the most urgent for all developed countries of the world community recently. The attempts to solve the problem in question by means of the increased consumption of oil, gas and coal as the main energy resources over the last 10-15 years have proven to result in heightened international tension and lost common survival principles for the world community. The seemingly paradoxical downturn in energy prices lower the reasonable level witnessed over the last three to four years can only be explained by the hypothesis that this downturn in oil prices is speculative in its character. The speculative character of this downturn is the only explanation of the situation academic economists can suggest implying both direct and figurative meanings of this word. That is why the main directions of searching for an efficient solution to the problem of secure energy supply seem to be rather difficult to predict in the decades to come.

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However, there is a sphere developing which any country can surely raise the level of its energy supply security. This is energy efficiency. Such energy efficiency problems are not possible to solve applying only an empirical approach. It is necessary to develop a theory which can create premises to solve concrete practical problems. In relation to the problem of heat supply to industrial and public facilities such a theory is a theory of heat transfer providing a possibility of justified choice of the heat source density taking into consideration the main significant factors determining the energy rejection rate from the area the thermal conditions of which are necessary to provide.

Such a theory can only be developed taking into account spatial temperature and heat transfer rate inhomogeneity in the analysis area which is the heat supply object. Besides, it is necessary to describe the conduction and convection processes in the air filling a particular area as well as the conduction processes in the building envelope. Such problems in the heat transfer theory are commonly called conjugate [1]. According to the model analysis [2] heat transmission capacity and the capability of the building envelope to accumulate heat influence the heat transfer rate in the air surrounding the main heat supply object which is the human being.

The problems of natural convection in the areas of different configuration have been intensively solved over the last 30 to 40 years. Among several thousand articles devoted to natural convection published over the last years typical examples are models [3-6]. Much less attention has been paid to the problems of conjugate heat transfer under the conditions of natural convection. However, a number of solutions being of interest both from the fundamental and practical points of view have been obtained for the small characteristic dimensions areas.

The number of problems of conjugate heat transfer under the conditions of natural convection solved for the large characteristic dimensions areas which are typical heat supply objects is really very small [7-10]. In general it is connected with the necessity to simulate the processes of turbulent natural convection of high Grashof or Rayleigh numbers. The difficulties in solving such problems are objective and known [11-14] even while using relatively simple turbulence models of κ - ε or κ - ω types. Calculation for any simulation of the processes of turbulent natural convection in the areas with characteristic dimensions of one to ten meters is very time consuming. At the same time simple algebraic turbulence models [12-14] efficient while solving a number of heat transfer problems were developed quite a long time ago. In this respect it appears relevant to analyze the possibility to use algebraic turbulence models to solve the conjugate problems of heat transfer under the conditions of turbulent natural convection in the areas with characteristic dimensions of one to ten meters. Besides, it is necessary to take into account the results of the experimental research into such processes [15, 16] which show that under real-life conditions of heat transfer there appear time-variable large temperature gradients both in the air and in the building envelope. That is why the simulation seems appropriate at least in a flat unsteady design.

The purpose of this work is to justify the possibility and conditions of using algebraic turbulence models to solve the conjugate problems of heat transfer under the conditions of turbulent heat-gravitational convection in the areas with characteristic dimensions corresponding to those of typical heat supply objects which are industrial and public facilities.

Problem statement and solution method

The heat transfer problem is defined for a rectangular area including, fig. 1, an air-filled cavity (1) and a building envelope (2), *i. e.* walls of final thickness which conduct and accumulate heat.

$$\frac{\Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y} = \frac{\partial}{\partial X} \left[\left(\frac{1}{\text{Pr} \sqrt{\text{Gr}}} + \frac{1}{\text{Pr}_t \text{Re}_t} \right) \frac{\partial \Theta}{\partial X} \right] + \frac{\partial}{\partial Y} \left[\left(\frac{1}{\text{Pr} \sqrt{\text{Gr}}} + \frac{1}{\text{Pr}_t \text{Re}_t} \right) \frac{\partial \Theta}{\partial Y} \right] \quad (2)$$

