

Numerical Solution of Contaminated Oil Along a Vertical Wavy Frustum of a Cone

Sadia Siddiqua^{*,1}, Naheed Begum^{*}, A. Waheed^{*}, Md. A. Hossain[‡], Rama Subba Reddy Gorla[§]

^{*}*Department of Mathematics, COMSATS Institute of Information Technology, Kamra Road, Attock, Pakistan*

[‡]*UGC Professor, University of Dhaka, Dhaka, Bangladesh*

[§]*Department of Mechanical Engineering, University of Akron, Akron, USA*

Abstract: The effect of wavy surface on natural convection flow of two-phase dusty fluid over a vertical frustum of a cone is presented in this paper. The boundary layer regime having the large value of Grashof number Gr is considered and the wavy surface is assumed to have wavelength and amplitude of $O(1)$. A sinusoidal surface is used as a particular example to elucidate the heat transfer mechanism near such surfaces. The transformed boundary layer equations are solved numerically and detailed results for the skin friction and rate of heat transfer coefficients are presented for a selected parameters: wavy surface amplitude, half cone angle, mass concentration parameter and dust parameter. Comprehensive flow formations of the contaminated oil having Pr equal to 500.0, are given with the aim to predict the enhancement of heat transport across the heated wavy frustum of the cone.

Keywords: Natural Convection, Two-Phase, Fluid-particle Flow, Frustum of a Cone, Wavy Surface

1 Introduction

The analysis of the flow of fluids with suspended particles or gas-particle mixture have received notable attention due to its practical applications in various problem of atmospheric, engineering and physiological fields. Typical examples occurring in nature are dust storms, forest-fire smoke and the dispersion of the solid pollutants in atmosphere. In addition, solid rocket exhaust nozzles, fluidization in chemical reactors with gas-solid feeds, ablation cooling, combustion chambers, blast waves moving over the Earth's surface, conveying of powdered materials, fluidized beds, environmental pollutants, petroleum industry, purification of crude oil, physiological flows and other technological fields (see [1]) are some of the practical problems where the dusty viscous flow found its applications. Other important applications involving dust particles in boundary layers include soil salvation by natural winds, lunar surface erosion by the exhaust of a landing vehicle and dust entrainment in a cloud formed during a nuclear explosion. For theoretical investigation Saffman [2] proposed the model initially and later on several attempts were made to conclude the physical insight of such two-phase flows (for example see [3-8]).

Irregular surfaces, say, vertical or horizontal wavy surfaces have been considered vastly in the literature [9-15]. Through these analysis it has been reported that heat transfer along these surfaces increases considerably, which ultimately serves practically in engineering applications (for instance in solar collectors, industrial heat exchangers and condensers

¹Corresponding author.

Email: saadiasiddiqua@gmail.com, Ph: +923335297152

in refrigerators). However, the problem of convection flow from a vertical wavy cone and/or a frustum was discussed by very few authors [16-20]. Natural convection over a vertical cone can be dealt with similarity methods whereas that over a frustum cone cannot be solved by using similarity methods and hence some alternative methods has to be used, such as, finite difference methods. Since heat transfer characteristics from the surface of the cone and frustum cone depend on the properties of the boundary layer, the accuracy with which these properties are known are important for engineering applications.

The behavior of two-phase flows along a wavy frustum of a cone is not yet discussed in the literature. The class of solutions of gas boundary layer containing uniform, spherical solid particles from a sinusoidal wavy frustum of a cone is therefore investigated in this article. We will discuss the solutions of the basic dusty fluid flows by considering the fact that the processes of drag and heat transfer are responsible for the coupling of particles and the gas. From the present analysis we will interrogate whether the presence of roughness element disturbs the two phase flow and alter the physical characteristics associates with the wavy frustum or not? The solutions are obtained numerically and presented in the form of wall shear stress, heat transfer rate, velocity and temperature profiles by varying several controlling parameters.

2 Analysis

Consider steady two dimensional natural convection fluid-particle flow along the vertical wavy frustum of a heated cone. the boundary layer is assumed to develop at the leading edge ($\hat{x} = \hat{x}_0$), which means the temperature at the circular base is assumed to be at the same temperature as the temperature of the surrounding fluid. Due to the difference in temperature between the surface and the surrounding fluid, an upward flow is created as a result of buoyancy. For the boundary layer analysis outlined, the vertical frustum of the cone is assumed to exhibits the transverse sinusoidal waves having the amplitude \hat{a} and the characteristic length L as depicted in Fig. 1. In particular, the wavy surface profile is mathematically expressed as:

$$\hat{y}_w = \sigma(\hat{x}) = \hat{a} \sin\left(\frac{\pi(\hat{x} - \hat{x}_0)}{L}\right) \quad (1)$$

where \hat{x}_0 is slant height of the lower end of the cone and L is the characteristic length associated with the uneven surface (also known as half of the wavelength of the uneven surface). We have considered dusty fluid which is originally at rest along a vertical heated wavy frustum of the cone. Initially, the system is having a uniform temperature T_∞ . Suddenly, the surface of the cone $\hat{y} = 0$ is heated to a temperature $T + \Delta T$ and natural convection starts due to this. Following (see Ref. [2], [14]), the current analysis is made by keeping in view the underlying assumptions:

