UNSTEADY MAGNETOHYDRODYNAMIC THERMAL AND DIFFUSION BOUNDARY LAYER FROM A HORIZONTAL CIRCULAR CYLINDER

by

Aleksandar Z. BORIČIĆ*, Miloš M. JOVANOVIĆ, and Branko Z. BORIČIĆ

Faculty of Mechanical Engineering, University of Nis, Nis, Serbia

The unsteady 2-D dynamic, thermal, and diffusion magnetohydrodynamic laminar boundary layer flow over a horizontal cylinder of incompressible and electrical conductivity fluid, in mixed convection in the presence of heat source or sink and chemical reactions. The present magnetic field is homogenous and perpendicular to the body surface. It is assumed that induction of outer magnetic field is a function of longitudinal co-ordinate outer electric field is neglected and magnetic Reynolds number is significantly lower than one, i. e. considered the problem is in approximation without induction. Fluid electrical conductivity is constant. Free stream velocity, temperature, and concentration on the body are functions of longitudinal co-ordinate. The developed governing boundary layer equations and associated boundary conditions are made dimensionless using a suitable similarity transformation and similarity parameters. System of non--dimensionless equations is solved using the implicit finite difference three-diagonal and iteration method. Numerical results are obtained and presented for different Prandtl, Eckart, and Schmidt numbers, and values: magnetic parameter, temperature, and diffusion parameters, buoyancy temperature parameters, thermal parameter, and chemical reaction parameter. Variation of velocity profiles, temperature and diffusion distributions, and many integral and differential characteristics, boundary layer, are evaluated numerically for different values of the magnetic field. Transient effects of velocity, temperature and diffusion are analyzed. A part of obtained results is given in the form of figures and corresponding conclusions.

Key words: *buoyancy forces, chemical reaction, circular horizontal cylinder, heat source or sink, magnetohydrodynamic boundary layer, mass and heat transfer, mixed convection.*

Introduction

Flow of electrically conducting fluid in magnetohydrodynamic (MHD) boundary layers, especially in cases where it is necessary, next to the field velocity, know the temperature and concentration fields, and when they are in fact present, and many outside influences on their development, is described, as shown very complex system of non-linear partial differential equation whose solution is connected with significant difficulties. The study of MHD incompressible viscous flow has many important engineering applications in devices such

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* Corresponding author; e-mails: aboricic@masfak.ni.ac.rs

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MHD power generator, the cooling of nuclear reactor, geothermal system, aerodynamic processes and the heat exchange designs, pumps and flow meters, in space vehicle propulsion, control and re-entry, in creating novel power generating systems and developing confinement schemes for controlled fusion. Besides the analysis of general circulation of conductive fluid under the influence of magnetic field, special interest is the study of the movement of fluids around the body boundary layer [1-3].

Analytical and numerical solutions of equations of unsteady MHD laminar boundary layer flow of an incompressible conducting fluid, there are a relatively small number of special cases which do not nearly cover all of those to engineering and technology major problems, for which there is a need to solve this system of MHD equations. Having regard to the intensive development of numerous, new, modern field of technology, which has been studied movement and non-Newtonian conducting fluid, highly complex physical and chemical characteristics, the presence of numerous effects of mass and heat transfer, the presence of chemical reactions, *etc.*, were is increasingly necessary to deal with the problem by studying non-similarity convection flow around the body complex forms. It should be noted, that the way of solving this set of tasks MHD, also analogous to the study of boundary layers of non-conducting fluid, used, many exact and approximate methods, developed in the classical theory of the boundary layer. So many of these methods, and transferred to the study of more complex problems MHD boundary layers [4-12].
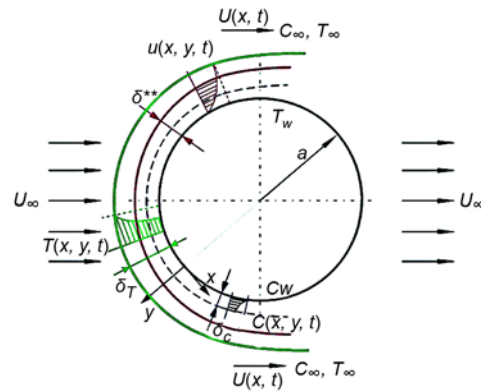
Significantly, the presence of non-stationary problems in technical and technological practices, their insufficient exploration, particularly in the area of MHD boundary layer, the need for management and control processes, which take place convection flow profile, especially when the problems are complex physical, or when equally present problems heat and mass transfer, indicate the need to intensify such research. The last twenty years, in the world literature there is a number of works that explore these issues. However, in most of these works were mainly addressed specifically defined main missions given profile: flat plate, positioned horizontally or vertically wedge both horizontally or vertical circular cylinder, *etc.,* with different boundary conditions for the temperature and concentration at the surface and in the outer flow [13-19]. Special attention was paid to various forms of profile body whereby the special interest shown by the analysis of MHD flow around a horizontal circular cylinder [20-27].

The subject of this paper will be exploring the effects of mass and heat transfer in a plane unsteady laminar MHD, the temperature and diffusion boundary layer, electrical conductive incompressible fluid around a horizontal circular cylinder, with arbitrarily given boundary conditions for temperature and concentration on the body, in the presence of volumetric buoyancy force resulting from the concentration difference and the Lorentz force. It is assumed, that induction of outer magnetic field is function of longitudinal co-ordinate with force lines perpendicular to the body surface on which boundary layer forms. Outer electric field is neglected and magnetic Reynolds number is significantly lower than one *i. e.* considered problem is in approximation without induction. It discusses the influence of influence of buoyancy forces, which are the result of concentration differences, present source/sink heat and the effect of chemical reactions, on the development of velocity, temperature and concentration fields. Dimensionless equations will be solved numerically using a finite difference and iteration methods. The results will be presented over a number of graphics of the skin friction coefficient, the rate of the mass transfer-concentration, velocity, temperature and concentration distributions for difference values the magnetic field, and introduced of the work of the similarity parameters.

Considering the previous, in this studied process the system of dynamic equations of MHD, of temperature, and diffusion boundary layer, which describes non-similarity problems, and that there will be a new approach to solving [26, 27], which, by their character-nature, can to a certain extent, divided into new methods for solving MHD boundary layer equations.

Formulation of the problem – mathematical analysis

A mathematical model of this problem, unsteady 2-D dynamics, thermal and diffusion MHD laminar boundary layer, of incompressible fluid, over a horizontal circular cylinder,

Flow in boundary layer

when the uniform external magnetic field, $B(x)$, is applied perpendicular to the surface of the body (the magnetic Reynolds number is significantly lower than one-considered problem is in approximation without induction), and the free stream velocity, $U(x, t)$, ambient temperature, T_∞, and concentrations, c_∞, is defined with system of for simultaneous equations: continuity equation and equations of dynamic, diffusion, and thermal boundary layer.

– continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \tag{1}$$

– momentum equation:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + \frac{\sigma B^2}{\rho} (U - u) + g\beta_T (T - T_\infty) \sin \alpha \tag{2}$$

– energy equation:

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{\lambda}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{1}{c_p} \left(\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \right)(U - u) + \frac{v}{c_p} \left(\frac{\partial u}{\partial y} \right)^2 +$$

$$+ \left(\frac{\sigma B^2}{\rho c_p} \right) u(u - U) + Q(T - T_\infty) \tag{3}$$

– mass-diffusion equation: $\quad \dfrac{\partial c}{\partial t} + u \dfrac{\partial c}{\partial x} + v \dfrac{\partial c}{\partial y} = D \dfrac{\partial^2 c}{\partial y^2} + k_h (c - c_\infty) \tag{4}$

and corresponding boundary and initial conditions:

$$u = 0, \qquad v = 0, \qquad T = T_w(x), \qquad c = c_w(x) \text{ for } y = 0$$

$$u = U(x, t), \qquad T = T_\infty, \qquad c = c_\infty \qquad \text{for } y \to \infty \tag{5}$$

$$u = u_0(x, y), \qquad T = T_0(x, y), \qquad c = c_0(x, y) \qquad \text{for } t = t_0$$

$$u = u_1(t, y), \qquad c = c_1(y) \qquad \text{for } x = x_0$$

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In previous system of equations and boundary conditions the parameter labeling used is common for the MHD boundary layer theory is: *x, y* – the longitudinal and transversal co-ordinate, *u, v* – the longitudinal and transversal velocity components, ν – the kinematics viscosity of fluid, σ – the fluid electrical conductivity, ρ – the fluid density, λ – the the thermal conductivity, c_p – the the specific heat at constant pressure, *T* – the fluid temperature, k_h – the chemical reaction parameter, *c* – the concentration of ionic species in solution, $T_w(x)$, $c_w(x)$ – the body surface temperature and concentration of ionic species, *D* – the diffusion coefficient. The term $Q(T - T_\infty)$ is assumed to be the amount of heat generated or absorbed per unit volume. *Q* is a heat generation/absorption constant.

If we introduce stream function – $\psi(x, y, t)$, the usual manner: $u = \partial\psi/\partial y$, $v = -\partial\psi/\partial x$, and new general similarities variables dimensionless transversal co-ordinate, velocity, temperature, and concentration [26]:

$$x = x, \quad t = t, \quad \eta(x, y, t) = \frac{D_0}{h(x, t)}\, y, \quad \phi(x, \eta, t) = \frac{D_0}{U(x, t)h(x, t)}\, \psi(x, y, t),$$

$$C(x, \eta, t) = \frac{c_w - c}{c_w - c_\infty}, \quad \theta(x, \eta, t) = \frac{T_w - T}{T_w - T_\infty}, \quad \varphi(\eta, f_{10}, g_{10}) = \phi_\eta = \frac{u}{U} \tag{6}$$

$$\mathrm{Ec} = \frac{U^2}{c_p(T_w - T_\infty)} - \text{Ecart number} \quad \mathrm{Pr} = \frac{\nu\rho c_p}{\lambda} - \text{Prandtl number}$$

$$\mathrm{Sc} = \frac{\nu}{D} - \text{Schmith number, and } Z(x, t) = \frac{\delta^{**2}}{\nu} \tag{7}$$

The system of eqs. (1)-(5), after a number of transformations translate into a form the governing boundary layer equations and associated boundary conditions (1)-(5), are converted into a non-dimensional form:

$$D_0^2 \varphi_{\eta\eta} + \frac{f(x, t) + 2f_1(x, t)}{2}\, \phi\varphi_\eta + \frac{\eta\dot{Z}(x, t)}{2}\, \varphi_\eta + D_0 v_{00}(x, t)\varphi_\eta + f_1(x, t)(1 - \varphi^2) +$$

$$+ \alpha_1^T(x, t)\left(1 - \theta\right) + [f_0(x, t) + \mathrm{g}(x, t)](x, t) = Z\varphi_{\eta t} + f_2(x, t)(\varphi\varphi_x - \phi_x\varphi_\eta) \tag{8}$$

$$\frac{D_0^2}{\mathrm{Pr}^*}\, \theta_{\eta\eta} + \frac{f(x, t) + 2f_1(x, t)}{2}\, \phi\theta_\eta + \frac{\eta\dot{Z}(x, t)}{2}\, \theta_\eta + D_0 v_{00}\theta_\eta - \mathrm{Ec}\, \mathrm{g}(x, t)(1 - \varphi)\varphi -$$

$$- \mathrm{Ec}\left[f_1(x, t) + f_0(x, t)\right]\left(1 - \varphi\right) + \left[l_1(x, t)\varphi - q(x, t)\right]\left(1 - \theta\right) -$$

$$- D_0^2\, \mathrm{Ec}(\varphi_{\eta\eta})^2 = Z(x, t)\theta_t + f_2(x, t)(\varphi\theta_x - \phi_x\theta_\eta) \tag{9}$$

$$\frac{D_0^2}{\mathrm{Sc}}\, C_{\eta\eta} + \frac{f(x, t) + 2f_1(x, t)}{2}\, \phi C_\eta + \frac{\eta\dot{Z}(x, t)}{2}\, C_\eta + D_0 v_{00}(x, t)C_\eta +$$

$$+ [c_1(x, t)\varphi - r(x, t)](1 - C) = ZC_t + f_2(x, t)(\varphi C_x - \phi_x C_\eta) \tag{10}$$

with boundary conditions: $\phi = \partial\phi/\partial\eta = \varphi = 0$, $\theta = 0$ and $C = 0$ for $\eta = 0$:

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$$\phi_\eta = \varphi \to 1, \quad \theta \to 1 \quad \text{and} \quad C \to 1 \to 1 \quad \text{for } \eta \to \infty \tag{11}$$

Where dimensionless parameters similarity are introduced in the following form:
– dynamical parameter

$$f(x,\, t) = U(x,\, t)\, Z'(x,\, t), \quad f_1(x,\, t) = U'(x,\, t)\, Z(x,\, t)$$

$$f_0(x,t) = \frac{U(\dot{x},t)}{U(x,t)}\, Z(x,t) \quad , f_2(x,\, t) = U(x,\, t)\, Z(x,\, t)$$

– temperature buoyancy-parameter

$$\alpha_1^{\mathrm{T}} = UZ\alpha^{\mathrm{T}}, \qquad \alpha^{\mathrm{T}}(x,t) = \frac{\mathrm{g}\beta_{\mathrm{T}}\, \sin\, \alpha(x)}{c_{\mathrm{T}} E_{\mathrm{T}x}^{\mathrm{T}}} = \alpha^{\mathrm{TT}}\, \sin\, \alpha(x)$$