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega \quad (3)$$

$$\begin{aligned} \frac{\partial E}{\partial \tau} + U \frac{\partial E}{\partial X} + V \frac{\partial E}{\partial Y} = \frac{\partial}{\partial X} \left[\left(\frac{1}{\sqrt{\text{Gr}}} + \frac{1}{\sigma_\varepsilon \text{Re}_t} \right) \frac{\partial E}{\partial X} \right] + \frac{\partial}{\partial Y} \left[\left(\frac{1}{\sqrt{\text{Gr}}} + \frac{1}{\sigma_\varepsilon \text{Re}_t} \right) \frac{\partial E}{\partial Y} \right] + \\ + c_{1\varepsilon} (\tilde{P}_k + c_{3\varepsilon} \tilde{G}_k) \frac{E}{K} - c_{2\varepsilon} \frac{E^2}{K} \end{aligned} \quad (4)$$

$$\frac{\partial K}{\partial \tau} + U \frac{\partial K}{\partial X} + V \frac{\partial K}{\partial Y} = \frac{\partial}{\partial X} \left[\left(\frac{1}{\sqrt{\text{Gr}}} + \frac{1}{\sigma_k \text{Re}_t} \right) \frac{\partial K}{\partial X} \right] + \frac{\partial}{\partial Y} \left[\left(\frac{1}{\sqrt{\text{Gr}}} + \frac{1}{\sigma_k \text{Re}_t} \right) \frac{\partial K}{\partial Y} \right] + \tilde{P}_k + \tilde{G}_k - E \quad (5)$$

where $\tilde{P}_k = \frac{1}{\text{Re}_t} \left[2 \left(\frac{\partial U}{\partial X} \right)^2 + 2 \left(\frac{\partial V}{\partial X} \right)^2 + \left(\frac{\partial U}{\partial X} + \frac{\partial V}{\partial X} \right)^2 \right]$ and $\tilde{G}_k = -\frac{1}{\text{Re}_t \text{Pr}_t} \frac{\partial \Theta}{\partial Y}$

– for the solid walls:

$$\frac{1}{\text{Fo}} \frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \quad (6)$$

The model constants are given as: $c_\mu = 0.09$; $c_{1\varepsilon} = 1.44$; $c_{2\varepsilon} = 1.92$; $c_{3\varepsilon} = 0.8$; $\sigma_\varepsilon = 1.3$; $\sigma_k = 1$.

Initial conditions:

$$\Psi(X, Y, 0) = \Omega(X, Y, 0) = \Theta(X, Y, 0) = K(X, Y, 0) = E(X, Y, 0) = 0 \quad (7)$$

Boundary conditions:

– on the external area contour except for the left border the boundary conditions of the second kind are set:

$$\frac{\partial \Theta}{\partial \vec{n}} = 0 \quad (8)$$

– on the left outer boundary:

$$\frac{\partial \Theta}{\partial X} = \text{Bi} \Theta + \text{Bi}(\Theta_e) + N \left[(\Theta)^4 - (\Theta_e)^4 \right] \quad (9)$$

– at the wall-gas interface the following conditions are set:

$$\Psi = 0, \quad \frac{\partial \Psi}{\partial \vec{n}} = 0, \quad \begin{cases} \Theta_1 = \Theta_2 \\ \frac{\partial \Theta_1}{\partial X} = \lambda_{1,2} \frac{\partial \Theta_2}{\partial X} \end{cases} \quad (10)$$

– on the surface of the heater:

$$\Theta = 1 \quad (11)$$

– for the κ - ε model near the solid surface the following is accepted:

$$\frac{\partial K}{\partial n} = 0, \quad E = \frac{c_\mu^{3/4} K^{3/2}}{k \Delta n}$$

For describing the processes of natural convection in the closed domain, fig. 1, under the conditions of the turbulent regime within the framework of algebraic model was used the system of Navier-Stokes differential equations with conditions (7)-(11). The turbulent component was taken into account by introducing the coefficient of turbulent viscosity, eq. (12).

