

Invited paper

## THE ORIGIN OF TURBULENCE IN WALL-BOUNDED FLOWS

by

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*The motion of liquids and gases can be either laminar, flowing slowly in orderly parallel and continuous layers of fluid that cannot mix, or turbulent in which motion exhibits disorder in time and space with the ability to promote mixing. Breakdown of ordered to disordered motion can follow different scenarios so that no universal mechanism can be identified even in similar flow configurations [1]. Only under very special circumstances can the mechanism associated with the appearance of turbulence be studied within the deterministic theory of hydrodynamic stability [2] or employing direct numerical simulations [3] which themselves cannot provide the necessary understanding [4]. Here we show that the representative mechanism responsible for the origin of turbulence in wall-bounded flows is associated with large variations of anisotropy in the disturbances [5]. During the breakdown process, anisotropy decays from a maximum towards its minimum value, inducing the explosive production of the dissipation which logically leads to the appearance of small-scale three-dimensional motions. By projecting the sequence of events leading to turbulence in the space which emphasizes the anisotropic nature in the disturbances [6], we explain why, demonstrate how and present what can be achieved if the process is treated analytically using statistical techniques [7]. It is shown that the statistical approach provides not only predictions of the breakdown phenomena which are in fair agreement with available data but also requirements which ensure persistence of the laminar regime up to very high Reynolds numbers.*

