MATHEMATICAL MODELING OF FISH BURGER BAKING USING FRACTIONAL CALCULUS

by

Eduarda M. BAINY [a], ***Ervin K. LENZI*** [b], ***Marcos L. CORAZZA*** [c], ***and Marcelo K. LENZI*** [c*]

[a] Departament of Food Engineering, Federal University of Southerm Brazil, Laranjeiras do Sul, Parana, Brazil
[b] Departament of Phisics, State University of Ponta Grossa, Ponta Grossa, Parana, Brazil
[c] Departament of Engineering Chemistry, Federal University of Parana, Curitiba, Parana, Brazil

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Tilapia (Oreochromis sp.) is the most important and abundant fish species in Brazil due to its adaptability to different environments. The development of tilapia-based products could be an alternative in order to aggregate value and increase fish meat consumption. However, there is little information available on fish burger freezing and cooking in the literature. In this work, the mathematical modeling of the fish burger baking was studied. Previously to the baking process, the fish burgers were assembled in cylindrical shape of height equal to 8 mm and diameter 100 mm and then baked in an electrical oven with forced heat convection at 150 ºC. A T-type thermocouple was inserted in the burger to obtain its temperature profile at the central position. In order to describe the temperature of the burger during the baking process, lumped-parameter models of integer and fractional order and also a non-linear model due to heat capacity temperature dependence were considered. The burger physical properties were obtained from the literature. After proper parameter estimation tasks and statistical validation, the fractional order model could better describe the experimental temperature behavior, a value of 0.91 ± 0.02 was obtained for the fractional order of the system with correlation coefficient of 0.99. Therefore, with the better temperature prediction, process control, and economic optimization studies of the baking process can be conducted.

Key words: *fish burger, tilapia fractional calculus, mathematical model*

Introduction

There is a major concern towards the adoption of balanced diet for people worldwide. In this sense, the fish meat plays an important role due to its dietary features [1]. To bridge this gap, the fish Tilapia (*Oreochromis* sp.) is one of the most interesting alternatives as it is on of most abundant species in Brazil also presenting a relative low price [2]. On the other hand, fish meat needs to be processed to suitably preserve its properties and quality. Recently, the development of burgers based on fish meat has drawn attention due to the facility of handling,

* Corresponding author, e-mail: lenzi@ufpr.br

storing, and preparing [3]. Moreover, the baking process represents an important tool for food preparation as it does not incorporate unhealthy trans fatty acids [4].

For further and in-depth studies aimed at process optimization and control, the use of mathematical models represents a fundamental role [5]. The literature presents several applications of modeling in food processing [6]. The reported applications usually concern the use of first principles models [7] and experimental validation using different food systems, such as chicken [8], pork [9], fish [10], and meat [11]. However, depending on the application, lumped models may be an important alternative, due to the simple mathematical structure, possibility of analytical solution and necessity of low computation time [12].

Fractional calculus represents a very broad research field dealing with derivatives of arbitrary order [13]. Applications can be found in different research fields, for example, in systems engineering [14], diffusive systems [15], and surfactant adsorption at interfaces [16, 17]. Recently, the use of the fractional calculus has attracted the attention of researchers concerning food systems [18], mainly with applications in diffusion and rheology [19].

This manuscript deals with the analysis and formulation of mathematical models for describing the baking temperature of fish burger. The models here considered are lumped, taking into account the temperature dependence only on the time. Three models are analyzed:
(1) linear with integer order derivative,
(2) linear with fractional order derivative, and
(3) a non-linear with integer order derivative.

Materials and methods

The fish burger main ingredient is fresh Tilapia (*Oreochromis niloticus*). The fillets were obtained in local market in the city of Curitiba (Parana, Brazil, 25°26’05.6”S 49°15’24.8”W) and properly used to produce the burger meat, which was stored under refrigeration. Afterwards, the burger meat was molded with the aid of a manual burger mold, leading to burgers like the one presented in fig. 1, with (100 ± 1) mm of diameter and (8.0 ± 2) mm of thickness. Besides the Tilapia meat obtained from the fillets, which represents 72% of the formulation, the burger contains also other ingredients, such as textured soy protein (3%), condiments (2%), and binding agents (23%). Further details regarding the manufacturing process, composition, and microbiological analysis were reported by Bainy *et al.* [1, 2].