The turbulization of heated air-flow was described in accordance with the main provisions [12-14] on the change in viscosity (ν_t) with increasing air velocity. The calculation of ν_t was carried out within the framework of improved Prandtl model (Van Driest) [12-14]:

$$\nu_t = l_m^2 \left[\left(\frac{\partial \bar{v}_x}{\partial y} \right)^2 + \left(\frac{\partial \bar{v}_y}{\partial x} \right)^2 \right]^{1/2} \quad (12)$$

In addition, the numerical investigations within the framework of Prandtl-Reichardt model are carried out:

$$\nu_t = \chi [U_{\max} - U_{\min}] \delta(x) \quad (13)$$

The results of a numerical solution of these two tasks are given as follows.

Equations (1)-(6) with the corresponding initial and boundary conditions (7)-(11) were solved by the method of finite differences. The algorithm [7, 19], developed for the numerical solution of the tasks of natural convection in a closed rectangular regions with a local energy sources was used.

For eqs. (1) and (3) used a difference scheme of alternating directions. In order that the circuit does not depend on the sign of the velocity, the approximation of the convective terms was considered averaged over U and $|U|$ (V and $|V|$). For the eqs. (2), (4), and (5), the splitting scheme was used. The non-linear boundary condition was realized by a simple iteration method. The equations were solved sequentially, each time step began with the calculation of the temperature field in the gas and solid phases, and Poisson's equations were used to determine the vector potential. Then the vorticity vector components used in the boundary conditions (Woods condition) were calculated and the equations of motion were solved.

Model approval

The developed method and the algorithm to solve problems of turbulent natural convection, namely algebraic models and the standard κ - ϵ model, were tested in solving well-known model problems [11, 17].

A rectangular area with sides of equal height and width was considered. The conditions of heat insulation were set on the upper and lower boundaries, the first kind conditions were set on the vertical boundaries. The solutions were obtained for different Rayleigh numbers ($Ra = GrPr$) corresponding to both laminar ($Ra = 10^4$) and turbulent ($Ra = 10^9$) air-flow.

Figure 2 shows the solution to the problem of heat transfer in a closed system applying three different turbulence models.

In addition to the analysis of temperature fields, a comparison of the Nusselt number for a vertical wall are carried out, table 1.

Table 1. Comparison of present predictions of average Nusselt number with published solutions for turbulent natural convection

| Ra | Present study | | [17] | [11] |
|--------|------------------------|--|-------|-------|
| | k - ϵ model | The algebraic Prandtl-Reichardt model (Van Driest) | | |
| 10^4 | 2.273 | 2.256 (2.264) | – | 2.286 |
| 10^9 | 56.77 | 55.12 (55.38) | 54.49 | 57.35 |