- The number density of particles is uniform in the suspension.
- The dust particles are sparsely distributed in the base fluid and they are non-interacting. As a result, there will be lack in randomness of the local particle movement and hence the associated pressure with the particles will be negligible and will be equal to the pressure of base fluid.

Now the governing system of equations is as under:

For the gas phase:

$$\frac{\partial(\hat{r}\hat{u})}{\partial\hat{x}} + \frac{\partial(\hat{r}\hat{v})}{\partial\hat{y}} = 0 \quad (2)$$

$$\rho \left(\hat{u} \frac{\partial\hat{u}}{\partial\hat{x}} + \hat{v} \frac{\partial\hat{u}}{\partial\hat{y}} \right) = -\frac{\partial\hat{p}}{\partial\hat{x}} + \mu \nabla^2 \hat{u} + g\beta (T - T_\infty) \cos \phi + \frac{\rho_p}{\tau_m} (\hat{u}_p - \hat{u}) \quad (3)$$

$$\rho \left(\hat{u} \frac{\partial\hat{v}}{\partial\hat{x}} + \hat{v} \frac{\partial\hat{v}}{\partial\hat{y}} \right) = -\frac{\partial\hat{p}}{\partial\hat{y}} + \mu \nabla^2 \hat{v} - g\beta (T - T_\infty) \sin \phi + \frac{\rho_p}{\tau_m} (\hat{v}_p - \hat{v}) \quad (4)$$

$$\rho c_p \left(\hat{u} \frac{\partial T}{\partial\hat{x}} + \hat{v} \frac{\partial T}{\partial\hat{y}} \right) = \kappa \nabla^2 T + \frac{\rho_p c_s}{\tau_T} (T_p - T) \quad (5)$$

For the particle phase:

$$\frac{\partial(\hat{r}\hat{u}_p)}{\partial\hat{x}} + \frac{\partial(\hat{r}\hat{v}_p)}{\partial\hat{y}} = 0 \quad (6)$$

$$\rho_p \left(\hat{u}_p \frac{\partial\hat{u}_p}{\partial\hat{x}} + \hat{v}_p \frac{\partial\hat{u}_p}{\partial\hat{y}} \right) = -\frac{\partial\hat{p}_p}{\partial\hat{x}} - \frac{\rho_p}{\tau_m} (\hat{u}_p - \hat{u}) \quad (7)$$

$$\rho_p \left(\hat{u}_p \frac{\partial\hat{v}_p}{\partial\hat{x}} + \hat{v}_p \frac{\partial\hat{v}_p}{\partial\hat{y}} \right) = -\frac{\partial\hat{p}_p}{\partial\hat{y}} - \frac{\rho_p}{\tau_m} (\hat{v}_p - \hat{v}) \quad (8)$$

$$\rho_p c_s \left(\hat{u}_p \frac{\partial T_p}{\partial\hat{x}} + \hat{v}_p \frac{\partial T_p}{\partial\hat{y}} \right) = -\frac{\rho_p c_s}{\tau_T} (T_p - T) \quad (9)$$

where (\hat{u}, \hat{v}) , T , \hat{p} , ρ , c_p , β , κ , μ are respectively the velocity vector in the (\hat{x}, \hat{y}) direction, temperature, pressure, density, specific heat at constant pressure, volumetric expansion coefficient, thermal conductivity and coefficient of viscosity of the fluid/carrier phase. Similarly, (\hat{u}_p, \hat{v}_p) , T_p , \hat{p}_p , ρ_p and c_s corresponds to the velocity vector, temperature, pressure, density and specific heat for the particle phase. Furthermore, $\vec{g} = (g \cos \phi, g \sin \phi)$ is the gravitational acceleration along the (\hat{x}, \hat{y}) directions respectively, and τ_m (τ_T) is the momentum relaxation time (thermal relaxation time) during which the velocity (temperature) of the particle phase relative to the fluid is reduced to $1/e$ times its initial value. In addition, ϕ is the half angle and \hat{r} is the local radius of the frustum of a cone which is given as follows:

$$\hat{r} = (\hat{x} + \hat{x}_0) \sin \phi \quad (10)$$

The fundamental equations stated above are to be solved under appropriate boundary conditions to determine the flow fields of the fluid and the dust particles. Therefore, respective boundary conditions for the gas and particle phase are:

$$\begin{aligned} \hat{u}(\hat{x}, \hat{y}_w) = \hat{v}(\hat{x}, \hat{y}_w) = T(\hat{x}, \hat{y}_w) - T_w = 0 \\ \hat{u}(\hat{x}, \infty) = T(\hat{x}, \infty) - T_\infty = 0 \end{aligned} \quad (11)$$

and

$$\begin{aligned} \hat{u}_p(\hat{x}, \hat{y}_w) = \hat{v}_p(\hat{x}, \hat{y}_w) = T_p(\hat{x}, \hat{y}_w) - T_w = 0 \\ \hat{u}_p(\hat{x}, \infty) = T_p(\hat{x}, \infty) - T_\infty = 0 \end{aligned} \quad (12)$$

where T_w is the constant temperature of the heated cone which is higher than the ambient fluid temperature T_∞ , *i.e.* ($T_w \gg T_\infty$). For large Gr , the system of equation Eqs. (2)-(12)

can be transformed into boundary layer equations as follows:

For the gas phase:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \quad (13)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \sigma_x Gr^{1/4} \frac{\partial p}{\partial y} + (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} + \theta + D_\rho \alpha_d (u_p - u) \quad (14)$$

$$\sigma_x \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) + \sigma_{xx} u^2 = -Gr^{1/4} \frac{\partial p}{\partial y} + \sigma_x (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} - \tan \phi \theta + \sigma_x D_\rho \alpha_d (u_p - u) \quad (15)$$

$$u \frac{\partial \theta}{\partial x} + v \frac{\partial \theta}{\partial y} = \frac{1}{\text{Pr}} \left((1 + \sigma_x^2) \frac{\partial^2 \theta}{\partial y^2} + \frac{2}{3} D_\rho \alpha_d (\theta_p - \theta) \right) \quad (16)$$

For the particle phase:

$$\frac{\partial(ru_p)}{\partial x} + \frac{\partial(rv_p)}{\partial y} = 0 \quad (17)$$

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} = -\frac{\partial p_p}{\partial x} + \sigma_x Gr^{1/4} \frac{\partial p_p}{\partial y} - \alpha_d (u_p - u) \quad (18)$$

$$\sigma_x \left(u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} \right) + u_p^2 \sigma_{xx} = -Gr^{1/4} \frac{\partial p_p}{\partial y} - \alpha_d \sigma_x (u_p - u) \quad (19)$$

$$u_p \frac{\partial \theta_p}{\partial x} + v_p \frac{\partial \theta_p}{\partial y} = -\frac{2}{3\gamma \text{Pr}} \alpha_d (\theta_p - \theta) \quad (20)$$

Associated boundary conditions for above phases are:

$$\begin{aligned} u(x, 0) = u_p(x, 0) = v(x, 0) = v_p(x, 0) = \theta(x, 0) - 1 = \theta_p(x, 0) - 1 = 0 \\ u(x, \infty) = u_p(x, \infty) = \theta(x, \infty) = \theta_p(x, \infty) = 0 \end{aligned} \quad (21)$$

where

$$\begin{aligned} (u, u_p) = \frac{\rho L}{\mu} Gr^{-1/2} (\hat{u}, \hat{u}_p), \quad (v, v_p) = \frac{\rho L}{\mu} Gr^{-1/4} ((\hat{v}, \hat{v}_p) - \sigma_x (\hat{u}, \hat{u}_p)), \quad x = \frac{\hat{x}}{L} \\ r = \frac{\hat{r}}{L}, \quad y = \frac{\hat{y} - \sigma(\hat{x})}{L} Gr^{1/4}, \quad (\theta, \theta_p) = \frac{(T, T_p) - T_\infty}{T_w - T_\infty}, \quad (p, p_p) = \frac{L^2}{\rho \nu^2 Gr} (\hat{p}, \hat{p}_p) \\ a = \frac{\hat{a}}{L}, \quad \sigma_x = \frac{d\hat{\sigma}}{d\hat{x}} = \frac{d\sigma}{dx}, \quad \sigma(x) = \frac{\sigma(\hat{x})}{L}, \quad Gr = \frac{g\beta L^3 (T_w - T_\infty) \cos \phi}{\nu^2} \\ x_0 = \frac{\hat{x}_0}{L}, \quad \nu = \frac{\mu}{\rho}, \quad \text{Pr} = \frac{\mu c_p}{\kappa} \end{aligned} \quad (22)$$

The carrier phase and particle phase are interacted through the terms: γ , D_ρ , τ_T and α_d . Particularly, $\gamma (= c_s/c_p)$ is the specific heat ratio of the mixture. For different gas-particle combinations, γ may vary between 0.1 and 10.0, and in such cases, either the temperature or the velocity tends to reach equilibrium faster (see [1]). $D_\rho (= \rho_p/\rho)$ is the mass concentration of particle phase or the ratio of densities of the fluid and the dust particles. $\tau_T = 1.5\gamma\tau_m Pr$ is the relation between thermal relaxation time (τ_T) and velocity relaxation time (τ_m); indicating that τ_T is obeying the Stokes law. It should be noted that, for most gases, the Prandtl number, Pr is near 2/3 and the specific heat mixture parameter (γ) is typically near one, which means that for this particular situation temperature and velocity relaxation become equivalent. Finally, $\alpha_d = L^2/\nu\tau_m Gr^{1/2}$ is the parameter depending on the relaxation time of the particles and the buoyancy force.

