From the Guest editors

CONTEMPORARY MODELLING METHODS IN HEAT, MASS, AND FLUID FLOW

Special Collection of Articles

Modelling transport processes in heat, mass, and fluid flow are always at the focus of the science in the modern era of rapid technological development. Generally, new problems emerging in science and technology invoke immediately the need to create mathematical models thus allowing understanding the basic relationship controlling the phenomena at issue. Moreover, exploring the models, even quite approximate, we may go deep into the physics of the processes and discover features which, to some extent, never would be detected by direct experimental observations.

The power of mathematical modelling is well demonstrated by the analytical models, which as a rule, simplify or reduce the initial models after scaling and establish general relationships upon imposed constrains. Numerical approaches in solution are powerful tools but without a preliminary analytical background the final outcomes of such calculations could not be adequately physically interpreted.

The modern science faces many non-linear problems that need new approaches in interpretation when the mathematic model should be build-up. With these words we emphasize the attention on the non-linear relaxations processes which comes from the departure of the classical Gaussian distributions (adequately modelled for large times by the classical parabolic models [1, 2]) and invoke applications of non-local (fractional) derivatives.

Due to technical publication problems and the proliferation of scientific reports the present collection was split into two parts. The first part consisting of 14 articles is presented in this issue. Another 6 papers of the collection will be published in the next issue of the journal (No. 2 of 2017).

This collection conveys strong, reliable, efficient, and promising developments of articles on contemporary modelling problems in heat, mass and fluid flow. Now, we will try to present briefly the main results of the articles forming the Part 1.

Fractional calculus is a hot topic in modern modelling studies *via* accounts of relaxation problems adequately modelled by fractional operators. This collection includes several studies on practically relevant problems employing factional derivatives with singular and no-singular kernels in the fading memories.

A new trend in modelling relaxation process by fractional derivatives is application of fractional operator with non-singular kernels. Applying the newly defined Caputo-Fabrizio derivative with exponential kernel, the work of Alkahtani and Atangana (in this issue, p. 1) presents an effective numerical solution by three different numerical schemes including the explicit, implicit, and Crank-Nicholson approaches of the newly developed Cattaneo-Hristov model (in this issue, p. 1) of transient heat diffusion in rigid conductors with non-singular fading memory. In fact this is his first attempt to find a solution of this Cattaneo-Hristov model.

Avci *et al.* (in this issue, p. 9) employ a conformable, local, well-behaved and limit-based definition, to obtain a local generalized form of advection-diffusion equation. This study is devoted to give a local generalized description to the combination of diffusive flux governed by Fick's law and the advection flux associated with the velocity field. As a result, the constitutive conformable advection-diffusion equation can be easily achieved. A Dirichlet problem for conformable advection-diffusion equation is derived by applying fractional Laplace transform with respect to time, and finite sin-Fourier transform with respect to spatial coordinate.

Povstenko *et al.* (in this issue, p. 19) present a control problem of thermal stresses in an infinite axisymmetric cylindrical body by a time-fractional heat conduction equation with the Caputo derivative of the order $0 < \alpha \leq 2$. The control function defined by the solution (using integral transform technique) guarantees the distribution of the stress component in some section of a body and at some time at a prescribed level.

The study of Bakhti *et al.* (in this issue, p. 29) focuses the attention on an important biophysical problem, especially a pulsatile flow of blood in stenosised arteries. The well-known constitutive models describing the viscoelasticity of blood, generalized by the Oldroyd-B model, are used. Numerical approximations of the axial velocity and wall shear stress use an implicit finite-difference scheme. The presented mathematical model provides realistic results that will help medical practitioners and it has direct applications in the treatment of cardiovascular diseases.

Bainy *et al.* (in this issue, p. 41) present an interesting study on baking fish burgers in electrical oven. Three mathematical models were developed from energy balance equations considering a lumped system and also considering the oven temperature described by a delayed Heaviside function. The proposed models could adequately describe the experimental data, however, the model based on fractional order derivatives presented the best data fit, characterized by the lowest value of the objective function, parameters with statistical significance and residues statistically equal to zero. The best fit probably occurred due to the memory effects provided by the fractional calculus, allowing the development of an accurate model for further studies of the fish burger baking process.

The article of Liu *et al.* (in this issue, p. 51) demonstrates the application of a variable-order fractal derivative model for anomalous diffusion transport. This study makes an attempt to introduce a new model, where the index of fractal derivative depends on temporal moment or spatial position. The main advantages of the article are the physical explanations of new model are explored by numerical simulation.

The work of Yang *et al.* (in this issue, p. 61) considers a fractional flow model of a time-variant behavior of non-Newtonian substances as well as thixotropic and anti-thixotropic phenomena of non-Newtonian flow. The special case demonstrating the practical relevance of the models is the behaviors of cellulose suspensions and SMS pastes under constant shear rate.

The study of Hristov (in this issue, p. 69) applies successfully the powerful engineering approach of the integral-balance method [3, 4] to a time-fractional diffusion equation (with Riemann-Liouville time derivative of order $0 < \mu < 1$) and a time-dependent diffusion power-law coefficient $D(t) = D_0 t^\beta$. The results confirm the analytical results of Fa and Lenzi [5] that its second moment is of power-law type with exponent $\mu + \beta$. The solution can present subdiffusive ($0 < \mu + \beta < 1$), superdiffusive ($\mu + \beta > 1$) and Gaussian transport ($\mu + \beta = 1$). The optimal exponents of the approximate parabolic profiles have been determined by minimization the mean squared error of approximation over the penetration depth. The results confirm the principle statement [5, 6] that in case of subdiffusive regime the first passage time is infinite.