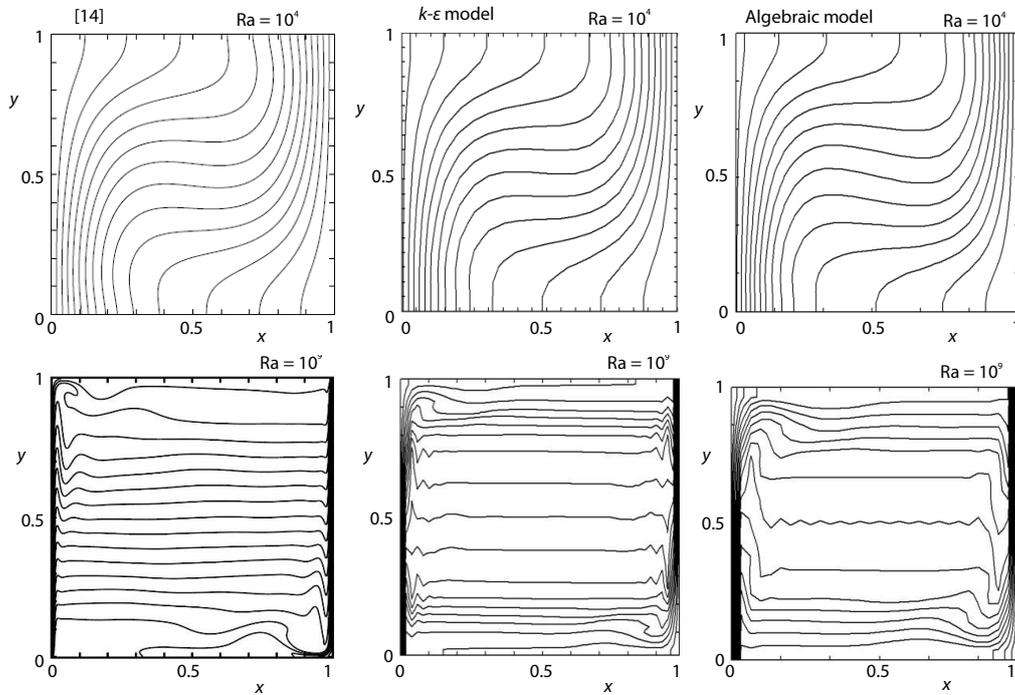


Figure 2. The temperature fields, corresponding to the problem [14] (different models of turbulence)

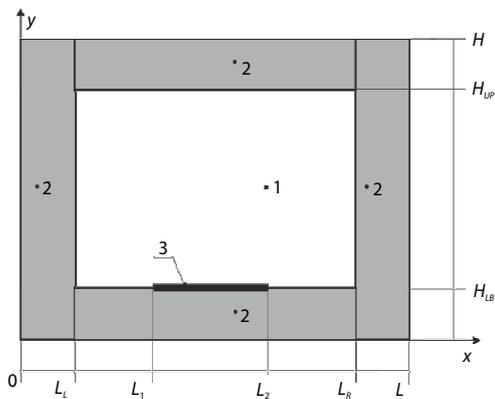


Figure 3. The area of the solution to the problem; 1 – air, 2 – enclosing structures, 3 – heat source

The values of Nusselt numbers (fig. 2, tab. 1) are in good agreement with the known solutions to the considered model problem of natural convection both for laminar and turbulent modes [11].

To assess the possibility to apply algebraic turbulent models while simulating conjugate heat and mass transfer in large heated production buildings ($Ra > 10^{10}$) there was performed numerical investigation in a rectangular enclosed area with a local heat source and solid walls of finite dimensions, fig. 3, applying both Prandtl-Reichardt and Van Driest algebraic turbulence models and $k-\varepsilon$ turbulence models.

The temperature fields, fig. 4, obtained by means of two different approaches to the description of turbulence are quite similar in their structure. The analysis of fig. 4 shows that warm air heated from the heat source goes up and then being cooled while moving along the upper and side boundaries goes down dividing the area into two convection zones different in size and intensity. This results primarily from the differences in heat transfer conditions on the left and right outer boundaries.

While analyzing the numerical investigation results it appears necessary to compare integral parameters such as, for example, the Nusselt number which is of the greatest interest for the inner interface and mean air temperatures, fig. 5, as it is done in [11, 17, 19, 20].

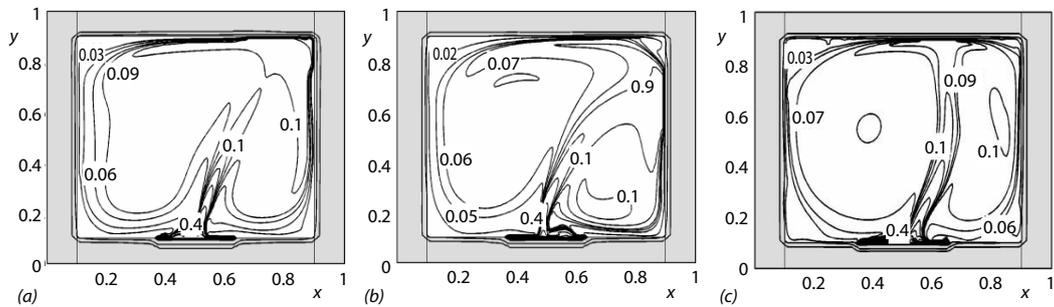


Figure 4. The temperature fields obtained with using three models of turbulence at $\tau = 500$;
 (a) Prandtl-Reichardt, (b) Van Driest, (c) – $k-\varepsilon$ model

The average Nusselt number was proven to grow with time in all three cases, fig. 5a. Non-monotone variation of the integral heat exchange coefficient was observed while applying algebraic turbulence models. Average temperatures of the considered air-filled area were also proven to grow, fig. 5(b). The deviations of θ values obtained applying different turbulence models do not exceed 10%.