The pressure gradient of the gas and the particle phase along the y direction are of $O(Gr^{-1/4})$, as depicted from the Eqs. (14) and (18), respectively, which implies that the lowest order pressure gradient of both phases along the x direction can be determined from the inviscid-flow solution. In the present problem, this pressure gradient is zero because there is no externally induced free stream. Eq. (14) further shows that $Gr^{1/4}\partial p/\partial y$ is $O(1)$ and can be determined by the left-hand side of this equation. Thus, the elimination of $\partial p/\partial y$ from (14) and (15) leads to:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\sigma_x \sigma_{xx}}{(1 + \sigma_x^2)} u^2 = (1 + \sigma_x^2) \frac{\partial^2 u}{\partial y^2} + \frac{1 - \sigma_x \tan \phi}{(1 + \sigma_x^2)} \theta + D_\rho \alpha_d (u_p - u) \quad (23)$$

The same reason holds for dusty phase, therefore, from Eqs. (18) and (19) one gets:

$$u_p \frac{\partial u_p}{\partial x} + v_p \frac{\partial u_p}{\partial y} + \frac{\sigma_x \sigma_{xx}}{(1 + \sigma_x^2)} u_p^2 = -D_\rho \alpha_d (u_p - u) \quad (24)$$

To establish the solutions of the coupled equations (13), (16), (17), (20), (23), (24) along with the boundary conditions (21) we switch into another system of equations with the help of primitive variable formulations. For this, the following set of continuous transformations are introduced:

$$(u, u_p) = x^{\frac{1}{2}}(U, U_p), \quad (v, v_p) = x^{-\frac{1}{4}}(V, V_p), \quad (\theta, \theta_p) = (\Theta, \Theta_p) \quad (25)$$

$$y = x^{\frac{1}{4}}Y, \quad r = (x + x_0) \sin \phi$$

Upon substitution of (25) in Eqs. (13), (16), (17), (20), (23), (24) we get:

$$X \frac{\partial U}{\partial X} + \frac{3X + X_o}{2(X + X_o)} U - \frac{1}{4} Y \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial Y} = 0 \quad (26)$$

$$\left(\frac{1}{2} + \frac{X \sigma_X \sigma_{XX}}{(1 + \sigma_X^2)} \right) U^2 + XU \frac{\partial U}{\partial X} + \left(V - \frac{1}{4} Y U \right) \frac{\partial U}{\partial Y} = (1 + \sigma_X^2) \frac{\partial^2 U}{\partial Y^2} + \frac{1 - \sigma_X \tan \phi}{(1 + \sigma_X^2)} \Theta + D_\rho \alpha_d X^{1/2} (U_p - U) \quad (27)$$

$$XU \frac{\partial \Theta}{\partial X} + \left(V - \frac{1}{4} Y U \right) \frac{\partial \Theta}{\partial Y} = \frac{1}{\text{Pr}} \left[(1 + \sigma_X^2) \frac{\partial^2 \Theta}{\partial Y^2} + \frac{2}{3} D_\rho \alpha_d X^{1/2} (\Theta_p - \Theta) \right] \quad (28)$$

$$X \frac{\partial U_p}{\partial X} + \frac{3X + X_o}{2(X + X_o)} U_p - \frac{1}{4} Y \frac{\partial U_p}{\partial Y} + \frac{\partial V_p}{\partial Y} = 0 \quad (29)$$

$$\left(\frac{1}{2} + \frac{X \sigma_X \sigma_{XX}}{(1 + \sigma_X^2)} \right) U_p^2 + XU_p \frac{\partial U_p}{\partial X} + \left(V_p - \frac{1}{4} Y U_p \right) \frac{\partial U_p}{\partial Y} = -\alpha_d X^{1/2} (U_p - U) \quad (30)$$

$$XU_p \frac{\partial \Theta_p}{\partial X} + \left(V_p - \frac{1}{4} Y U_p \right) \frac{\partial \Theta_p}{\partial Y} = -\frac{2}{3\gamma \text{Pr}} \alpha_d X^{1/2} (\Theta_p - \Theta) \quad (31)$$