Inverse heat conduction problem was successfully solved in case of space-fractional transport (with Riemann-Liouville space derivative) by the work of Brociek and Slota (in this issue, p. 81). The thermal conductivity coefficient was recovered by a minimization procedure regarding a functional and applying the Real Ant Colony Optimization (RealACO) algorithm.

In the model we apply the Riemann-Liouville fractional derivative. The paper presents also some examples to illustrate the accuracy and stability of the presented algorithm.

Classical calculus with integer-order derivatives is forever influential tool in modelling of non-linear problems in the era of rapidly developing microelectronics and material science, and are challenging for developing new models and solutions. This collection contains several nice studies which will be commented next

Bennacer *et al.* (in this issue, p. 89) applied the multiple-relaxation-time (MRT) lattice-Boltzmann method to investigate combined natural and forced convection occurring in a two-dimensional square cavity. The solution is performed for a left vertical wall at a constant temperature, which is higher than of the right wall. This yields a "cooperating" case, in which dynamic and buoyancy forces are added together. The double distribution model used in lattice Boltzmann methods is adopted to simulate the hydrodynamic and thermal fields. Simulations have been conducted over a wide range of Rayleigh (Ra) and Reynolds (Re) numbers, and the features of dynamic and thermal fields are presented for the spectra of this mixed convection phenomenon.

Hetmaniok *et al.* (in this issue, p. 105) present an efficient solution of inverse heat transfer problem of shrinking bodies during solidifications occurring within the casting mould. Mathematically the process was modeled by means of the so called solidification in the temperature interval that is with the aid of the heat conduction equation with the substitute thermal capacity describing the distribution of temperature in the solid phase, two-phase zone and liquid phase. The calculations were performed for three versions of the velocity vector included in the governing equation and for various grids introduced in the considered domain to examine the influence of the grid density on the quality of results.

Laraqi *et al.* (in this issue, p. 117) considered a non-linear heat transfer problem in the highly important industrial problem of lubrication where the viscosity of fluid is a determining factor in the thermal behavior of lubricant and solid surfaces in friction. The study is interested in the effect of temperature on the viscosity and the thermal behavior of the lubricant and solves the problem by assuming an exponential relationship between the viscosity and the temperature. The analytical solution developed is compared to a numerical modeling using a finite difference method.

Heat conduction with non-homogeneous boundary condition was successfully solved by Laraqi *et al.* (in this issue, p. 125). These problems are difficult to solve by using the classical methods such as integral transforms or separation of variables. These methods lead to solving of dual integral equations or Fredholm integral equations, which are not easy to use. The study addresses the thermal resistance of a finite medium submitted to conjugate surface with Neumann and Dirichlet conditions, which are defined by a band-shape heat source and a uniform temperature. The opposite surface is subjected to a homogeneous boundary condition such uniform temperature, or insulation. The proposed solving process is based on simple and accurate correlations that provide the thermal resistance as a function of the ratio of the size of heat source and the depth of the medium. A judicious scale analysis is performed in order to fix the asymptotic behavior at the limits of the value of the geometric parameter.

In the short communication of Yang and Gao (in this issue, p. 133) a new approach to solve the classical diffusion equation (the case of heat conduction) is proposed. The solution technology combines the variational iterative method and an integral transform similar to Sumudu transform.

The special collection *Contemporary Modelling Methods in Heat, Mass and Fluid Flow* provides a collection of solved problems serving as steps ahead to innovative schemes and elaborated solutions. We hope the articles will provide to the readers new ideas and may be a good source for further inspirations in the thermal science problems.

Last but not least, we like to express our gratitude to all authors who trusted and contributed the collection, the reviewers offering their time to increase the quality of the contents and the editors of *Thermal Science*, as well, for the support of our initiative.

References

[1] Carslaw, H. S., Jaeger, J. C., *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, Oxford, UK, 1992
[2] Crank, J., *Free and Moving Boundary Problems*, Clarendon Press, Oxford, UK, 1984
[3] Goodman, T. R., Application of Integral Methods to Transient Nonlinear Heat Transfer, in: *Advances in Heat Transfer,* (Eds. Irvine T. F., Hartnett J. P.,), Academic Press, San Diego, Cal., USA, Vol. 1, 1964, pp. 51-122
[4] Hristov, J., Approximate Solutions to Time-Fractional Models by Integral Balance Approach, in: *Fractals and Fractional Dynamics*, (Eds. Cattani, C., Srivastava, H. M., Yang, X.-J.,), De Gruyter Open, Warsaw, 2015, pp. 78-109
[5] Fa, K. S., Lenzi, E. K., Time-Fractional Diffusion Equation with Time Dependent Diffusion Coefficient, *Physica A, 72* (2005), ID 011107, doi: /10.1103/PhysRevE.72.011107
[6] Fa, K. S., Lenzi, E. K., Power Law Diffusion Coefficient and Anomalous Diffusion: Analysis of Solutions and the First Passage Time, *Physical Review E, 67* (2003), ID 061105, doi: 10.1103/PhysRevE.67.061105

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