– magnetic parameter

$$g(x,t) = N(x,t)Z(x,t), \quad N(x,t) = \frac{\sigma B(x,t)^2}{\rho}$$

– temperature parameter

$$l_1(x,t) = \frac{T_w'(x)}{T_w(x) - T_\infty}\, U(x,t)Z(x,t)$$

– thermal parameter

$$q(x,t) = QZ(x,t)$$

– diffusion parameter

$$c_1(x,t) = \frac{c_w'(x,t)}{c_w(x,t) - c_\infty}\, U(x,t)Z(x,t)$$

– chemical reaction parameter

$$r(x,\, t) = kZ(x,\, t) \tag{12}$$

where D_0 is the normalizing constant, which is determined from the condition that the first equation of system of eq. (3), is reduced to the case of flow past a flat plate:

$$\frac{\mathrm{d}^3\varphi_0}{\mathrm{d}\eta^3} + \varphi_0\, \frac{\mathrm{d}^2\varphi_0}{\mathrm{d}\eta^2} = 0 \qquad \text{from which follows} \qquad D_0 = \sqrt{\xi_0} = 0.469 \tag{13}$$

To the system of eqs. (9), where the unknown function $\phi(x,\, t,\, \eta)$, $\theta(x,\, t,\, \eta)$, $C(x,\, t,\, \eta)$, and $Z(x,\, t)$, it is necessary to add one of the appropriate integral equations. In this paper is as integral equation, using – the integral impulse equation:

$$\frac{H^*}{2}\, \frac{\partial Z(x,t)}{\partial t} + \frac{U(x,t)}{2}\, \frac{\partial Z(x,t)}{\partial x} = \varsigma - \left\{ \begin{bmatrix} \dfrac{U(\dot{x},t)}{U(x,t)} + U'(x,t) + N(x,t) \end{bmatrix} H^* \\ +2U'(x,t) + U\alpha^{\mathrm{T}} H_{\mathrm{T}} \end{array} \right\} Z(x,t) \tag{14}$$

with initial and boundary conditions: $\partial Z/\partial t = 0$, for $t = 0$, and $\partial Z/\partial x = 0$ for $x = 0$.

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Where the dimensionless characteristic functions of the boundary layer are introduced in the following form:

$$\zeta = \left[\frac{\partial\left(\dfrac{u}{U}\right)}{\partial\left(\dfrac{y}{\delta^{**}}\right)} \right]_{y=0}, \qquad H = \frac{\delta^*}{\delta^{**}} = \frac{1}{D_0}\int_0^\infty (1-\varphi_\eta)\,\mathrm{d}\eta, \qquad H_{\mathrm{T}} = \frac{\delta_{\mathrm{T}}}{\delta^{**}},$$

$$\delta^* = \int_0^\infty (1-\varphi_y)\,\mathrm{d}y, \qquad \delta^{**} = \int_0^\infty \varphi_y(1-\varphi_y)\,\mathrm{d}y, \qquad \delta_{\mathrm{T}} = \int_0^\infty (1-\theta)\,\mathrm{d}y \qquad (15)$$

System of eqs. (8)-(10), (14), with corresponding boundary conditions, with group of independent parameters (12), can be now applied to any concrete example, defined profile of the body, given boundary, and initial conditions, of temperature and concentration on the surface of the body and velocity outer flow.

Equations (8)-(10), (14), with the corresponding boundary conditions are applied in this paper to the example of the flow around a horizontal circular cylinder. In this case, the analysis will be carried in non-dimensional form, where longitudinal co-ordinate and velocity, will be scaled in relation to velocity of outer flow $U(x, t)$ and the radius of the cylinder, a, and the co-ordinate and velocity, perpendicular to the boundary layer, will be divided with $\mathrm{Re}^{1/2}$. With defined free stream velocity, externally uniform magnetic field strength, $B(x)$, which is in this case taken as constant, and boundary conditions for temperature and concentration and friction on the body $-\tau_w = \tau(x, t)$, determined with the expressions:

$$\hat{U}(\hat{x}, \hat{t}) = (1 + a_1\hat{t}^2)\sin\hat{x}, \qquad \alpha^{\mathrm{T}}(\hat{x}, \hat{t}) = \alpha^{\mathrm{TT}}\sin\hat{x} \quad \alpha^c(x) = \alpha^{cc}\sin x,$$

$$T_w(\hat{x}) = T_\infty + T_0(1 + a_2\hat{x}), \qquad c_w(\hat{x}) = c_\infty + c_0(1 + a_3\hat{x}), \qquad \tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_0 \qquad (16)$$

where $\hat{x}$ is the angular co-ordinate of arbitrary points on the cylinder measured from the stagnation point in radians, and $\hat{t}$ is the dimensionless size of the time.

Size of the coefficients a_1, a_2, and a_3, are arbitrary positive ore negative dimensionless constants. Positive constant value a_1 corresponding to accelerated fluid flow, a negative, to decelerated fluid flow. Positive ore negative value constants a_2, a_3, indicate increase or decrease the temperature and concentration in the direction along the cylinder profile.

In this example the introduced parameters similarity (12), are transformed into the:

$$f(x, t) = (1 + a_1 t^2)\sin x Z'(x, t), \qquad f_1(x, t) = (1 + a_1 t^2)\cos x Z(x, t),$$

$$f_0(x, t) = \frac{2t}{1 + a_1 t^2} Z(x, t), \qquad g(x, t) = N(x, t)Z(x, t),$$

$$\alpha_1^{\mathrm{T}} = (1 + a_1 t^2)\sin^2 x\, \alpha^{\mathrm{TT}}Z(x, t), \qquad l_0(x, t) = (1 + a_1 x^2)\sin x \frac{a_2}{1 + a_2 x} Z(x, t), \qquad (17)$$

$$c_0(x, t) = (1 + a_1 x^2)\sin x \frac{a_3}{1 + a_3 x} Z(x, t), \qquad q(x, t) = QZ(x, t), \qquad r(x, t) = k_h Z(x, t)$$

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Method of solution, results and discussion

Non-dimensional system of parabolic differential eqs. (8)-(10), and momentum eq. (14), with boundary conditions, numerically is solved, using the finite difference method with appropriate indirect block scheme defined by the five points. Getting highly accurate solutions, achieved on the one hand, as a result of a range of possibilities provided by finite difference method, and, on the other hand, as result of an iterative process which allows approximate solutions with arbitrary accuracy given. Approximation of non-linear differential equations is performed by a system of algebraic equations, defined on a discrete set of points of integration network as defined in the plane –x, t, η. At this staged network integration, discrete values of the dependent and independent values in the node identified by the node index (m, n, k) are:

$$x_n = n\Delta x, \quad t_k = k\Delta t, \quad \eta_m = m\Delta\eta$$

$$\phi_{m,n}^k = \phi(\eta_m, x_n, t_k), \quad \varphi_{m,n}^k = \varphi(\eta_m, x_n, t_k), \quad \theta_{m,n}^k = \theta(\eta_m, x_n, t_k), \quad C_{m,n}^k = C(\eta_m, x_n, t_k),$$

$$f_n^k = f(x_n, t_k), \quad H_{c,n}^k = H_c(x_n, t_k), \quad H_n^{*k} = H^*(x_n, t_k),$$

$$\varsigma_n^k = \varsigma(x_n, t_k), \quad \varsigma_{T,n}^k = \varsigma_T(x_n, t_k), \quad \varsigma_{c,n}^k = \varsigma_c(x_n, t_k), \quad Z_n^k = Z(x_n, t_k), \qquad (18)$$