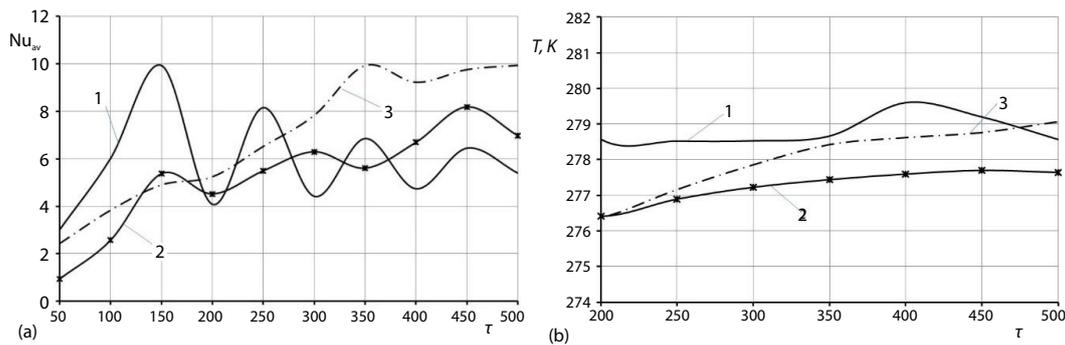


Figure 5. Dependence of the average Nusselt number on time for the border $X = L_R$ (a) and the average air temperature (b) on time; 1 – the Prandtl-Reichardt model, 2 – the Van Driest model, 3 – the $k-\varepsilon$ model

The mathematic simulation results, fig. 5, allow making a conclusion about the possibility to apply Prandtl algebraic models to describe the integral parameters of turbulent heat transfer under conditions of natural convection in a geometrically simple rectangular area. It can also be concluded that a simple model [12-14] can be applied when the movement of air heated from the heat source is provided only by buoyancy.

Due to this, the processes of heat transfer in a typical heat supply object, fig. 1, are simulated applying Prandtl (Van Driest) algebraic turbulence models.

Results and discussion

The numerical solution to the problem (1)-(4) and (7)-(12) was obtained at non-dimensional temperatures: $\theta_{hs} = 1$; $\theta_0 = 0$; $\theta_c = -0.1$. Special attention was paid to stream functions, fig. 6, determining temperature fields, and analyzing the non-stationarity factor determining the variation of convective heat transfer in many practically significant cases. The Rayleigh number in the considered problem setting was 10^{13} which corresponds to developed turbulent natural convection.

The validity of the numerical simulation results was evaluated testing the conservatism of the applied difference scheme by an algorithm developed to solve time-dependent non-linear heat transfer problems [17].

The temperature fields analysis, fig. 6, allows defining the following peculiarities of the considered processes. During the initial period two specific convection zones are formed, figs. 6(a) and 6(c). Further they are transformed as the building envelope is heated.

Due to heat removal from the $X = 0$ boundary into the external low-temperature environment the air in the upper left corner of the considered area, fig. 6, remains heated up to the temperatures about the same as the initial ones.

