Introduction

Theory of local fractional calculus is a useful tool to model the non-differential problems in mathematical physics [1-3]. The local fractional partial differential equations (PDE) [4] are classified as one of three types, namely parabolic, hyperbolic, and elliptic. For example, the diffusion equation [5-9] is a parabolic local fractional PDE. The local fractional wave equation [10, 11] is hyperbolic local fractional PDE. The local fractional Laplace equation [12-14] is elliptic local fractional PDE.

Many technologies for solving the local fractional PDE were developed, e.g., the functional method, Similarity variable method, differential transform method, Laplace variational iteration method, Laplace series expansion method, and so on. In this paper, we consider the local fractional Korteweg-de Vries equation [1]:

\[
\frac{\partial^\vartheta \Lambda(\psi, \tau)}{\partial \psi^\vartheta} + \frac{\partial^\vartheta \Lambda(\psi, \tau)}{\partial \psi^{3\vartheta}} = 0
\]  

(1)

where the local fractional partial derivative of \(\Lambda(\psi, \tau)\) of order \(\vartheta (0 < \vartheta < 1)\) at the point \(\psi = \psi_0\) is given by [1, 4]:

\[
\frac{\partial^\vartheta \Lambda(\psi_0, \tau)}{\partial \psi^\vartheta} = \frac{\Delta^\vartheta [\Lambda(\psi, \tau) - \Lambda(\psi_0, \tau)]}{(\psi - \psi_0)^\vartheta}
\]

(2)

with \(\Delta^\vartheta [\Lambda(\psi, \tau) - \Lambda(\psi_0, \tau)] \equiv \Gamma(1 + \vartheta)\Lambda(\psi, \tau) - \Lambda(\psi_0, \tau)\)

The local fractional Laplace transform (LFLT) of \(\Omega(\tau)\) is defined [1]:

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\[
\bar{\Phi}_\theta(\Omega(\tau)) = \bar{\Phi}_\theta(\Omega_y(y)) = \frac{1}{\Gamma(1+\theta)} \int_0^\infty E_\theta(-y^\theta \tau^\theta)\Omega(\tau)(d\tau)^\theta, \quad 0 < \theta \leq 1
\]  

(3)

The inverse LFLT is defined [1]:

\[
\Omega(\tau) = Y^{-1}_\theta \{ \Omega_{\theta\theta}(y) \} = \frac{1}{(2\pi)^\theta} \int_{\mu-i\infty}^{\mu+i\infty} E_\theta(y^\theta \tau^\theta)\Omega_{\theta\theta}(y)(dy)^\theta
\]  

(4)

where \( y^\theta = \mu^\theta + i^n x^\theta \) and \( \text{Re}(y^\theta) = \mu^\theta \). The properties of the LFLT were listed in [1].

The aim of the paper is to solve the local fractional Korteweg-de Vries equation by using the local fractional Laplace series expansion method.

**The non-differentiable solution for local fractional Korteweg-de Vries equation**

By using the technology [15], we present a multi-term separated function given by:

\[
E_\theta(\tau^\theta) = \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)}
\]  

(5)

which leads to the solution of eq. (1), namely:

\[
\Lambda(\psi, \tau) = \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} H_k(\psi)
\]  

(6)

where \( H_k(\psi) \) is the non-differentiable functions with respect to \( \psi \).

With the help of eq. (6), we have:

\[
\frac{\partial^\theta \Lambda(\psi, \tau)}{\partial \tau^\theta} = \frac{\partial^\theta}{\partial \tau^\theta} \left[ \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} H_k(\psi) \right] = \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} H_{k+1}(\psi)
\]  

(7)

\[
\frac{\partial^\theta \Lambda(\psi, \tau)}{\partial \psi^\theta} = \frac{\partial^\theta}{\partial \psi^\theta} \left[ \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} H_k(\psi) \right] = \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} \frac{\partial^\theta H_k(\psi)}{\partial \psi^\theta}
\]  

(8)

and

\[
\frac{\partial^{3\theta} \Lambda(\psi, \tau)}{\partial \psi^{3\theta}} = \frac{\partial^{3\theta}}{\partial \psi^{3\theta}} \left[ \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} H_k(\psi) \right] = \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} \frac{\partial^{3\theta} H_k(\psi)}{\partial \psi^{3\theta}}
\]  

(9)

After substituting eqs. (7)-(9) into eq. (1), we have:

\[
\sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} H_{k+1}(\psi) + \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} \frac{\partial^\theta H_k(\psi)}{\partial \psi^\theta} + \sum_{k=0}^\infty \frac{\tau^{k\theta}}{\Gamma(1+k\theta)} \frac{\partial^{3\theta} H_k(\psi)}{\partial \psi^{3\theta}} = 0
\]  

(10)

which leads to the operator equation:

\[
H_{k+1}(\psi) = \left[ \frac{\partial^\theta H_k(\psi)}{\partial \psi^\theta} + \frac{\partial^{3\theta} H_k(\psi)}{\partial \psi^{3\theta}} \right]
\]  

(11)
We now consider the initial value condition of eq. (1), namely:
\[ \Lambda(\psi, 0) = E_\phi(-\psi^\varphi) \] (12)

Thus, we have the following iterative formula:
\[
\begin{align*}
H_{k+1}(\psi) &= -\left[ \frac{\partial^\varphi H_k(\psi)}{\partial \psi^\varphi} + \frac{\partial^{3\varphi} H_k(\psi)}{\partial \psi^{3\varphi}} \right] \\
H_0(\psi) &= E_\phi(-\psi^\varphi)
\end{align*}
\] (13)

Taking the LFLT of eq. (13), we present a new iteration formula:
\[
\begin{align*}
H_{k+1}(y) &= (-2)^k H_k(y) \\
H_0(y) &= \frac{1}{1 + y^\varphi}
\end{align*}
\] (14)

Therefore, we obtain from eq. (13) the following values:
\[
\begin{align*}
H_1(y) &= -\frac{2}{1 + y^\varphi} \\
H_2(y) &= \frac{2^2}{1 + y^\varphi} \\
H_k(y) &= -\frac{2^k}{1 + y^\varphi}
\end{align*}
\] (15, 16, 17)

and others.

Thus, we obtain the local fractional Laplace series of the solution of eq. (1), namely:
\[
\Lambda(y, \tau) = \sum_{k=0}^{\infty} -\frac{2^k \tau^{k\varphi}}{\Gamma(1 + k\varphi)} \frac{1}{1 + y^\varphi} = E_\phi(-2\tau^\varphi) \frac{1}{1 + y^\varphi}
\] (18)

Taking the inverse LFLT of eq. (18), we give the local fractional series solution:
\[
\Lambda(\psi, \tau) = E_\phi(-2\tau^\varphi) E_\phi(\psi^\varphi)
\] (19)

and its corresponding graph represents in fig. 1.

**Conclusion**

We use the local fractional Laplace series expansion method to deal with the local fractional Korteweg-de Vries equation. The non-differentiable solution when $\varphi = \ln 2/\ln 3$ is obtained. The new technology is accurate and efficient for solving the PDE in mathematical physics.

**Figure 1. The series solution of local fractional Korteweg-de Vries equation when $\varphi = \ln 2/\ln 3$**
Nomenclature

ψ – space co-ordinate, [m]
ϑ – fractal dimensional order, [-]
\( Y_{\vartheta \tau \Omega} \) – LFLT of \( \Omega(\tau) \), [-]
\( Y_{\vartheta \tau \Omega}^{-1} \) – inverse LFLT, [-]

References