$$f_n^k = f(x_n, t_k), \quad f_{1,n}^k = f_1(x_n, t_k), \quad f_{0,n}^k = f_0(x_n, t_k), \quad v_{0w,n}^k = v_{0w}(x_n, t_k),$$

$$\alpha_{1,n}^{\mathrm{T},k} = \alpha_{1,n}^{\mathrm{T}}(x_n, t_k), \quad \mathrm{g}_n^k = \mathrm{g}(x_n, t_k), \quad l_{1,}^k = l_1(x_n),$$

$$q_n^k = q(x_n, t_k), \quad c_1^k = c_1(x_n), \quad r_n^k = r(x_n, t_k)$$

First and second order derivatives are replaced by following equations:

$$\varphi_{\eta\eta} = \frac{\varphi_{m+1,n+1}^{k+1,i} - 2\varphi_{m,n+1}^{k+1,i} + \varphi_{m-1,n+1}^{k+1}}{(\Delta\eta)^2},$$

$$\theta_{\eta\eta} = \frac{\theta_{m+1,n+1}^{k+1,i} - 2\theta_{m,n+1}^{k+1,i} + \theta_{m-1,n+1}^{k+1}}{(\Delta\eta)^2}, \qquad C_{\eta\eta} = \frac{C_{m+1,n+1}^{k+1,i} - 2C_{m,n+1}^{k+1,i} + C_{m-1,n+1}^{k+1}}{(\Delta\eta)^2},$$

$$\phi_\eta = \frac{\phi_{m+1,n+1}^{k+1,i} - \phi_{m-1,n+1}^{k+1}}{2\Delta\eta}, \qquad \theta_\eta = \frac{\theta_{m+1,n+1}^{k+1,i} - \theta_{m-1,n+1}^{k+1}}{2\Delta\eta}, \qquad C_\eta = \frac{C_{m+1,n+1}^{k+1,i} - C_{m-1,n+1}^{k+1}}{2\Delta\eta}, \quad (19)$$

$$\phi_{f_{10}} = \frac{\phi_{m,n+1}^{k+1,i} - \phi_{m,n}^{k+1}}{\Delta x}, \quad \theta_{f_{10}} = \frac{\theta_{m,n+1}^{k+1,i} - \theta_{m,n}^{k+1}}{\Delta x}, \quad C_{f_{10}} = \frac{C_{m,n+1}^{k+1,i} - C_{m,n}^{k+1}}{\Delta x}, \quad \varphi_{f_{10}} = \frac{\varphi_{m,n+1}^{k+1,i} - \varphi_{m,n}^{k+1}}{\Delta x},$$

$$\phi_{g_{10}} = \frac{\phi_{m,n+1}^{k+1,i} - \phi_{m,n+1}^k}{\Delta t}, \qquad \theta_{g_{10}} = \frac{\theta_{m,n+1}^{k+1,i} - \theta_{m,n+1}^k}{\Delta t}, \qquad C_{g_{10}} = \frac{C_{m,n+1}^{k+1,i} - C_{m,n+1}^k}{\Delta t},$$

$$\varphi_{g_{10}} = \frac{\varphi_{m,n+1}^{k+1,i} - \varphi_{m,n+1}^k}{\Delta t}$$

where index *j* indicates the current number of iterations.

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Using eq. (19), system of partial differential eqs. (8)-(10), is reduced to the following system of algebraic equations – dynamic equations of the boundary layer, equations of temperature boundary layer, diffusion boundary layer equations, and algebraic, linearized momentum eq. (14). Linearization of the non-linear terms in the equations are resolved using a combination of methods, such as values, which now form the equation coefficients, taken from the previous network layer or in the previous iteration and the current iteration are well-known value. In this paper, we get the system of algebraic equations, which is suitable for further calculations, is resolved using the three diagonal methods.
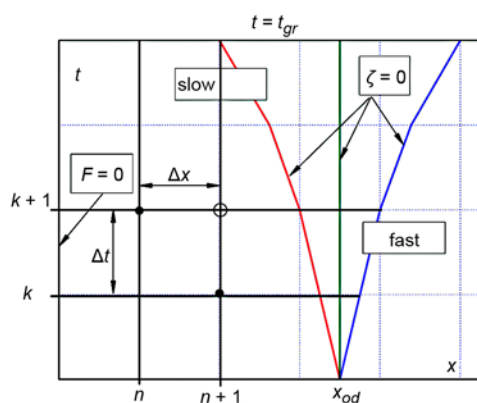

Figure 1. Planar integration network and integration area

In this way the obtained systems of simultaneous equations, each of which contains three unknown functions of the velocity ratio – non-dimensional temperature – concentration levels can be solved, because the number of equations of each system is equal to the number of unknown functions. Special design matrix system of algebraic equations, the three-diagonal matrix, is used for solving simple direct method, which for its implementation does not require the creation of inverse matrix. System of algebraic equations can be solved by applying the three-diagonal method. The integration of the equations is performed on the adopted integration network, in a limited part of the space, of the first 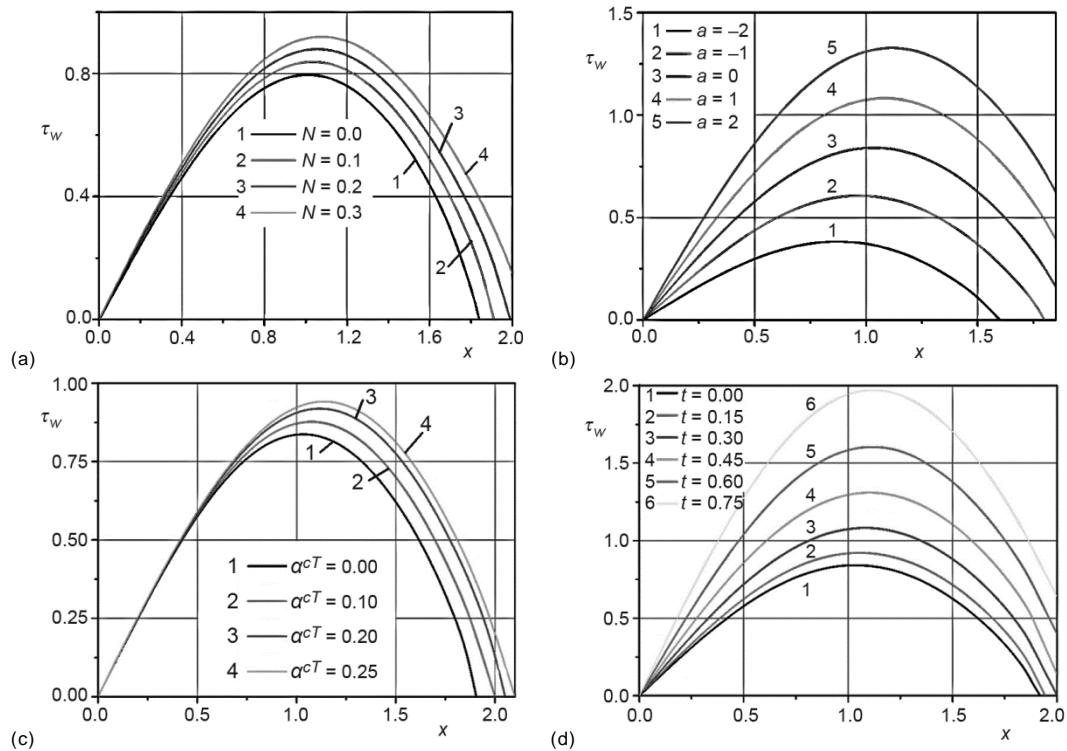octant (x, η, t) of the co-ordinate plane $(t, 0, \eta)$, which physically represents unsteady flow around the front of the stop point $(f = F = 0, x = 0)$ the surface defined by the separation point of the boundary layer $(\varsigma = 0)$, whose size varies with the taken, values ratio a_1 and size (N, α^{cc}) (fig. 1).

