THE LOCAL FRACTIONAL ITERATION SOLUTION FOR THE DIFFUSION PROBLEM IN FRACTAL MEDIA

by

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In this paper, we address the coupling method for the local fractional variational iteration algorithm III and local fractional Laplace transform for the first time, which is called as the local fractional Laplace transform variational iteration algorithm III. The proposed technology is used to find the local fractional iteration solution for the diffusion problem in fractal media via local fractional derivative.

Key words: *diffusion, heat conduction, local fractional Laplace transform, local fractional Laplace transform variational iteration algorithm III, analytical solution, local fractional derivative*

Introduction

Local fractional variational iteration method [1] was used to solve the partial differential equations (PDE) in mathematical physics, *e. g.*, heat-conduction [2, 3], wave [4], diffusion [5], Tricomi [6], and Fokker-Planck [7] equations. Recently, the new technology based on the theory of the of the local fractional variational iteration method, which is called as the local fractional Laplace transform variational iteration method, was developed in [1, 8-12]. It was used to solve the PDE via local fractional derivative. Li *et al.* [9] considered the fractal vehicular traffic flow. Jassim *et al.* [10] solved the diffusion and wave equations defined on Cantor sets. Goswami *et al.* [11] and Yang *et al.* [12], respectively, reported the non-differentiable solutions of the local fractional differential equations. Other technologies, such as similarity variable [13], Cole-Hopf transformation [14], homotopy perturbation [15], asymptotic perturbation [16], and extended differential transform [17] methods, were reported. The local fractional variational iteration algorithms were reported in [18-20]. The main aim of the manuscript is to present the local fractional Laplace transform variational iteration algorithm III to solve the diffusion equation in the fractal heat-conduction problem.

Analysis of the mathematical method

Let us consider the local fractional differential equation:

$$L_\xi \psi_\xi(x, \tau) = P\psi_\xi(x, \tau) \tag{1}$$

where L_ξ and P are local fractional differential operators [1, 2, 5, 18-20].

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We can write the correction functional in the form:

$$\psi_{\xi,n+2}(x,\tau)=\psi_{\xi,n+1}(x,\tau)+{}_0I_\tau^{(\zeta)}\{\lambda P[\tilde{\psi}_{\xi,n+1}(x,\tau)-\tilde{\psi}_{\xi,n}(x,\tau)]\} \tag{2}$$

where ${}_0I_\tau^{(\zeta)}$ is the local fractional integral operator [1-2, 12] and $\tilde{\psi}_{\xi,n}$ is considered as a restricted local fractional variation, *i. e.*, $\delta^\xi\tilde{\psi}_{\xi,n}=0$.

The local fractional variational iteration algorithm-III is presented:

$$\psi_{\xi,n+2}(x,\tau)=\psi_{\xi,n+1}(x,\tau)+{}_0I_\tau^{(\zeta)}\{\lambda P[\psi_{\xi,n+1}(x,\tau)-\psi_{\xi,n}(x,\tau)]\} \tag{3}$$

where λ is the identified fractal Lagrange multiplier.

Taking the local fractional Laplace transform (LLT) of eq. (3) gives:

$$\psi_{\xi,n+2}(x,s)=\mathrm{LLT}_\xi[\psi_{\xi,n+2}(x,\tau)]=\psi_{\xi,n+1}(x,s)+s^\xi\{\lambda P[\psi_{\xi,n+1}(x,s)-\psi_{\xi,n}(x,s)]\} \tag{4}$$

where the local fractional Laplace transform is given by [1] $\mathrm{LLT}_\xi[\varpi(t)]=\varpi(s)$.

Finally, we get the local fractional Laplace series solution in the form:

$$\psi_\xi(x,s)=\sum_{i=0}^{\infty}\psi_{i,\xi}(x,s) \tag{5}$$

Taking the inverse local fractional Laplace transform of eq. (5), we obtain:

$$\psi_\xi(x,t)=\mathrm{LLT}_\xi^{-1}[\psi_\xi(x,s)]=\mathrm{LLT}_\xi^{-1}\left[\sum_{i=0}^{\infty}\psi_{i,\xi}(x,s)\right]=\sum_{i=0}^{\infty}\psi_{i,\xi}(x,t) \tag{6}$$

where the local fractional Laplace transform is given by [1] $\mathrm{LLT}_\xi^{-1}[\varpi(s)]=\varpi(t)$.

The technology is called as the local fractional Laplace transform variational iteration algorithm III.

Solving the local fractional diffusion equation

We now consider the local fractional diffusion equation arising in the fractal heat-conduction problem [2, 13-16]:

$$\frac{\partial^\xi\psi_\xi(x,\tau)}{\partial\tau^\xi}=\frac{\partial^{2\xi}\psi_\xi(x,\tau)}{\partial x^{2\xi}} \tag{7}$$

subject to the initial condition:

$$\psi_\xi(x,0)=\mathrm{E}_\xi(x^\xi) \tag{8}$$

According to the result [2], we have:

$$\lambda=-1 \tag{9}$$

such that the local fractional variational iteration algorithm-III is given:

$$\begin{cases} \psi_{\xi,n+2}(x,s) = \psi_{\xi,n+1}(x,s) - s^{\xi} \left\{ \dfrac{\partial^{2\xi}}{\partial x^{2\xi}} P[\psi_{\xi,n+1}(x,s) - \psi_{\xi,n}(x,s)] \right\} \\ \psi_{\xi,1}(x,s) = \left(\dfrac{1}{s^{\xi}} + \dfrac{1}{s^{2\xi}} \right) \mathrm{E}_{\xi}(x^{\xi}) \\ \psi_{\xi,0}(x,s) = \dfrac{1}{s^{\xi}} \mathrm{E}_{\xi}(x^{\xi}) \end{cases} \tag{10}$$

Thus, we obtain the local fractional Laplace transfer series solution:

$$\psi_{\xi}(x,s) = \sum_{i=0}^{\infty} \psi_{i,\xi}(x,s) = \left(\frac{1}{s^{\xi}} + \frac{1}{s^{2\xi}} + \frac{1}{s^{3\xi}} + ... + \frac{1}{s^{(i+1)\xi}} \right) \mathrm{E}_{\xi}(x^{\xi}) \tag{11}$$

which leads to:

$$\psi_{\xi}(x,s) = \mathrm{LLT}_{\xi}^{-1} \left[\sum_{i=0}^{\infty} \psi_{i,\xi}(x,s) \right] = \mathrm{E}_{\xi}(\tau^{\xi}) \mathrm{E}_{\xi}(x^{\xi}) \tag{12}$$

Equation (12) is the local fractional iteration solution of eq. (7).

Conclusions

We proposed the local fractional Laplace transform variational iteration algorithm III. We used it to solve the local fractional diffusion equation arising in the fractal heat-conduction problem. The obtained result shows that the proposed technology is accurate and efficient to handle the fractal heat-conduction problem via local fractional calculus.

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Nomenclature

x – space co-ordinate, [m]

Greek symbols

ξ – fractal dimension, [–]
$\psi_{\xi}(x, \tau)$ – concentration, [–]
τ – time, [s]

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