Subject to the conditions:

$$U(X, 0) = V(X, 0) = \Theta(X, 0) - 1 = U_p(X, 0) = V_p(X, 0) = \Theta_p(X, 0) - 1 = 0 \quad (32)$$

$$U(X, \infty) = U_p(X, \infty) = \Theta(X, \infty) = \Theta_p(X, \infty) = 0$$

A numerical solution for the coupled system of non linear partial differential Eqs. (26)-(32) by a finite difference method is straightforward, since the computational grids can be fitted to the body shape in (X, Y) coordinates. The discretization process is carried out by exploiting the central difference quotients for diffusion terms, and the forward difference for the convection terms. The computational process is started at $X = 0.01$ as the singularity at this point has been removed by the scaling. At every X station, the computations are iterated until the difference of the results, of two successive iterations become less or equal to 10^{-6} . In order to get accurate results, we have compared the results at different grid size in Y direction and reached at the conclusion to chose $\Delta Y = 0.005$. In this integration, the maximum value of Y is taken to be 90.0. A detail description of discretization procedure and numerical scheme is presented in [19].

Once the unknown variables of both phases are obtained, the quantities of interest: local skin friction coefficient, τ_w , and rate of heat transfer, Q_w can be computed from the following relations:

$$C_f = \frac{\tau_w}{\rho(\nu_\infty/L)^2}, \quad Nu = \frac{LQ_w}{\kappa(T_w - T_\infty)} \quad (33)$$

where

$$\tau_w = \mu(\hat{\mathbf{n}} \cdot \nabla \hat{u})_{\hat{y}=0}, \quad Q_w = -\kappa(\hat{\mathbf{n}} \cdot \nabla T)_{\hat{y}=0} \quad (34)$$

Here $\hat{\mathbf{n}}$ is the unit vector normal to the wavy surface and defined as:

$$\hat{\mathbf{n}} = \left(-\frac{\sigma_x}{\sqrt{1 + \sigma_x^2}}, \frac{1}{\sqrt{1 + \sigma_x^2}} \right) \quad (35)$$

Now by using the transformations in Eqs. (22) and (25) in (33), we get:

$$\begin{aligned} \tau_w &= C_f \left(\frac{Gr^{-3}}{X} \right)^{1/4} = \sqrt{1 + \sigma_X^2} \left(\frac{\partial U}{\partial Y} \right)_{Y=0} \\ Q_w &= Nu \left(\frac{Gr}{X} \right)^{-1/4} = -\sqrt{1 + \sigma_X^2} \left(\frac{\partial \Theta}{\partial Y} \right)_{Y=0} \end{aligned} \quad (36)$$

3 Numerical Results and Discussion

The system of equations for the two phase model obtained in (27)-(32) are solved numerically by the two-point implicit finite difference method. From the problem under consideration, a number of results can be retrieved that are discussed in the literature by various authors. For example, if $\alpha_d = 0$, $D_\rho = 0$ and $x_0 = 0$ then governing equations will refer to the model of full wavy cone discussed by Pop and Na [16]. Similarly, if $\alpha_d = 0$, $D_\rho = 0$, $x_0 = 0$ and $a = 0$ then our model will describe the problem of natural convection flow over a flat frustum cone which has been solved in [20]. This analysis also retrieves the results of Pop and Na [17] in the absence of $\alpha_d = 0$ and $D_\rho = 0$. In reference [17], the authors adopted stream function formulation and solved via Keller box method, while on the other hand, present authors formulate the physical situation through primitive variable formulation and solved via Thomas algorithm. In spite of different formulations and methods as well; the computational results are compared in Fig. 2 by keeping the parameters as: $a = 0.0, 0.25, 0.5$, $D_\rho = 0.0$, $Pr = 6.7$, $\alpha_d = 0.0$, $x_0 = 1.0$ and $\phi = 10^\circ$. Both results agree well with each other and the comparison shows good accuracy.

For the two-phase problem under consideration, numerical results are presented for Prandtl number, $Pr = 500.0$ and $x_0 = 1.0$. The value of mass concentration parameter D_ρ is taken as 10.0 and the value of γ is assumed to vary from 0.1 to 1.0 that helps in considering the orders of magnitude of the dimensionless numbers. Particularly, solutions are presented by keeping in view mixture of oil with metal particles and these solutions are presented through variation of wall shear stress and heat transfer, velocity and temperature profile, streamlines and isotherms.