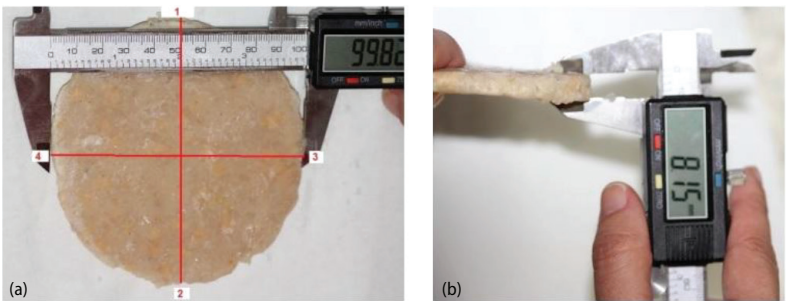

Figure 1. Tilapia fish burger dimensions

The fish burgers were cooked with the aid of a temperature controlled electrical oven (Forno vipinho 0448, Perfecta Curitiba, Curitiba-PR) using air forced convection. Firstly, the oven temperature was set to 150 ºC. After reaching this value, the oven was open and two fish burgers, each one containing a thermocouple (T-type) inserted into the geometric center,

were place over a metal dish. After closing the oven door, it was observed that the temperature of the oven fallen to 110 ºC and very quickly reached the desired set-point of 150 ºC. A second thermocouple was used to measure air temperature inside the oven, located approximately 2 cm from the sample. The thermocouples were wired to a data acquisition system (FieldLogger, Novus, Porto Alegre, Brazil) connected to a computer. Temperature data were recorded every 10 seconds during the baking process. The fish burger was considered baked when the measured temperature at its central position reached 76 ºC. Three experimental runs were carried out. The average temperatures of the oven and the burgers, as reported by Bainy *et al.* [1], were used in this modeling study. Further details regarding the cooking process were reported by Bainy *et al.* [1].

Theoretical framework

Mathematical model

The modeling studies were aimed at a proper description of the experimental dynamic behavior of the burger temperature. The temperature of the oven was adequately described by a delayed Heaviside function. The modeling approach here considered, consists in the analysis of the energy balance on the burger taking into account two basic hypotheses during the model derivation: (1) the fish burger mass did not considerably change, (2) the presence of the burger did not influence the oven temperature heating dynamics, (3) the temperature of the burger did not depend on spatial variables, it depended only on time, (4) there were no chemical reactions taking place in the burger, and (5) forced convection driven the heat transfer. Consequently, in this scenario, the following energy balance can be written [7]:

$$\frac{\mathrm{d}\left(m_{\text{burger}}\hat{H}_{\text{burger}}\right)}{\mathrm{d}t} = hA_{\text{transfer}}\left[T_{\text{oven}}(t) - T_{\text{burger}}(t)\right] \qquad T_{\text{burger}}(t=0) = T_{\text{burger}0} \tag{1}$$

In the energy balance, the enthalpy and temperature of the oven can be described by the following expressions, respectively:

$$\hat{H}_{\text{burger}} = \int_{T_{\text{ref}}}^{T_{\text{burger}}(t)} c_{p\,\text{burger}}(T)\,\mathrm{d}T \tag{2}$$

$$T_{\text{oven}}(t) = T_{\text{oven}0} + M\,\text{Heaviside}(t-\phi) \tag{3}$$

From the energy balance, three different modeling scenarios are obtained to describe the temperature of the burger. The first one, named MODEL1, is the simplest as the heat capacity, c_p, is assumed to be constant, therefore $\hat{H}_{\text{burger}} = c_{p0}T_{\text{burger}}(t)$. The MODEL2 provides a generalization of MODEL1 by considering the fractional calculus derivative based on Caputo approach [20], due to the facility to handle integer order initial conditions, typically present in engineering problems [21]. Finally, MODEL3 provides a generalization of MODEL1 by considering the heat capacity as a linear function of the temperature, $c_{p\,\text{burger}}(T) = c_{p0} + c_{p1}T_{\text{burger}}(t)$, therefore, leading to the following expression for enthalpy $\hat{H}_{\text{burger}} = c_{p0}T_{\text{burger}}(t) + (c_{p1}/2)\left[T_{\text{burger}}(t)\right]^2$. Higher order temperature dependence of the heat capacity has been extensively reported in the literature [7, 22]. However, for the fish burger, these values are not known, being need to be estimated from experimental data, consequently, to avoid over parametrization issues, the linear dependence was chosen. The MODEL1 and MODEL2 represent linear models and were solved by the Laplace transform method [23]. As previously mentioned, in order to adequately solve MODEL2, the

Caputo approach [24] was considered to represent the fractional derivative. This was the chosen approach as it effectively handles initial conditions of integer order, which are commonly present in physical scenarios, as the one reported in this study. It must be emphasized that the solution of MODEL2 is represented with the aid of two-parameter Mittag-Leffler function.