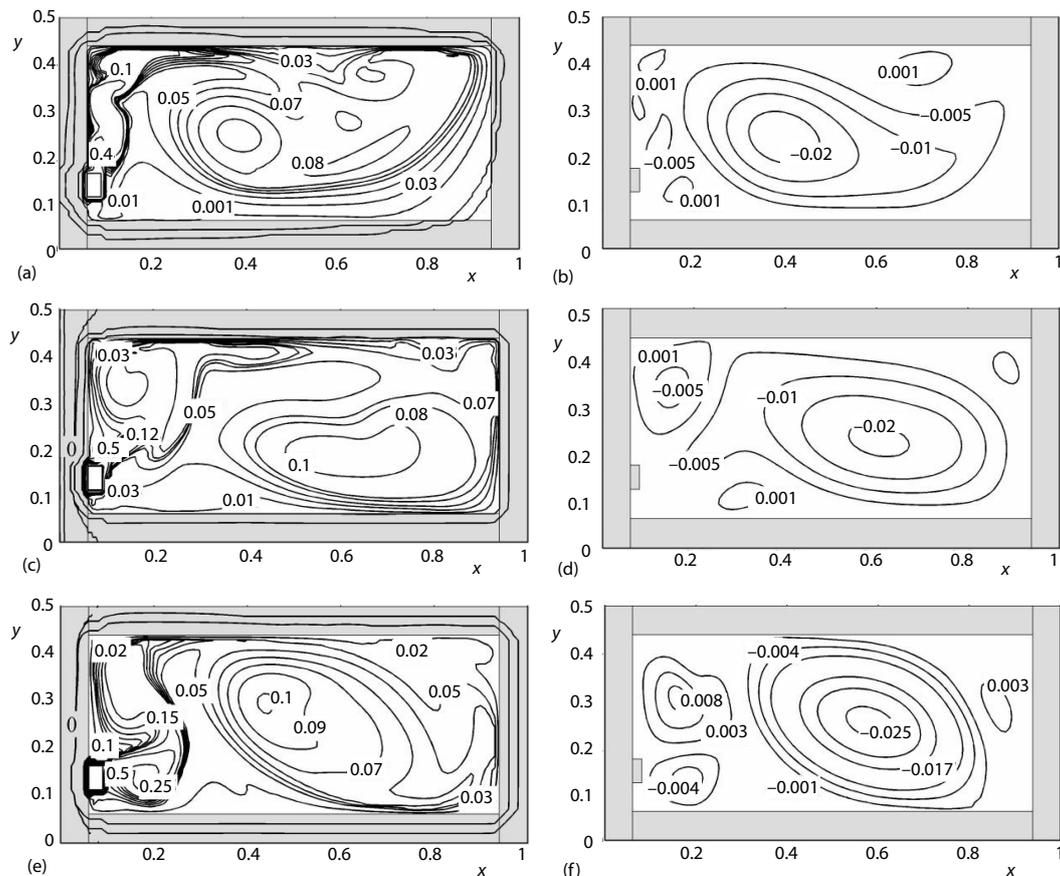


Figure 6. Temperature fields (a, c, e) and the contours of stream function (b, d, f) at different values of τ ; $\tau = 50000$ (a, b), $\tau = 150000$ (c, d), $\tau = 240000$ (e, f)

Figure 7 shows the time distribution of the average Nusselt number on the vertical ($X = L_R$) and horizontal ($Y = H_{UP}$) interface calculated with the equations [17]:

$$\text{Nu}_{any} = \frac{1}{H_{UP} - H_R} \int_{H_R}^{H_{UP}} \left. \frac{\partial \Theta}{\partial X} \right|_{X=L_R} dY \quad \text{Nu}_{anx} = \frac{1}{L_R - L_L} \int_{L_L}^{L_R} \left. \frac{\partial \Theta}{\partial Y} \right|_{Y=H_{UP}} dX$$

The obtained results demonstrate the stabilization of heat transfer modes at the air-sol-id wall interface at $\tau = 2.2 \cdot 10^5$. The values of the heat transfer integral coefficient grow with time, fig. 7. The variation interval for numerical simulation is chosen in the way that the process of heat transfer in wall-adjacent areas reaches the stationary regime.

The $Nu(\tau)$ dependence characterizes significant non-stationarity of the considered process of the temperature field formation in the heat area in quite a long period of time. The latter results from heating the solid walls with subsequent heat accumulation within them during a long time interval. With the time scale of $t_0 = 0.015$ s physical stationarity time reaches 3300 s. Relatively equal values of the Nu numbers on the vertical and horizontal boundaries most likely result from the air-flow character in the right part of the air cavity, fig. 7.