Graphical presentation of skin friction coefficient, τ_w , and rate of heat transfer coefficient, Q_w is given in Fig. 3 for oil particulate suspension. The skin friction coefficient decreases when the value of mass concentration parameter D_ρ increases from 0.0 to 10.0. The carrier phase loses some kinetic energy from the particles through the interaction and as a result the velocity gradient for the carrier fluid decreases at the surface of the wavy frustum of vertical cone. On the contrary, rate of heat transfer coefficient drastically increases. It is worthy to mention here that Q_w is mainly influenced due to particle impingement on the sediment layer. For higher values of D_ρ , the dusty oil gains the kinetic and thermal energy from the particles and the relative velocity of particle phase increases and the particles moves through the thermal boundary layer region with less time, thus preserving the lower temperature of the regions away from the interface. This tends to increase the temperature gradient between the impinging particles and the sediment and consequently the rate of heat transfer coefficient is promoted.

Fig. 4 displays the variation in quantities: τ_w and Q_w that are brought by changing the value of specific heat ratio parameter (γ) of the suspension. The skin friction coefficient remains insensitive but rate of heat transfer get stronger within the boundary layer region when γ increases. The case for $\gamma = 1.0$ corresponds to the physical situation when the specific of the carrier phase, c_s , becomes equal to that of particle phase, c_p , due to which the oil particulate suspension gains more thermal energy which give rise to the temperature gradient and hence assist the rate of heat transfer to enhance.

The influence of the dust parameter, α_d , for oil particulate suspension on τ_w and Q_w , is depicted in Fig. 5. It is interesting to infer from this figure that the value of skin friction coefficient, τ_w , and rate of heat transfer coefficient, Q_w , increases owing to the increase in dust parameter, α_d . The presence of inert particles are responsible for the enhancement of both the quantities.

The effect of mass concentration parameter on velocity and temperature profiles is illustrated in Fig. 6. For comparison, suspension without particle cloud (pure oil) is also presented. The curve in Fig. 6(a) show that by loading the dust particles in the oil mixture the fluid and temperature profiles of both phases diminishes considerably. For nonzero value of D_ρ the velocity and temperature profiles of carrier and dusty phase decays quickly to its asymptotical values and approaches to zero in the free stream region. The presence of inert particles are responsible for this behavior as they increase frictional forces. Clearly, it is seen from this figure that the thickness of momentum and thermal boundary layers increases. The dust particles near the surface are quit fine, that is, mass of the dust particles is negligibly small, then the relaxation time of dust particle decreases, and as a result the velocities and temperature profiles overlaps in the vicinity of the frustum cone. Further, the fluid, which is nearer to the axis of flow, moves with a greater velocity than the dust particles. This is because the fluid velocity is the source of the duct particle velocity.

4 Conclusion

The present analysis aims to compute the numerical results of two-phase boundary layer flow of dusty fluid along a vertical wavy frustum of a heated cone. Primitive variable formulations is adopted to transform the dimensionless boundary layer equations into convenient form and then the resulting nonlinear system of boundary layer equations are iteratively solved step by step by using implicit finite difference method along with tri-diagonal solver. The problem is investigated to predict the characteristics of oil particulate suspension moving along a vertical wavy frustum of a cone. Results are interpreted by considering base fluid/continuous fluid as oil, while particle cloud contains metal as a solid phase under the influence of several physically important parameters, such as, D_ρ , a , α_d and γ . From detailed numerical results, it is found that the skin friction coefficient decreases when metal particles are loaded into the oil and remains almost invariant by increasing the value of γ . On the contrary, the rate of heat transfer coefficient drastically increases when metal particles are increased within the oil.

References

- [1] Rudinger, G., Fundamentals of gas-particle flow, Elsevier Scientific Publishing Co., Amsterdam, 1980.
- [2] Saffman, P. G., On the stability of laminar flow of a dusty gas, *J. Fluid Mech.*, **13**, 1962, 120-128.
- [3] Marble, F. E., Dynamics of a gas containing small solid particles, combustion and propulsion, 5th AGARD colloquium, Pergamon press, 1963.
- [4] Singleton, R. E., Fluid mechanics of gas-solid particle flow in boundary layers, Ph.D. Thesis, California Institute of Technology, 1964.
- [5] Michael, D. H. and Miller, D. A., Plane parallel flow of a dusty gas, *Mathematica*, **13**, 1966, 97-109.
- [6] Datta, N. and Mishra, S. K., Boundary layer flow of a dusty fluid over a semi-infinite flat plate, *Acta Mechanica*, **42**, 1982, 71-83.
- [7] Siddiqua, S., Hossain, M. A. and Saha, S. C., Two-phase natural convection flow of a dusty fluid, *Int. J. Numer. Methods Heat Fluid Flow*, **45**, 2015, 1542-1556.
- [8] Yao, L. S., Natural convection along a vertical wavy surface, *J. Heat Transfer*, **105**, (1983) 465-468.
- [9] Moulic, S. G., Yao, L. S., Natural convection along a wavy surface with uniform heat flux, *J. Heat Transfer*, **111**, (1989) 1106-1108.
- [10] Rees, D. A. S., Pop, I., Free convection induced by a vertical wavy surface with uniform heat flux in a porous medium, *J. Heat Transfer*, **117**, (1995) 545-550.
- [11] Hossain, M. A., Pop, I., Magnetohydrodynamic boundary layer flow and heat transfer on a continuous moving wavy surface, *Arch. Mechanics*, **48**, (1996) 813-823.