Finally, MODEL3 represents a non-linear differential equation, which was solved by separation method [25]. The solution was obtained in terms of an implicit expression based on $T_{\text{burger}}(t)$ and t. However, by employing the LambertW function [26], $T_{\text{burger}}(t)$ can be obtained as an explicit function of time. Table 1 presents the differential equations derived from the energy balance and the respective solution. Finally, it is important to stress that MODEL2 corresponds to a fractionalization of the derivative of MODEL1. This feature may provide a better mathematical modeling as the burger structure is porous and heterogeneous, thus being important to take into account memory effects, which is an intrinsic feature of fractional derivatives [27]. Therefore, it is important to note that in MODEL1, the time constant, τ_{int}, has units of (time). On the other hand, in MODEL2, this time constant, labeled τ_{frac}, has units of (time)$^{\beta}$.

Table 1. Mathematical models

MODEL1	
$$\frac{\mathrm{d}\left[T_{\text{burger}}(t)\right]}{\mathrm{d}t}=\left[\frac{hA_{\text{transfer}}}{m_{\text{burger}}c_{p0}}\right]\left\{\left[T_{\text{oven0}}+M\,\text{Heaviside}(t-\phi)\right]-T_{\text{burger}}(t)\right\}$$	(4a)
$$\frac{\mathrm{d}\left[T_{\text{burger}}(t)\right]}{\mathrm{d}t}=\left[\frac{1}{\tau_{\text{int}}}\right]\left\{\left[T_{\text{oven0}}+M\,\text{Heaviside}(t-\phi)\right]-T_{\text{burger}}(t)\right\}$$	(4b)
$$T_{\text{burger}}(t)=T_{\text{burger0}}+\left(T_{\text{oven0}}-T_{\text{burger0}}\right)\left[1-\mathrm{e}^{-\left(\frac{t}{\tau_{\text{int}}}\right)}\right]-M\,\text{Heaviside}(t-\phi)\left[1-\mathrm{e}^{-\left(\frac{t-\phi}{\tau_{\text{int}}}\right)}\right]$$	(5)
MODEL2	
$$\frac{\mathrm{d}^{\beta}\left[T_{\text{burger}}(t)\right]}{\mathrm{d}t^{\beta}}=\left[\frac{1}{\tau_{\text{frac}}}\right]\left\{\left[T_{\text{oven0}}+M\,\text{Heaviside}(t-\phi)\right]-T_{\text{burger}}(t)\right\}$$	(6)
$$T_{\text{burger}}(t)=T_{\text{burger0}}+\frac{\left(T_{\text{oven0}}-T_{\text{burger0}}\right)}{\tau_{\text{frac}}}e_0\left[t,\left(\frac{-1}{\tau_{\text{frac}}}\right);\beta,\beta+1\right]-$$ $$-\frac{M}{\tau_{\text{frac}}}\text{Heaviside}(t-\phi)e_0\left[(t-\phi),\left(\frac{-1}{\tau_{\text{frac}}}\right);\beta,\beta+1\right]$$ $$e_0(t,m_1;m_2,m_3)=t^{m_3-1}E_{m_2,m_3}\left(m_1t^{m_3}\right);\quad E_{\text{k}_1,\text{k}_2}(k_3)=\sum_{j=0}^{\infty}\frac{(k_3)^j}{\Gamma(k_1j+k_2)}$$	(7)