Key words: *turbulence, wall bounded flows, anisotropy*

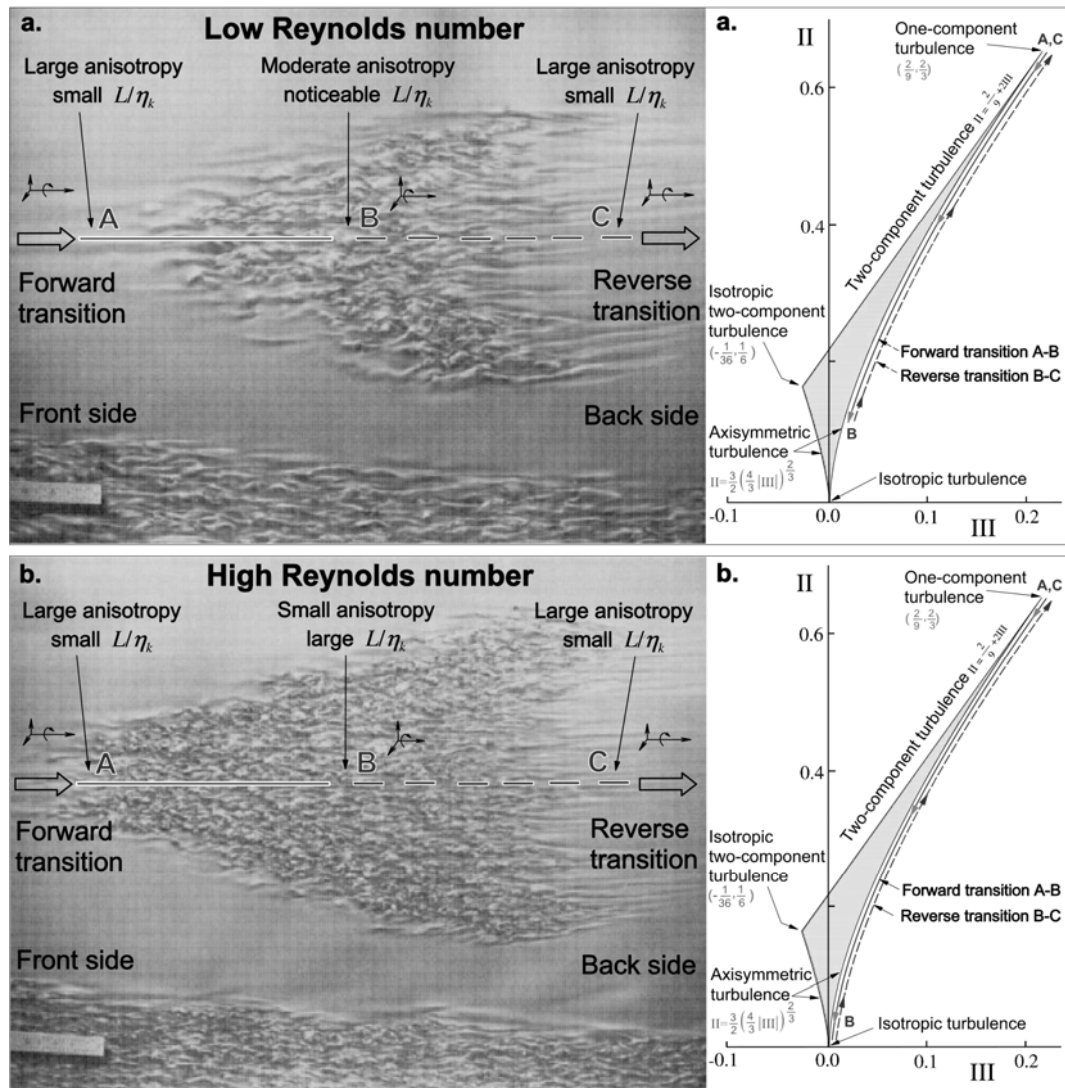
### Introduction

Since its accidental discovery [8] over 50 years ago, it is known that the most common and therefore representative scenario of breakdown of the laminar regime to turbulence in simple wall-bounded flows proceeds intermittently in the form of randomly appearing spots of turbulence as shown in fig. 1. The spots move slightly slower than the main stream and maintain their specific arrowhead shape, which can be clearly distinguished in flow visualization experiments [9-11]. In front of a spot, where the flow is still laminar, highly elongated flow patterns extending nearly to the size of the shear layer ( $L$ ) are observed. These patterns

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are usually termed streaks [12] in the literature. The streamwise streaks have a large length to width ratio and therefore are highly anisotropic.



**Figure 1.** Statistical interpretation of the breakdown leading to turbulence [10] and its projection on the anisotropy-invariant map; the influence of anisotropy in the disturbances is reflected in the evolution of spectral separation  $L/\eta_k$  in the flow; (a) Evolution of the turbulent spot at low Reynolds number reveals only a moderate level of anisotropy at the spot center and therefore only noticeable  $L/\eta_k$ , (b) As the Reynolds number increases, the level of anisotropy progressively decreases in the core region of the spot, which increases  $L/\eta_k$  (for color image see journal website)

The visual impression suggests that the streak pattern is statistically axisymmetric and corresponds to a system which is invariant under rotation about the axis aligned with the main flow. Downstream of the front side and towards the center of the spot, anisotropy de-

creases and is accompanied by the appearance of small-scale random motions which show nearly no preference for orientation in any specific direction. At the spot center, the patterns are almost isotropic, particularly at large Reynolds number, as can be seen from fig. 1(b). The sudden appearance of small-scale random motion implies that rapid decrease in anisotropy provokes explosive growth of the dissipation,  $e = \nu(\partial u_i / \partial x_j + \partial u_j / \partial x_i) \partial u_j / \partial x_i$ , which significantly reduces the smallest scale of motion defined by Kolmogorov's length scale,  $\eta_k = (\nu^3/\varepsilon)^{1/4}$ . In this way, an energy cascade is initiated, which promotes the spectral separation ( $L/\eta_k$ ) which is the essential feature of turbulence.

Further downstream towards the back side of the spot, where reverse transition from the turbulent to the laminar state occurs, anisotropy increases, leading to the reappearance of streaky patterns similar to those observed at the front side of the spot. This evidence indicates considerable similarity between the mechanisms associated with forward and reverse transitions and the role which anisotropy plays in the dynamics of the dissipation process, which can only be understood using statistical techniques. The origin of turbulence in wall-bounded flows can be further analyzed by looking into the evolution of anisotropy in the apparent stresses  $\overline{u_i u_j}$ , which can be quantified using the anisotropy tensor [6] defined as  $a_{ij} = \overline{u_i u_j} / q^2 - 1/3 \delta_{ij}$  (where  $q^2 = \overline{u_s u_s}$ ) and its scalar invariants  $\text{II} = a_{ij} a_{ji}$  and  $\text{III} = a_{ij} a_{jk} a_{ki}$ . A plot of  $\text{II}$  vs.  $\text{III}$  for axisymmetric disturbances,  $\text{II} = 3/2(4/3|\text{III}|)^{2/3}$ , and two-component disturbances,  $\text{II} = 2/9 + 2\text{III}$ , defines the anisotropy-invariant map shown in figs. 1(a) and 1(b) (right), which bounds all physically realizable disturbances [6]. The two curves in this figure represent axisymmetric disturbances. The right-hand curve corresponds to disturbances with the streamwise intensity component larger than in the other two directions,  $\overline{u_1^2} > \overline{u_2^2} = \overline{u_3^2}$  ( $\text{III} > 0$ ), and the left-hand curve corresponds to axisymmetric disturbances with  $\overline{u_1^2} < \overline{u_2^2} = \overline{u_3^2}$  ( $\text{III} < 0$ ). Along the straight line reside two-component disturbances. The limiting states of disturbances are located at the corner points on the right- and left-hand sides of the anisotropy-invariant map and correspond to one-component disturbances and isotropic two-component disturbances, respectively. Simple trajectories of joint variations of invariants across the anisotropy-invariant map which interconnect large variations of anisotropy during forward and reverse transitions are sketched in figs. 1(a) and 1(b) (right). These trajectories lie very close to the right-hand boundary of the map and suggest that the corresponding disturbances may be assumed to be axisymmetric with  $\text{III} \geq 0$ .