Numerical results are obtained and presented for different values of Prandtl, Eckart, and Schmidt numbers, dynamic parameters $f_i \rightarrow \pm a_1$, and also parameter of the thermal buoyancy parameter $\alpha_1^T \rightarrow \alpha^{cT}$, temperature parameter $l_0 \rightarrow \pm a_2$, diffusion parameter $c_0 \rightarrow \pm a_3$ thermal parameter $q \rightarrow Q$, and chemical reaction parameters $h \rightarrow k_h$. The solutions for the flow, temperature and diffusion transfer, and other integral characteristics boundary layer, are evaluated numerical for difference values the magnetic number N. Transient effects of velocity, temperature and diffusion was analyzed.

A part of the obtained results is presented in figs. 2-7. Figure 2 shows the variations of the variables τ_w, figs. 3-4 presented thickness variations of the dynamic temperature and diffusion boundary layer for different values of parameter N, a_1, and a^{cT}, fig. 5 variations $\theta_{\eta 0}$ and $C_{\eta 0}$, in function of the co-ordinate x. Figures 6-7 show the variations of dimensionless velocity, temperature, and concentration in the function of the co-ordinate η, in a certain section of the boundary layer, for several values of the magnetic parameter N, Prandtl, Eckart, and Schmidt numbers, and for several values of the thermal buoyancy-parameter a^{cT}, heat source/sink parameter, Q, and chemical reaction parameter k_h.

It can be noticed in fig. 2 (where Pr = 0.73, Ec = 0.3, Sc = 0.3, $Q = k_h = a_1 = \alpha^{cc} = a_2 = 0.1$), that the magnetic parameter N, influences considerably to the position of the boundary layer separation point. The increase in the magnetic parameter move the point of of boundary layer separation downstream and in that sense its influence can be considered positive.

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Figure 2. Stream function τ_w, for different values of magnetic parameter N, a_1, α^{cT} and t

These conclusions are valid also for values of the temperature buoyancy parameter α^{cT} ($N = 0.1$). It can be noticed from these figures that free stream acceleration causes a delay in boundary layer separation, *i. e.* it moves the boundary layer separation point. It can be noticed from these figures that free stream acceleration causes a delay in boundary layer separation, *i. e.* it moves the boundary layer separation point downstream ($a_1 = \pm1$), while free stream deceleration moves the boundary layer separation point upstream ($a_1 = \pm1$). Thus the influence of free stream acceleration is positive and the influence of free stream deceleration is negative. Figure 2(d) clearly recognize the positive impact of accelerating external current at the point separation of the boundary layer, which is becoming more expressed with increasing time.

Figures 3 and 4 show diagrams of changes in thickness of the dynamic, temperature and diffusion boundary layer in the function changes of parameters N, a_1, α^{cT}. From development presented curves clearly recognize the influence of increasing or decreasing size of the introduced parameters to increase or decrease in the thickness of the corresponding boundary layer

Figures 5(a) and 5(b) present the graphs of characteristic functions ς_T in function of co-ordinate x for different values of Prandtl number and time (where Ec = 0.7, Sc = 7.0, N = 0.2, $k = K_p = 0.1$, $a_1 = 1$, and $t = 0.1$). Figure 5(c) shows the graphs of characteristic functions ς_T for different values of parameter $a_{3x} \rightarrow a_3$ (where Ec = 0.7, Sc = 7.0, N = 0.2, $k = K_p = 0.1$, a_1 = 1, and $t = 0.1$).

Figure 6 shows change the dimensionless velocity in a certain section of the boundary layer ($x = 1.0$, $t = 0.06$) for different values magnetic field N and temperature buoyancy force α^{cT}. It is evident that, with increasing the magnetic field strength and buoyancy force, increase the size of the dimensionless velocity in the boundary layer.
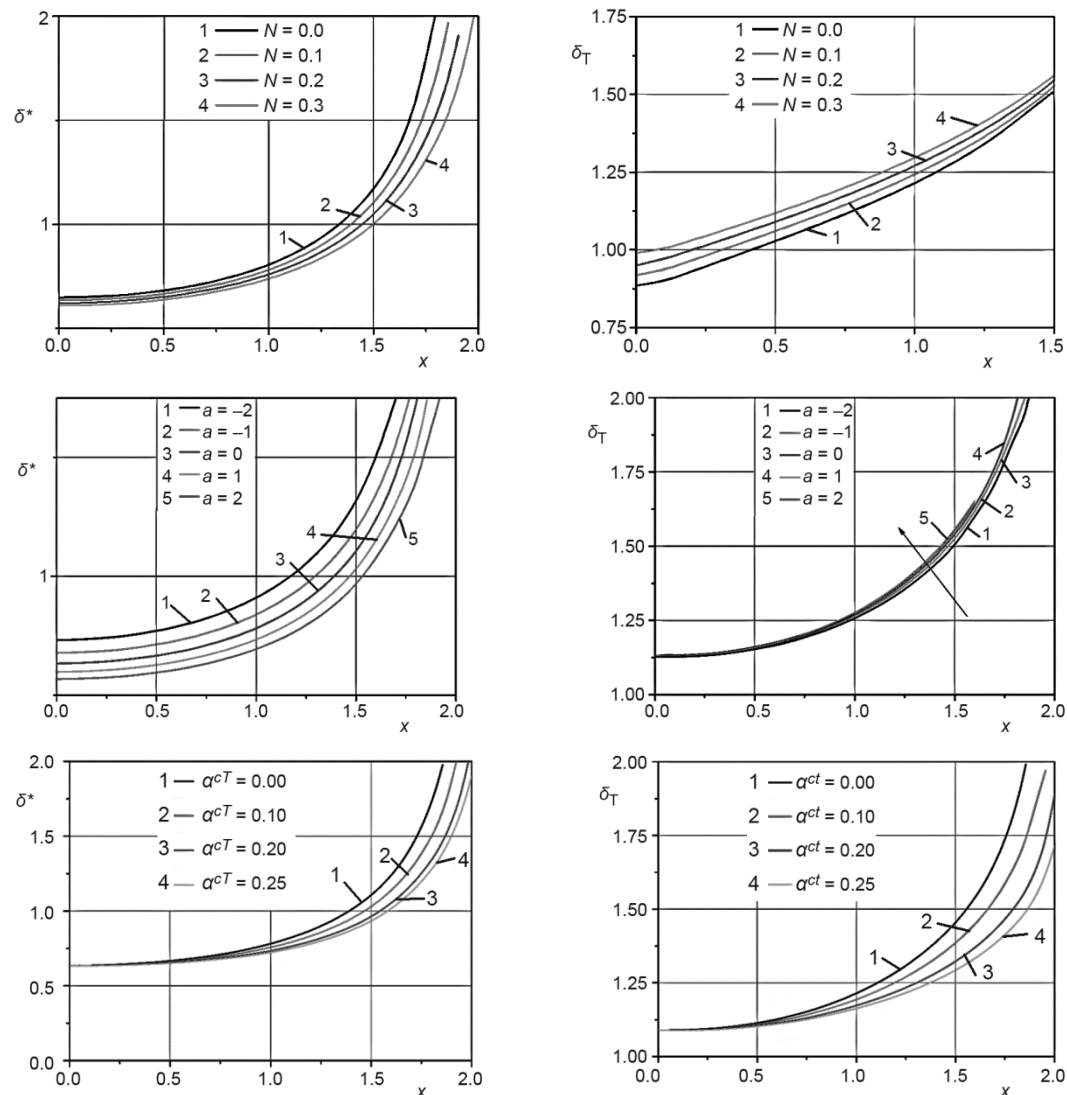
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Figure 3. Thickness of the dynamic, temperature boundary layer for different values of parameter N, a_1, and α^{cT}