MODEL3	
$$\left[c_{p0}+c_{p1}T_{\text{burger}}(t)\right]\frac{\mathrm{d}\left[T_{\text{burger}}(t)\right]}{\mathrm{d}t}=\left[\frac{hA_{\text{transfer}}}{m_{\text{burger}}}\right]\left\{\left[T_{\text{oven0}}+M\,\text{Heaviside}(t-\phi)\right]-T_{\text{burger}}(t)\right\}$$	(8)
Implicit solution $$\begin{cases}\left[c_{p1}T_{\text{burger}}(t)\right]+\left(c_{p1}T_{\text{burger0}}+c_{p0}\right)\ln\left[\dfrac{T_{\text{oven0}}-T_{\text{burger}}(t)}{T_{\text{oven0}}-T_{\text{burger0}}}\right]=-\Psi t & ;\ t<\phi\\ \left[c_{p1}T_{\text{burger}}(t)\right]+\left(c_{p1}T_{\text{burger0}}+c_{p0}\right)\ln\left[\dfrac{(T_{\text{oven0}}+M)-T_{\text{burger}}(t)}{(T_{\text{oven0}}+M)-T_{\text{burger0}}}\right]=-\Psi t & ;\ t\geq\phi\end{cases}$$	(9)
Explicit solution $$T_{\text{burger}}(t)=\begin{cases}T_{\text{oven0}}\left\{1-\mathrm{e}^{-\left[\text{LambertW}(R_1)+\frac{T_{\text{oven0}}c_{p1}+\Psi t}{T_{\text{oven0}}c_{p1}+c_{p0}}\right]}\right\}+ \\ +T_{\text{burger0}}\left\{\mathrm{e}^{-\left[\text{LambertW}(R_1)+\frac{T_{\text{oven0}}c_{p1}+\Psi t}{T_{\text{oven0}}c_{p1}+c_{p0}}\right]}\right\} & ;\ t<\phi\\ (T_{\text{oven0}}+M)\left\{1-\mathrm{e}^{-\left[\text{LambertW}(R_2)+\frac{(T_{\text{oven0}}+M)c_{p1}+\Psi t}{T_{\text{oven0}}c_{p1}+c_{p0}}\right]}\right\}+ \\ +T_{\text{burger0}}\left\{\mathrm{e}^{-\left[\text{LambertW}(R_2)+\frac{(T_{\text{oven0}}+M)c_{p1}+\Psi t}{T_{\text{oven0}}c_{p1}+c_{p0}}\right]}\right\} & ;\ t\geq\phi\end{cases}$$	(10)

initial condition for all models:

$$T_{\text{burger}}(t=0)=T_{\text{burger0}},\qquad \tau_{\text{int}}=\frac{m_{\text{burger}}c_{p0}}{hA_{\text{transfer}}},\quad \Psi=\frac{hA_{\text{transfer}}}{m_{\text{burger}}}$$

$$R_1=\left[\frac{c_{p1}\left(T_{\text{oven0}}-T_{\text{burger0}}\right)\mathrm{e}^{-\left(\frac{T_{\text{oven0}}c_{p1}+\Psi t}{T_{\text{oven0}}c_{p1}+c_{p0}}\right)}}{T_{\text{oven0}}c_{p1}+c_{p0}}\right],$$

$$R_2=-\left\{\frac{c_{p1}\left(T_{\text{burger0}}-T_{\text{oven0}}-M\right)\mathrm{e}^{-\left[\frac{(T_{\text{oven0}}+M)c_{p1}+\Psi t}{T_{\text{oven0}}c_{p1}+c_{p0}}\right]}}{T_{\text{oven0}}c_{p1}+c_{p0}}\right\}$$

Parameter estimation

The dynamic models reported by tab. 1 present parameters that need to be estimated. More specifically, one can observe that MODEL1 has τ_{int} as unknown parameter, MODEL2 has β and τ_{frac} as unknown parameters, and MODEL3 has c_{p0}, c_{p1}, and Ψ as unknown parameters. For proper parameter estimation, the sum of squared difference between experimental values and model predictions was used as objective function [23]:

$$F_{\text{OBJ}} = \sum_{i=1}^{\text{NE}} \left[T_{\text{burger}}(t)\Big|_i^{\text{experimental}} - T_{\text{burger}}(t)\Big|_i^{\text{model}} \right]^2 \tag{11}$$

In order to adequately solve the optimization problem, a hybrid approach consisting of a genetic algorithm and a gradient-based method was used [24]. The main advantage of this procedure is that the presence of the genetic algorithm probably avoided a local minimum value of the objective function.