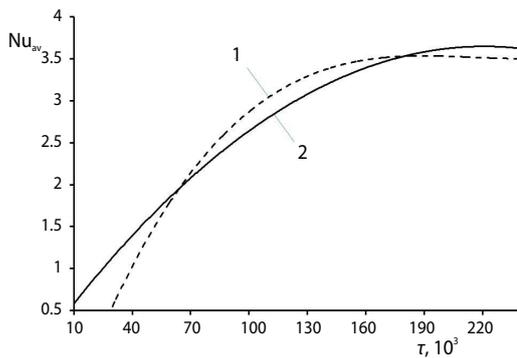


Figure 7. Dependence of the average Nusselt number on the time τ at the interfaces; 1 – $Y = H_{UP}$; 2 – $X = L_R$

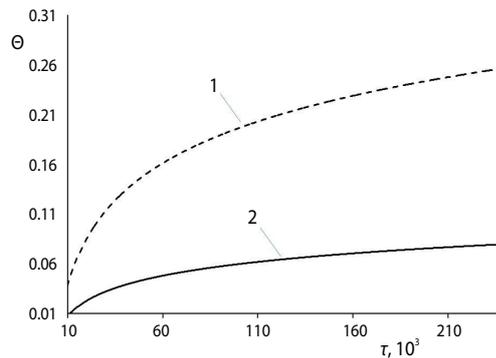


Figure 8. Dependence of the average dimensionless temperature θ on time τ ; 1 – solution based on the balance model [1, 2], 2 – solution of the system of equations (1)-(4) and (7)-(12)

The temperature gradients in wall air layers on the right vertical and upper horizontal boundaries at $\tau = 2.2 \cdot 10^5$ differ insignificantly. In general the comparison of the $\theta(X, Y)$ distribution in figs. 6(a), (c) and (e) allows making a conclusion about the stabilization of heat transfer in wall layers resulting from prolonged heating of solid walls. The isothermal lines shown in fig. 7 clearly demonstrate the movement of the high-temperature area in the building envelope as the τ parameter grows. To evaluate the prognostic potential of the developed heat transfer model in a typical heat supply object two significantly different approaches, fig. 8, namely a traditional balance approach [21-23] and an approach based on a system of eqs. (1)-(4) and (7)-(12), were compared.

It can be clearly seen, fig. 8, that the exclusion of the heat accumulation process in the building envelope from consideration results in real temperatures being more than three times different from those calculated applying a balance model. At the same time the deviation of the parameter with the growing θ parameter increases due to high heat accumulating capacity of solid walls. The numerical investigation results allow making a conclusion that balance models can reliably describe real temperatures of heat supply objects only at very large values of characteristic times of the considered processes. At such values the surface temperatures of the vertical (excluding the left one in fig. 1) and horizontal building envelope reach the values relatively equal to the air temperature. Correspondingly, the intensity of heat removal into that building envelope decreases to the lowest level and the average estimated temperatures obtained applying the two above mentioned different approaches will be relatively equal. In other words, balance models are similar to real processes only under the conditions of long periods of constant temperatures and cool air heat exchange with external surfaces of the building envelope.

Conclusions

The comparative analysis of the results of applying Prandtl-Reichardt and Van Driest algebraic turbulence models and the standard κ - ε turbulence model to solve the problems of convection heat transfer in large industrial heat supply objects showed the possibility of their application to describe the integral parameters of thermophysical processes under the conditions of developed natural turbulent convection ($10^8 < Ra < 10^{13}$) in enclosed conjugate rectangular areas.

Two significantly different approaches to the determination of average temperature in a heated area, namely a traditional balance approach and an approach based on the considered system of partial differential equations, were compared. The exclusion of the heat accumulation process in the building envelope from consideration (the balance model) while determining the thermal conditions of the heated area was proven to result in the average temperature values being more than three times different from those obtained from numerical simulation during the initial period which is quite long.