Figures 7(a, b, c) show the effects of Prandtl number, and parameter N, and parameter a_{2x} on the dimensionless temperature (for fixed values of Ec = 0.3, Sc = 0.3, N = 0.1, $k_h = a_2 = a_3 = 0,1$, $a_1 = 1$, $t = 0.02$). It is clear, that the dimensionless temperature at a point increases (fluid temperature decreases) with increase in Prandtl number. The increase of Prandtl number means that the thermal diffusivity decreases. So the rate of heat transfer is decreased due to the decrease of fluid temperature in the boundary-layer. Increase of heat generation/absorption constant, a_{2x}, from negative to positive values, has the opposite effect, since for this case, the temperature in the boundary layer grows while the dimensionless temperature decreases. Figure 7(d) shows the effects for different values the parameter k_h, on the dimensionless concentration C. Increase values of the parameter k, increases dimensionless concentration.
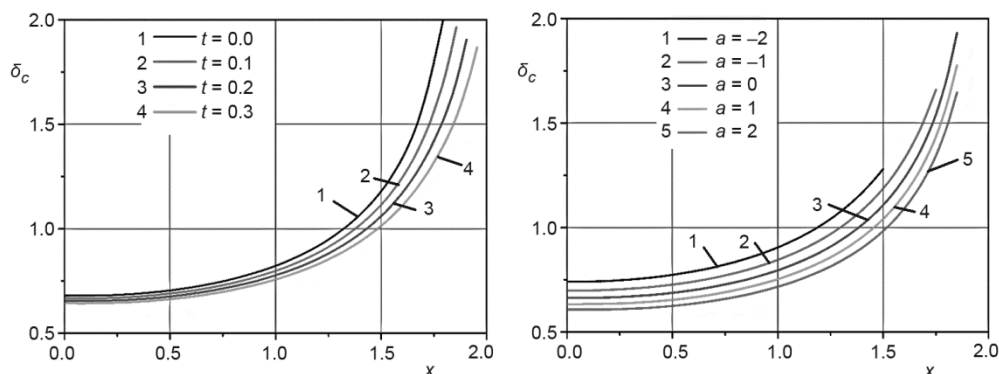
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Figure 4. Thickness of the diffusion boundary layer for different values of parameters N and a
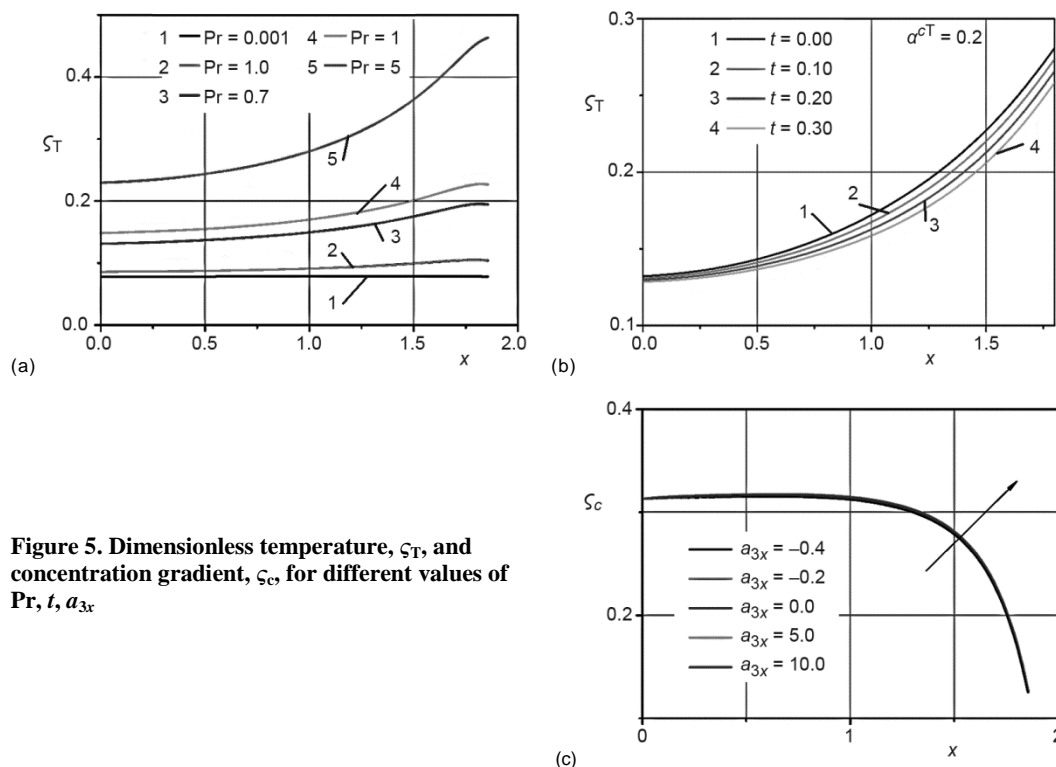
Figure 5. Dimensionless temperature, ς_T, and concentration gradient, ς_c, for different values of Pr, t, a_{3x}