Further details regarding the statistical analysis of the parameter estimates can be found in the literature [23, 24]. The parameter variance was calculated using the complete Hessian of the objective function [23], which is a recommended approach for non-linear estimation problems. Generically, the following matrixes were considered:

$$\left[\underline{\underline{\mathrm{V}}}_{\theta}\right]_{(\text{NP}\times\text{NP})} = \left[\left(\underline{\underline{\mathrm{H}}}_{\theta}\right)^{-1}\right]_{(\text{NP}\times\text{NP})} \cdot \left(\left[\underline{\underline{\mathrm{G}}}_{y}\right]_{(\text{NP}\times\text{NE})} \cdot \left[\underline{\underline{\mathrm{V}}}_{y^E}\right]_{(\text{NE}\times\text{NE})} \cdot \left[\underline{\underline{\mathrm{G}}}_{y}^{T}\right]_{(\text{NE}\times\text{NP})}\right) \cdot \left[\left(\underline{\underline{\mathrm{H}}}_{\theta}\right)^{-1}\right]_{(\text{NP}\times\text{NP})} \tag{12}$$

$$\left[\underline{\underline{\mathrm{V}}}_{\theta}\right]_{(\text{NP}\times\text{NP})} = \begin{bmatrix} \delta^2_{(\theta_1)} & \delta^2_{(\theta_1-\theta_2)} & \cdots & \delta^2_{(\theta_1-\theta_{\text{NP}})} \\ \delta^2_{(\theta_1-\theta_2)} & \delta^2_{(\theta_2)} & \cdots & \delta^2_{(\theta_2-\theta_{\text{NP}})} \\ \vdots & \vdots & \ddots & \vdots \\ \delta^2_{(\theta_1-\theta_{\text{NP}})} & \delta^2_{(\theta_2-\theta_{\text{NP}})} & \cdots & \delta^2_{(\theta_{\text{NP}})} \end{bmatrix} \tag{13}$$

$$\left[\underline{\underline{\mathrm{V}}}_{y^E}\right]_{(\text{NE}\times\text{NE})} = \delta^2_{\text{y}^{\text{E}}} \left[\underline{\underline{I}}\right]_{(\text{NE}\times\text{NE})} \quad , \quad \delta^2_{\text{y}^{\text{E}}} = \sum_{i=1}^{\text{NE}} \frac{\left[T_{\text{burger}}(t)\Big|_i^{\text{experimental}} - T_{\text{burger}}(t)\Big|_i^{\text{model}} \right]^2}{\text{NE} - \text{NP}} \tag{14}$$

$$\left[\underline{\underline{\mathrm{H}}}_{\theta}\right]_{(\text{NP}\times\text{NP})} = \begin{bmatrix} \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_1^{\,2}} & \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_1 \partial \theta_2} & \cdots & \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_1 \partial \theta_{\text{NP}}} \\ \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_1 \partial \theta_2} & \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_2^{\,2}} & \cdots & \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_2 \partial \theta_{\text{NP}}} \\ \vdots & \vdots & \ddots & \vdots \\ \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_1 \partial \theta_{\text{NP}}} & \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_2 \partial \theta_{\text{NP}}} & \cdots & \dfrac{\partial^2 F_{\text{OBJ}}}{\partial \theta_{\text{NP}}^{\,2}} \end{bmatrix} \tag{15}$$

$$\left[\underline{\underline{\mathrm{G}_y}}\right]_{(\mathrm{NP}\times\mathrm{NE})} = \begin{bmatrix} \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_1\partial y_1^{\mathrm{E}}} & \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_1\partial y_2^{\mathrm{E}}} & \cdots & \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_1\partial y_{\mathrm{NE}}^{\mathrm{E}}} \\ \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_2\partial y_1^{\mathrm{E}}} & \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_2\partial y_2^{\mathrm{E}}} & \cdots & \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_2\partial y_{\mathrm{NE}}^{\mathrm{E}}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_{\mathrm{NP}}\partial y_1^{\mathrm{E}}} & \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_{\mathrm{NP}}\partial y_2^{\mathrm{E}}} & \cdots & \frac{\partial^2 F_{\mathrm{OBJ}}}{\partial\theta_{\mathrm{NP}}\partial y_{\mathrm{NE}}^{\mathrm{E}}} \end{bmatrix} \tag{16}$$

Results and discussions

The results analyses start by listing in tab. 2 all the known variable values and the oven temperature dynamics, which considered a time delay equal to 0.38 min that was obtained directly from the experimental data observation [1, 2].