The values of temperatures and integral heat transfer parameters obtained while solving the problem of heat transfer in a typical heat supply object demonstrate the possibility to significantly increase the accuracy of predicting the parameters of thermal conditions of such objects applying the mathematic models taking into account combined convection and heat transfer capacity processes in the system of the air cavity and the building envelope, *i. e.* solving the conjugate problems of heat transfer.

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Nomenclature

| | | | |
|---|---|----------------------|---|
| A_0^+ | – closing ratio, [–] | T_0 | – temperature at the initial time, [K] |
| a | – thermal diffusivity, [m^2s^{-1}] | T_e | – environmental temperature, [K] |
| Bi | – Biot number ($= aL / \lambda_2$), [–] | T_{hs} | – scale of temperature, [K] |
| $c_{\mu}, c_{1\varepsilon}, c_{2\varepsilon}, c_{3\varepsilon}$ | – parameters in k - ε model, [–] | t_0 | – time scale, [s] |
| E | – dimensionless analog of the dissipation rate of turbulent kinetic energy, [–] | U, V | – dimensionless velocity, [–] |
| Fo | – Fourier number ($= a t_0 / L^2$), [–] | U_{max}, U_{min} | – maximum and minimum velocities in the layer, [m^2s^{-1}] |
| Gr | – Grashof number ($= g\beta L^3(T_{hs} - T_0) / \nu^2$), [–] | \bar{v} | – speed, [m^2s^{-1}] |
| g | – acceleration created by the mass forces, [ms^{-2}] | x, y | – co-ordinates, [m] |
| H | – size of the area in the directions Y , [–] | X, Y | – dimensionless Cartesian co-ordinates, [–] |
| K | – dimensionless analog of turbulent kinetic energy, [–] | y^+ | – dimensionless distance from the wall in the near-wall region, [–] |
| k | – Karman constant, [–] | <i>Greek symbols</i> | |
| L | – size of the area in the directions X | α | – coefficient of heat exchange between the external environment and the solution area under consideration |
| l_m | – the mixing length ($l_m = k\delta[1 - e^{-y^+/A_0^+}]$), [m] | β | – thermal coefficient of volume expansion, [K^{-1}] |
| N | – Starck number ($= \varepsilon\sigma L(T - T_c)^3 / \lambda_2$), [–] | δ | – half-width of the mixing layer, [m] |
| Nu | – average convective Nusselt number, [–] | ε | – reduced degree of blackness, [–] |
| \vec{n} | – vector normal to the surface | Θ | – dimensionless temperature, [–] |
| Δn | – distance between the first point of the inner gas area and the solid wall | $\lambda_{1,2}$ | – relative thermal conductivity coefficient ($= \lambda_1 / \lambda_2$), [–] |
| Re_t | – turbulent Reynolds number ($= E / c_{\mu} K^2$), [–] | | |
| Pr | – Prandtl number ($= \nu / a$), [–] | | |
| Pr_t | – turbulent Prandtl number ($= \nu_t / a_t$), [–] | | |
| T | – temperature, [K] | | |

| | | | |
|-------------------------------------|---|-------------------|--|
| λ_1 and λ_2 | – thermal conductivity of air and solid wall, respectively, [$\text{Wm}^{-1}\text{K}^{-1}$] | Ψ | – dimensionless analog of the stream function, [–] |
| ν | – coefficient of kinematic viscosity, [m^2s^{-1}] | Ω | – dimensionless analog of vorticity, [–] |
| ν_t | – coefficient of turbulent kinematic viscosity, [m^2s^{-1}] | <i>Subscripts</i> | |
| σ | – Stefan-Boltzmann constant, [$\text{Wm}^{-2}\text{K}^{-4}$] | <i>hs</i> | – heat source |
| $\sigma_\varepsilon, \sigma_\kappa$ | – parameters in k - ε model, [–] | <i>e</i> | – environment |
| τ | – dimensionless time, [–] | <i>L</i> | – left |
| χ | – empirical dimensionless parameter which is constant in the layer thickness, [–] | <i>R</i> | – right |
| | | <i>UP</i> | – upper |
| | | <i>LB</i> | – lower boundaries |

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