Based on the results skin friction on the body, the heat transfer coefficient and the concentration on the surface of the cylinder (Nusselt and Sherwood numbers), the thickness of the dynamic, of temperature and diffusion boundary layer, as well as profiles, the velocity ratio, dimensionless temperature, and concentration in certain sections of the boundary layer, it can be concluded about the possibilities introduced by the impact on the development of MHD boundary layers. Thus, the influence of the magnetic field, the size of the coefficient of the non-stationarity, and the size of dimensionless speed of suction/injection, can successfully manage the development of all three MHD boundary layers. The temperature parameters and
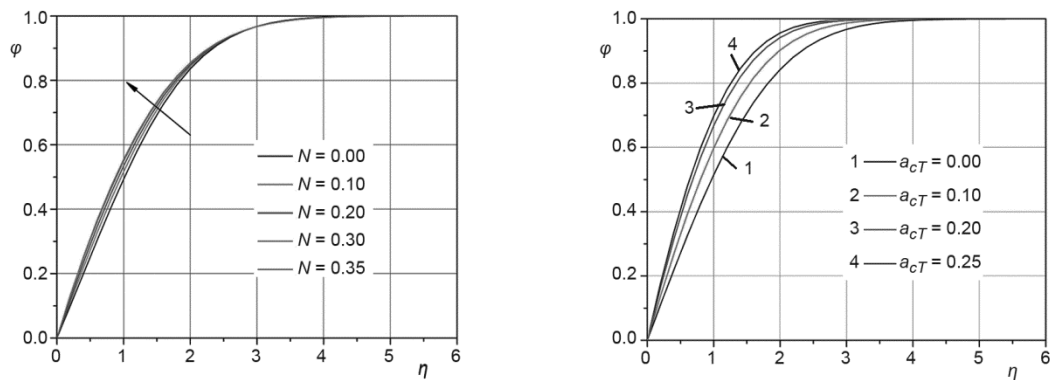
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Figure 6. Dimensionless velocity, φ, for different values of magnetic parameter N and α^{cT}
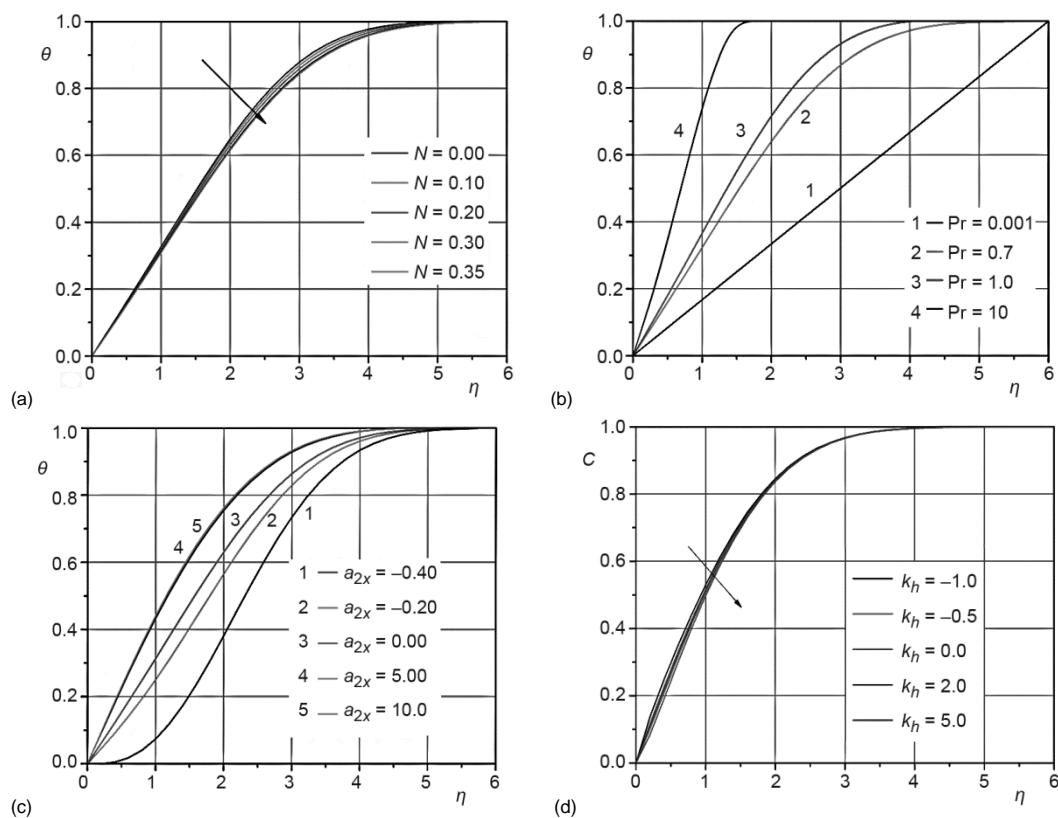
Figure 7. Dimensionless temperature, θ, dimensionless concentration, C, for different values of N, Pr, a_{2x}, and k_h

values of the coefficients and the extent and dimensions that characterize the sources/sinks of heat, extended Prandtl number, which contains the influence of heat radiation and Evart number, we can manage the development of the temperature boundary layer, and the diffusion parameters, namely the size of the coefficients and exponents, sizes that characterize a chemical reaction and Schmidt number, the development of the diffusion boundary layer.

Conclusion

This paper is devoted to the analysis of unsteady 2-D dynamic, thermal, and diffusion MHD laminar boundary layer flow over a horizontal circular cylinder, of incompressible and electrical conductivity fluid, in the presence of diffusion buoyancy effects, heat source or sink and chemical reactions. The developed governing boundary layer equations and associated boundary conditions are converted into a dimensionless form using a suitable similarity transformation, and similarity parameters. Numerical solution of dynamic, temperature, and diffusion boundary layer equations, with integral impulse equation are obtained by using the finite difference method, combined with the method of iteration. Numerical results for the solutions of the flow, temperature, and diffusion transfer and other integral characteristics boundary layer, are obtained and presented for different parameters such Schmidt, Prandtl, and Eckart numbers, magnetic number N, and dynamical parameters, diffusion buoyancy-parameter, temperature, thermal, diffusion, and chemical reaction parameters. Transient effects of velocity, temperature and diffusion are analyzed. A part of obtained results is given in the form of figures and corresponding conclusions. In this paper, the results of integral and differential characteristics of the boundary layer, show the effects of magnetic field and other parameters introduced similarity on the flow development around the circular cylinder surface. Results of friction skin on the body and the velocity distribution, show positive effect of magnetic field action, accelerating the outer flow and an increase in diffusive buoyancy force in the development of the boundary layer around the surface of the cylinder and moving the point of separation of the boundary layer along the profile of the body.