- [12] Hossain, M. A., Rees, D. A. S., Combined heat and mass transfer in natural convection flow from a vertical wavy surface, *Acta Mechanica*, **136**, (1999) 133-141.
- [13] Siddiqa, S., Hossain, M. A., Saha, S. C., Natural convection flow with surface radiation along a vertical wavy surface, *Numerical Heat Transfer, Part A: Applications*, **64**, (2013) 400-415.
- [14] Siddiqa, S., Hossain, M. A., Saha, S. C., The effect of thermal radiation on the natural convection boundary layer flow over a wavy horizontal surface, *Int. J. of Thermal Sciences*, **84**, (2104) 143-150.
- [15] Pop, I., Natural convection of a darcian fluid about a wavy cone, *Int. Commun. Heat Mass transfer*, **21**, (1994) 891-899.
- [16] Pop, I., Na, T. Y., Natural convection from a wavy cone, *Appl. Sci. Res.*, **54**, (1995) 125-136.
- [17] Pop, I., Na, T. Y., Natural convection over a vertical wavy frustum of a cone, *J. Nonlinear Mechanics*, **34**, (1999) 925-934.
- [18] Molla, M. M., Hossain, M. A., Gorla, R. S. R., Radiation effects on natural convection boundary layer flow over a vertical wavy frustum of a cone, *Proc. IMechE Part C: J. Mechanical Engineering Science*, **223**, (2009) 1605-1614.
- [19] Siddiqa, S., Hossain, M. A., Begum, N., Radiation effects on natural convection flow along a vertical wavy cone with variable fluid properties, submitted in: *J. of App. Math.*, (2015).
- [20] Na, T. Y., Chiou, J. P., Laminar natural convection over a frustum of a cone, *Applied Scientific Research*, **35**, (1979) 409-421.
- [21] Apazidis, N., Temperature distribution and heat transfer in a particle-fluid flow past a heated horizontal plate, *Int. J. Multiphase Flow*, **16**, (1990), 495-513.

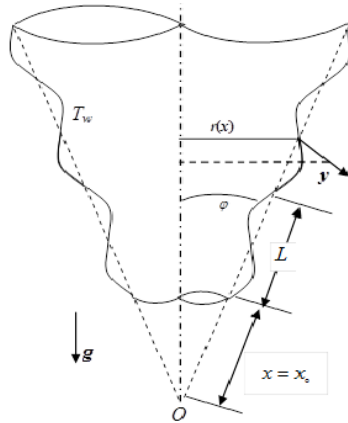


Fig. 1 Schematic of the problem.

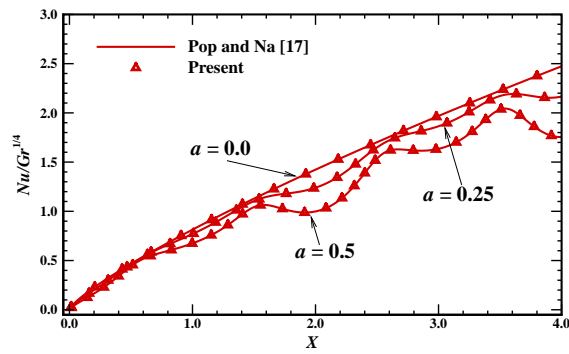


Fig. 2 Rate of heat transfer coefficient for $a = 0.0, 0.25, 0.5$, $D_\rho = 0.0$, $Pr = 6.7$, $\alpha_d = 0.0$, $x_0 = 1.0$ and $\phi = 10^\circ$.

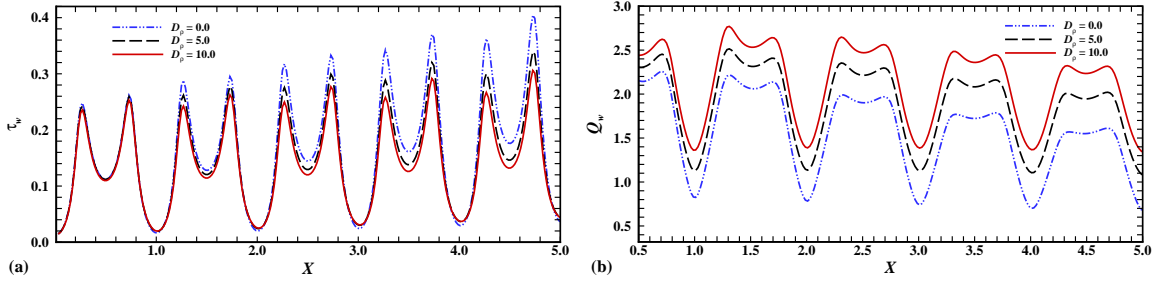


Fig. 3(a) Skin friction and (b) Rate of heat transfer coefficients for oil particle flow with $D_p = (0.0, 5.0, 10.0)$, $\gamma = 0.3$, $\alpha_d = 0.5$, $a = 0.3$, and $\phi = \pi/7$.