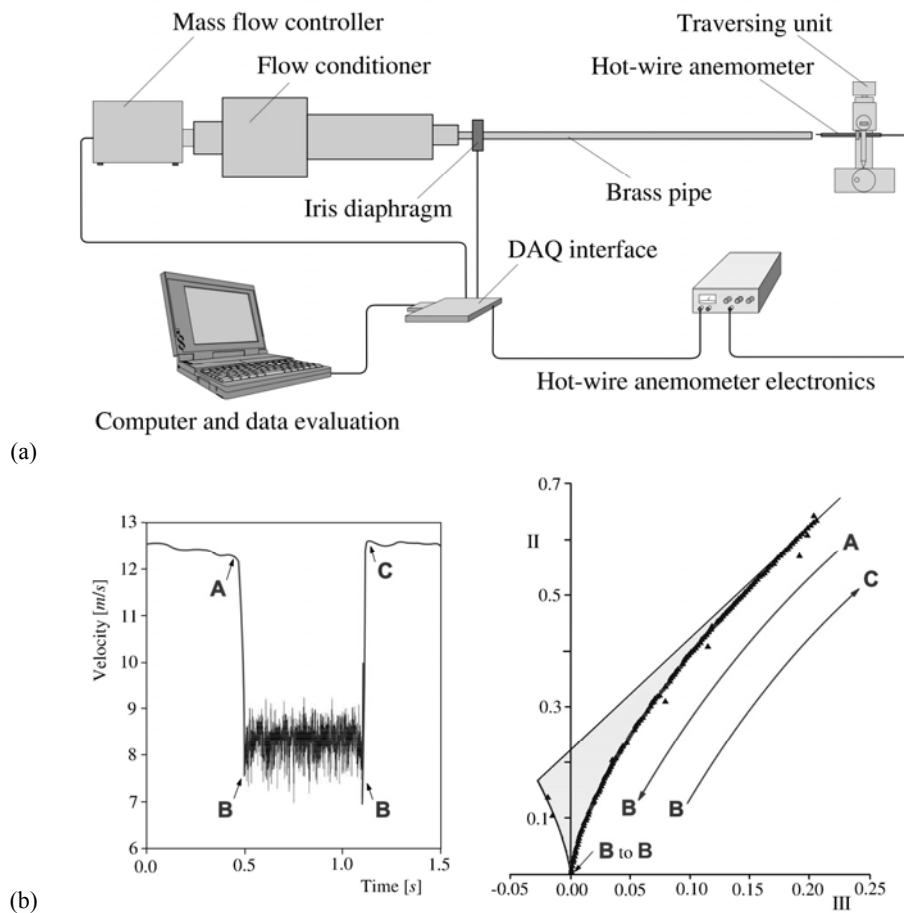
This conjecture was verified experimentally by measuring anisotropy and its scalar invariants in a specially designed test facility in which spots were generated periodically in a laminar pipe flow.

### Instrumentation and experimental results

Figure 2 shows experimental results which confirm the axisymmetric nature of the disturbances with large variations of anisotropy starting from its maximum value, which corresponds to the one-component state, towards the nearly isotropic state and *vice versa* during forward and reverse transitions, respectively. In providing further support for the above-discussed peculiarities of the processes involved during laminar-turbulent transitions, an important role is played by numerical simulations, which offer a physical understanding if the results are cast into an anisotropy-invariant map [7].