Nomenclature

B – magnetic field induction, [T]
C – dimensionless concentration function
c – concentration of the fluid
H – characteristic function, [–]
N – characteristic function, [s^{-1}]
T – fluid teperature, [K]
t – time, [s]
U – free stream velocity, [ms^{-1}]
u, v – longitudinal and transversal projection of velocities in boundary layer, [ms^{-1}]
w – surface conduction
x, t, η – differentiation with respect to x, t, η
x, y – longitudinal and transversal co-ordinates, [m]
Z – characteristic function, [s]

Greek symbols

δ – boundary layer thickness, [m]
η – dimensionless transversal co-ordinate, [–]
θ – dimensionless temperature function, [–]
τ – shear stress, [Pa]
φ – dimensionless stream function, [–]
ψ – stream function, [m^{2}s^{-1}]
ς – characteristic function, [–]

Subscripts

0 – initial time moment $t = t_0$
1 – boundary layer cross-section $x = x_0$
∞ – conductions far away from the surface

References

[1] Blum, E. J., Mihajlov, I. A., Heat Transfer in Electro Conductive Fluid in Presence of Transversal Magnetic Field, *Magnetohydrodinamics, 5* (1966), pp. 2-18
[2] Sparrow, E., Cess, R., Effect of Magnetic Field on Free Convection Heat Transfer, *Int. J. Heat and Mass Transfer, 3* (1961), 4, pp. 267-274
[3] Rosow, J., On Flow of Electrically Conducting Fluid over a Flat Plate in the Presence of a Transverse Magnetic Field, Report, No.1358, NASA, USA, 1958
[4] Yuferev, V., About an Approximate Method of Calculation of the Laminar Boundary Layer of the Conducting Liquid (in Russian), *Fluid Dynamics, 1* (1958), pp 124-127
[5] Karjakin, Y., MHD Laminar Boundary Layer Axisimetrical Flow with Spin (in Russian), *Magnetic Hydrodynamics, 2* (1968), pp 47-54
[6] Savić, S., *et al.*, Investigation of the Ionized Gas Flow Adjacent to Porous Wall in the Case when Electroconductivity is a Function of the Longitudinal Velocity Gradient, *Thermal Science, 14* (2010), 1, pp. 89-102

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Boričić, A. Z., *et al.*: Unsteady Magnetohydrodynamic Thermal and …
THERMAL SCIENCE, Year 2016, Vol. 20, Suppl. 5, pp. S1367-S1380

[7] Nikodijević, D., *et al*., Generalized Similarity Method in Unsteady Two-Dimensional MHD Boundary Layer on the Body wich Temperature Varies with Time, *Inter. Journal of Engineering, Science and Technology*, *1* (2009), 1, pp. 206-215

[8] Boričić, Z., *et al.,* Universal Equations of Unsteady Two-Dimensional MHD Boundary Layer on the Body with Temperature Gradient Along Sutface, *WSEAS Trans. on Fluid Mechanics*, *4* (2009), 1, pp. 97-106

[9] Nikodijević, D., *et al.*, Unsteady Plane MHD Bondary Layer Flow of a Fluid of Variable Electrical Conductivity, *Thermal Science, 14* (2010), Suppl. 2*,* pp. S171-S182

[10] Nikodijević, D., *at al.,* Parametric Method for Unsteady Two-Dimenssional MHD Boundary Layer on a Body whose the Temperature Varies with Time, *Archives of Mechanics, Polish Akademy of Science*, *v 63* (2011), 11, pp. 57-71

[11] Nikodievć, D., *et al.,* Application of Parametric Method to the Solution of Unsteady Temperature MHD Boundary Layer on the Porous Arbitraty Scope Body, *Proceedings,* 2nd International Conference, Mechanical Engineering in XXI Century, Nis, Serbia, 2013, pp. 139-144

[12] Chamkha, A., *et al.,* Thermal Radiation Effects on MHD Forced Convection Flow Adjacent to a Non- Isothermal Wedge in the Presence of a Heat Source or Sink, *Heat and Mass Tansfer*, *39* (2003), 4, pp. 305-312

[13] Chen, C., Heat and Mass Transfer in MHD Flow by Natural Convection from a Permeable, Inclined Surface with Wall Temperature and Concentration*, Acta Mechanica*, *172* (2004), 3-4, pp. 219-235

[14] Subhas, A., *et al.,* Buoyancy Force Thermal Radiation Effects in MHD Boundary Layer Visco-Elastic Fluid over Continuously Moving Stretching Surface*, International Journal of Thermal Science*, *44* (2005), 5, pp. 465-476

[15] Rashad, A., Bakier, A., MHD Effects on Non-Darcy Forced Convection Boundary Layer Flow Past a Permeable Wedge in Porous Medium with Uniform Heat Flux, *Nonlinear Analysis and Control, 14* (2009), 2, pp. 249-261

[16] Rajeswari, R., *et al.*, Chemical Reaction, Heat and Mass Transfer on Nonlinear MHD Boundary Layer Flow Through a Vertical Porous Surface in the Presence of Suction, *Applied Mathematical Sciences, 3* (2009), 20, pp. 2469-2480

[17] Saleh, M., *et al.,* Heat and Mass Transfer in MHD Visco-Elastic Fluid Flow Through a Porous Medium Over a Stretching Sheet with Chemical Reaction, *Applied Mathematics*, *1* (2010), 6, pp. 446-455

[18] Kandasamy, R., Muhamian, A., Scaling Transformation for the Effect of Temperature-Dependent Fluid Viscosity with Thermophoresis Particle Deposition on MHD Free Convection Heat and Mass Transfer over a Porous Stretching Surface, *Trans Porous Medium, 84* (2010), 2, pp. 549-568

[19] Miraj, A., *et al.,* Conjugate Effects of Radiation and Joule Heating on MHD Free Convection Flow Along a Sphere with Heat Generation, *American Journal of Computational Marhematics, 1* (2011), 1, pp. 18-25

[20] Aldoss, T. K., *et al.*, MHD Mixed Convection from a Horizontal Circular Cylinder, *Numerical Heat Transfer, 30* (1996), 4, pp. 379-396

[21] Yih, K., Effect of Uniform Blowing/Suction on MHD Natural Convecton over a Horizontal Cylinder: UWT or UHT, *Acta Mechanica, 144* (2000), 1-2, pp. 17-27

[22] Amin, M., Combined Effects of Viscous Dissipation and Joule Heating on MHD Forced Convection over a Nonisothermal Horizontal Cylinder Embeded in a Fluid Saturated Porous Medium, *Journal of Magnetism and Magnetic Materials 263* (2003), 3, pp. 337-343

[23] Nazar, R., *et al.,* Mixed Convection Boundary Layer Flow from a Horizontal Circular Cylinder a Constant Surface Heat Flux, *Heat and Mass Transfer, 40* (2004), 3-4, pp. 219-227

[24] Molla, M., *et al.*, MHD Natural Convection Flow from an Isothermal Horizontal Circular Cylinder under Consideration of Temperature Dependent Viscosity, *Enginering Computations*, *29* (2012), 8, pp. 875-887

[25] Boričić, A., *et al.,* MHD Dynamic and Diffusion Boundary Layer Flow of Variable Electrical Conductivity Fluid past a Circular Cylinder, *Procedeedings*, 15th Symposium on Thermal Science and Engineering of Serbia, Sokobanja, Serbia, SINTERM, 2011, pp. 66-762

[26] Boričić, A., *et al.,* Magnetohydrodynamic Effects on Unsteady Dynamic Thermal and Diffusion Boundary Layer Flow over a Horzontal Circular Cylinder, *Thermal Science, 12* (2012), Suppl. 2, pp. S311-S321

[27] Boričić, A., *et al.,* Heat and Mass Transfer on Unsteady MHD Dynamic, Temperature and Diffusion Boundary Layer Flow over a Horizontal Circular Cylinder, *Proceedings,* 2nd International Conference, Mechanical Engineering in XXI Century, Nis, Serbia, 2013, p. 145-150