Table 2. Variable values

T_{burger0} [°C]	T_{oven0} [°C]	M [°C]	ϕ [min]
19	110	40	0.38

Considering the parameter estimation approach reported in section *Parameter estimation*, tab. 3 presents the values of the estimated parameters of each model, with the respective uncertainty, the value of the objective function, F_{OBJ}, the value of the correlation coefficient, r, and the value of, δ^2_{yE}.

Table 3. Parameter estimation results

MODEL1	τ_{int}	5.5864
	δ_τ	0.0789
	F_{OBJ}	89.8
	r	0.99
	δ^2_{yE}	4.09
MODEL2	τ_{frac}	5.2516
	$\delta_{\tau,\mathrm{frac}}$	0.1052
	β	0.9139
	δ_β	0.02282
	F_{OBJ}	52.9
	r	0.99
	δ^2_{yE}	2.52
MODEL3	c_{p0}	2850
	δ_{cp0}	1673
	c_{p1}	2.81
	δ_{cp1}	1.55
	Ψ	461
	δ_Ψ	198
	F_{OBJ}	81.5
	r	0.99
	δ^2_{yE}	4.07

It can be observed that for all models, the value of, r, is close to 1, indicating a good correlation between experimental data and model predictions. Also, for all models, it can be observed that the parameter uncertainty, $\delta^2_{\theta_i}$, is smaller than the value of the respective parameter, indicating that statistical significance does exist. However, at a first glance, one would expect MODEL3 to provide the best fit, as it represents a non-linear reality, by considering a temperature dependent heat capacity. But a careful analysis may lead to the conclusion that although linear, MODEL2 presented the best data fit due to the presence of the fractional derivative. This can be observed as value of the objective function is almost 40% lower. Moreover, it is also important to take into account the number of parameters in the model, as the more parameters, the better the expected fit. Consequently, an important number for model discrimination is given by the value of δ^2_{yE}, which is the value of the objective function divided by the degree of freedom of the parameter estimation problem, given by NE-NP. This number shows that MODEL1 and MODEL3 have roughly the same value, but MODEL2 presents a lower value, leading to the conclusion that it is the best of the three analyzed models. Finally, it is important to emphasize that as mentioned before, the fish burger is not a smooth material, therefore, memory effects may be present, being adequately described by the fractional derivative. Figure 2 presents the dynamic behavior of the mathematical models, showing that MODEL2 looks closer to the experimental data when compared to the other models.

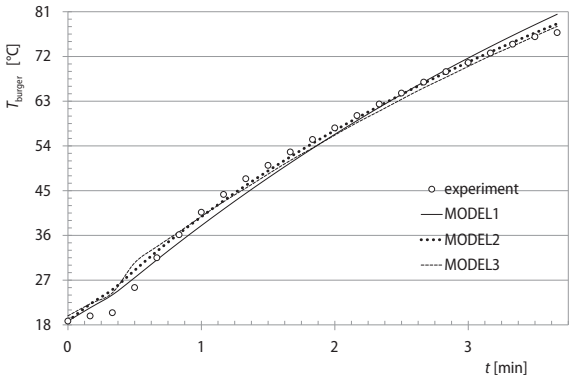

Figure 2. Dynamic behavior of the temperature inside the fish burger

As MODEL2 presented the best data fit, further analysis regarding its parameter estimates should be carried out. Figure 3 presents the experimental data plotted against the model predictions. It can be observed that the plot is close to the reference line of equal temperatures, especially for higher values. The lack of good prediction for lower temperatures was a common feature of all models, which probably happened due to the choice of the oven temperature. Finally, Figure 4 presents a histogram of the residues (experimental value – model prediction), showing that most part of the residues is close to zero, which is a good feature for the model estimates. The residues present mean equal to –0.28 and standard deviation equal to 1.52, allowing one to conclude that the mean is statically equal to 0, as reported by Isfer, *et al.* [21]. This is an important feature of the model, as the prediction errors are most likely to occur due to random errors than because of systematic errors, leading to an important conclusion that the model structure is fine, *i. e.*, no other terms or parameters need to be included.