Examination of anisotropy maps of turbulence utilizing numerical databases [13-17] of fully developed channel flow at low and moderate Reynolds numbers shown in fig. 3 suggest that invariants in the region very close to the wall, which must lie along the two-component limit, tend to move towards the right corner point of the map, which corresponds

to the one-component state, as the critical Reynolds number for breakdown to turbulence is approached,  $Re \rightarrow (Re)_{crit}$ . For this special situation, which is the realistic state of the disturbances preceding breakdown to turbulence, theoretical considerations and numerical simulations show that the dissipation rate must vanish at the wall [18],  $\varepsilon_{wall} \rightarrow 0$ . This fundamental deduction implies that as long as the disturbances assume the one-component state at the wall, the flow will remain laminar since the energy of the disturbances ( $k = 1/2q^2$ ) cannot be amplified since  $k$  grows as  $k \rightarrow (\varepsilon_{wall}/\nu)x_2^2/2$  as the wall is approached,  $x_2 \rightarrow 0$ .



**Figure 2.** Experimental verification of the axisymmetric nature of the disturbances during forward and reverse transitions in a laminar pipe flow; (a) Experimental set-up for measurements of ensemble-averaged records of fluctuating velocity components aligned with the triggering signal used to activate the iris diaphragm which produced turbulence spots under well controlled conditions, (b) Traces of joint variations of invariants obtained from ensemble-averaged hot-wire signals sampled during the sequence of events leading to forward (A-B) and reverse (B-C) transition (*for color image see journal website*)

The results of the invariant analysis shown in fig. 3 suggest that the stable laminar regime, which is fixed to the one-component state located at the wall, can be broken down to

turbulence only by decreasing the anisotropy at the outer flow boundary  $\Pi_\infty$  below the critical value  $(\Pi_\infty)_{\text{crit}}$ , which is dictated by the  $\varepsilon$  equation. These results further suggest that only a decrease in  $\Pi_\infty$  below  $(\Pi_\infty)_{\text{crit}}$  can induce a similar decrease in  $(\Pi)_{\text{wall}}$  along the two-component limit of fig. 3, which ensures amplification of the disturbances (as  $\varepsilon_{\text{wall}} > 0$ ) necessary to promote transition to turbulence. By extrapolation of the trend in the results of fig. 3 corresponding to the region around the channel centerline, the critical value  $(\Pi_\infty)_{\text{crit}} \approx 0.14$  is obtained. From fig. 3, we may conclude that for  $(\Pi)_{\text{wall}} = (\Pi)_{1C} = 2/3$  and  $\Pi_\infty > (\Pi_\infty)_{\text{crit}}$ , the laminar regime can persist up to very high Reynolds numbers. Following the deductions obtained from the results presented in figs. 1 and 3, we may attack the transition problem analytically using the Navier-Stokes and continuity equations.