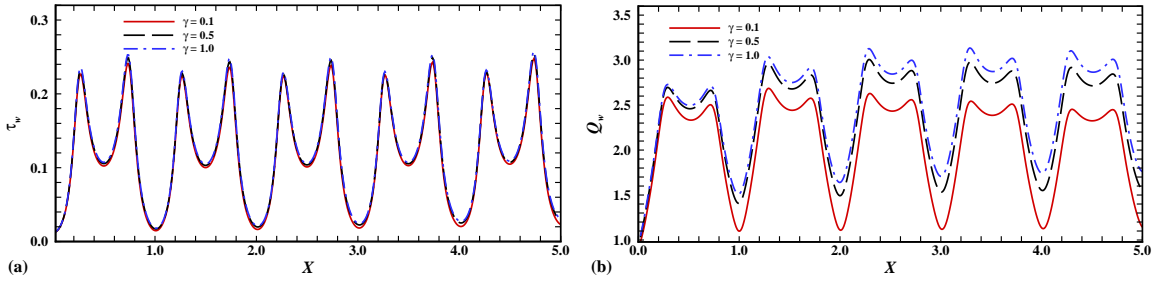


Fig. 4(a) Skin friction and (b) Rate of heat transfer coefficients for oil particle flow with $\gamma = (0.1, 0.5, 1.0)$, $D_p = 10.0$, $\alpha_d = 0.5$, $a = 0.3$, and $\phi = \pi/7$.

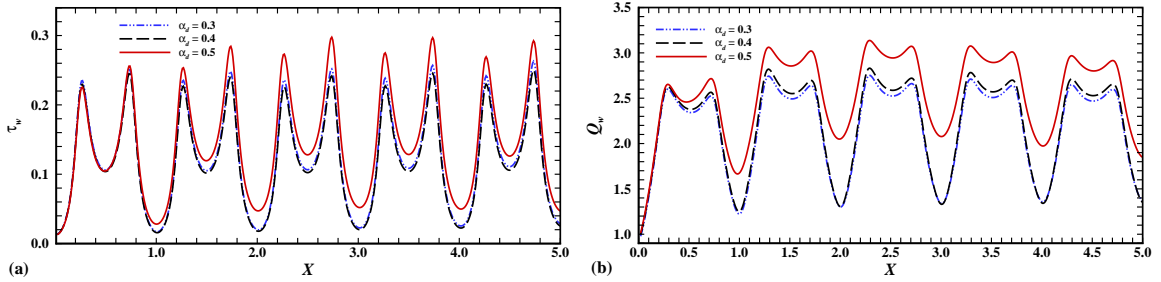


Fig. 5(a) Skin friction and (b) Rate of heat transfer coefficients for oil particle flow with $\alpha_d = (0.3, 0.4, 0.5)$, $D_p = 10.0$, $\gamma = 0.3$, $a = 0.3$, and $\phi = \pi/7$.

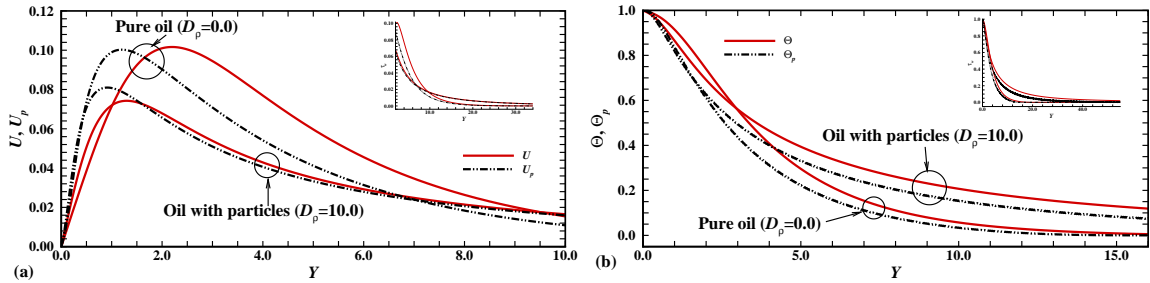


Fig. 6(a) Velocity and (b) Temperature profile for oil particle flow with $D_p = 10.0$, $\gamma = 0.3$, $\alpha_d = 0.5$, $a = 0.3$, $\phi = \pi/7$ and $X = 30.0$.