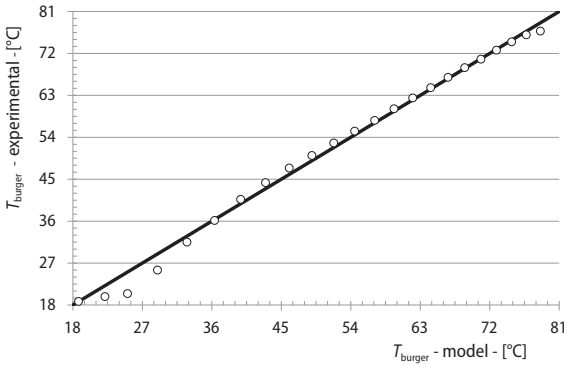

Figure 3. Experimental data *vs.* model predictions

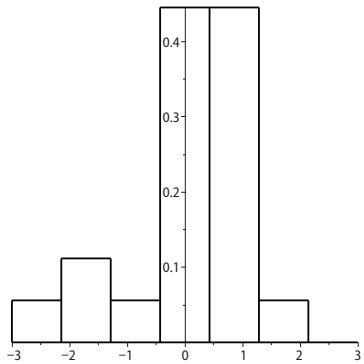

Figure 4. Residuals histogram

Conclusions

The baking process of fish burger is a challenging task. This work presents the model development of the baking process of fish burger produced from Tilapia (*Oreochromis* sp.) in an electrical oven. Three mathematical models were developed from energy balance equations considering a lumped system and also considering the oven temperature described by a delayed Heaviside function. The proposed models could adequately describe the experimental data, however, the model based on fractional order derivatives presented the best data fit, characterized by the lowest value of the objective function, parameters with statistical significance and residues statistically equal to zero. The best fit probably occurred due to the memory effects provided by the fractional calculus, allowing the development of an accurate model for further studies of the fish burger baking process.

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Nomenclature

A_{transfer} – available area for heat transfer, [m2]
$c_{p\text{burger}}$ – heat capacity of the burger, [Jkg^{-1}ºC^{-1}]
c_{p0} – first term of the heat capacity equation, [Jkg^{-1}ºC^{-1}]
c_{p1} – second term of the heat capacity equation, [Jkg^{-1}ºC^{-2}]
$E_{\text{k1,k2}}$(k3) – two-parameter (k1,k2) Mittag-Leffler function with argument k3
$\left[\underline{\underline{\mathrm{G_y}}}\right]_{(\mathrm{NP}\times\mathrm{NE})}$ – sensitivity matrix
$\hat{H}_{\text{burger}}$ – enthalpy of the burger, [Jkg^{-1}]
$\left[\underline{\underline{\mathrm{H}_\theta}}\right]_{(\mathrm{NP}\times\mathrm{NP})}$ – Hessian matrix
h – convection heat transfer coefficient, [Jmin^{-1}m^{-2}ºC^{-1}]
$\left[\underline{\underline{\mathrm{I}}}\right]_{(\mathrm{NE}\times\mathrm{NE})}$ – identity matrix
M – temperature variation of the oven, [ºC]
m_{burger} – mass of the burger, [kg]
NE – number of experiments, [–]
NP – number of parameters, [–]
T_{burger} – temperature of the burger, [ºC]
T_{burger0} – initial temperature of the burger, [ºC]
T_{oven} – temperature of the oven, [ºC]
T_{oven0} – initial temperature of the oven, [ºC]
T_{ref} – reference temperature for enthalpy, [ºC]
t – time, [min]
$\left[\underline{\underline{\mathrm{V}_\theta}}\right]_{(\mathrm{NP}\times\mathrm{NP})}$ – parametric variance matrix
$\left[\underline{\underline{\mathrm{V_{yE}}}}\right]_{(\mathrm{NE}\times\mathrm{NE})}$ – experimental variance matrix

Greek symbols

β – fractional derivative order, [–]
$\delta^2_{(\theta_i)}$ – variance of parameter θ_i
$\delta^2_{(\theta_i-\theta_j)}$ – covariance of parameters θ_i and θ_j
θ_i – i-th parameter to be estimated, [–]
τ_{frac} – time constant of MODEL2, [min]$^{\beta}$
τ_{int} – time constant of MODEL1, [min]
ϕ – time delay of the oven heating, [min]

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