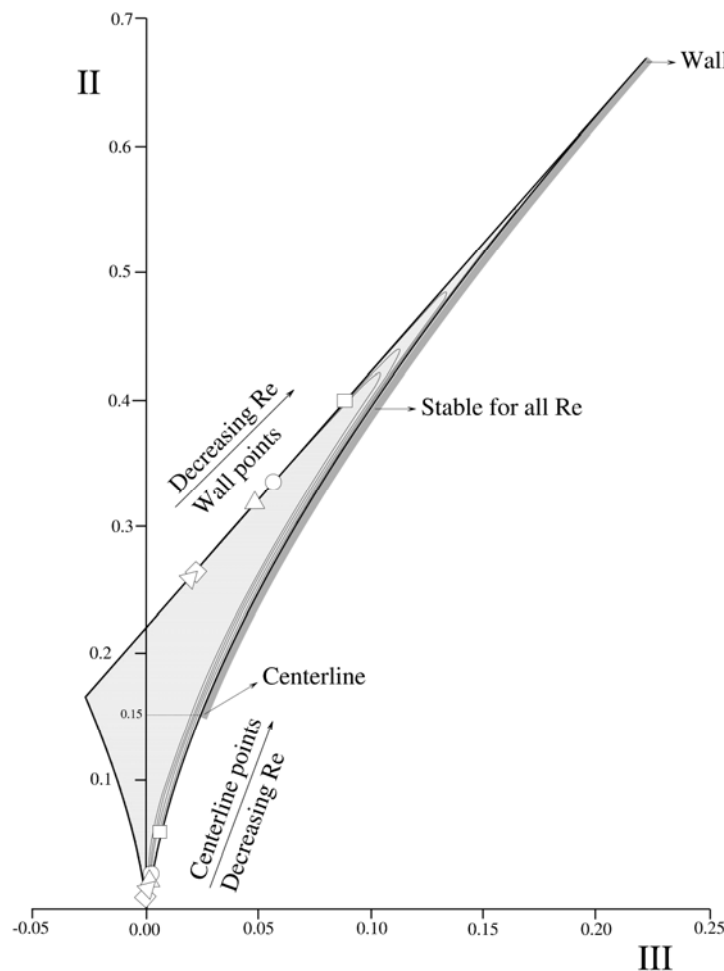


Figure 3. Anisotropy-invariant mapping of turbulence in a channel flow:  $\Delta$  [13],  $\diamond$  [14],  $\triangleleft$  [15],  $\circ$  [16], and  $\square$  [17]; data which correspond to low Reynolds number show the trend as  $Re \rightarrow (Re)_{\text{crit}}$  towards the theoretical solution valid for small, neutrally stable, statistically stationary axisymmetric disturbances; the shading on the right-hand boundary of the map indicates the area occupied by the stable disturbances; for such disturbances it is expected that the laminar regime in a flat plate boundary layer will persist up to very high Reynolds numbers (for color image see journal website)

By splitting instantaneous velocity and pressure fields into the mean-laminar flow and random disturbances  $u_i$  and  $p$  superimposed on it and assuming that the disturbances are small, transport equations for the statistical properties of the disturbances are obtained [5]. These equations are, however, not closed. For the case of axisymmetric disturbances, such as those marked with shading in fig. 3, the closure problem can be overcome on firm mathematical grounds utilizing the two-point correlation technique and the invariant theory [7, 19, 20]. Using the closure proposals suggested for such disturbances, after lengthy derivations the transport equations for the apparent stresses  $\overline{u_i u_j}$  and the dissipation rate  $\varepsilon$  emerge in a closed form:

$$\begin{aligned} \frac{\partial \overline{u_i u_j}}{\partial t} + U_k \frac{\partial \overline{u_i u_j}}{\partial x_k} &\simeq P_{ij} + a_{ij} P_{ss} + \mathcal{F} \left( \frac{1}{3} P_{ss} \delta_{ij} - P_{ij} \right) - 2 \mathcal{A} \varepsilon a_{ij} - \frac{2}{3} \varepsilon \delta_{ij} + \frac{1}{2} \nu \frac{\partial^2 \overline{u_i u_j}}{\partial x_k \partial x_k}, \\ \frac{\partial \varepsilon}{\partial t} + U_k \frac{\partial \varepsilon}{\partial x_k} &\simeq -2 \mathcal{A} \frac{\overline{\varepsilon u_i u_k}}{k} \frac{\partial U_i}{\partial x_k} - \psi \frac{\varepsilon^2}{k} + \frac{1}{2} \nu \frac{\partial^2 \varepsilon}{\partial x_k \partial x_k} \end{aligned}$$

where  $\overline{U_i}$  represents the velocity field of mean-laminar flow,  $x_i$  is the space coordinate,  $P_{ij} = -\overline{u_i u_j} \partial U_j / \partial x_k - \overline{u_j u_k} \partial U_i / \partial x_k$  can be interpreted as the production of  $\overline{u_i u_j}$  and  $F, A$  and  $\psi$  are the scalar functions that depend on the invariants II and III and the Reynolds number  $R_\lambda = q\lambda/\nu$  based on the Taylor micro-scale  $\lambda$ , which is related to  $\varepsilon$  by  $\varepsilon \simeq 5\nu q^2/\lambda^2$ .

If we consider transition of the Blasius flow [21] in a flat plate boundary layer, the energy equation (which is obtained by contraction of the equations for  $\overline{u_i u_j}$ ) immediately suggests stability towards small disturbances if the production  $P_k = P_{ss}/2$  is balanced by the dissipation  $P_k \simeq \varepsilon$ .

The equilibrium constraint leads to the energy equation, which is of boundary layer character [20] and does not allow amplification of statistically stationary disturbances in the boundary layer above corresponding values of the free stream. Inserting  $P_k \simeq \varepsilon$  into the dissipation rate equation and requiring that the dissipation rate must be always non-negative  $\varepsilon \geq 0$  (to satisfy realizability [6]) and at the critical point follows the energy (as emerges from the conjecture [22]  $\varepsilon \simeq Aq^3/l$ ), we deduce the transition criterion  $2A - \psi \geq 0$  in terms of the Reynolds number  $(R_\lambda)_T$  and the anisotropy in the disturbances.

For a certain magnitude (II) and character (III) of the anisotropy, the derived transition criterion suggests permissible magnitudes for the intensity and the length scale of disturbances  $R_\lambda \rightarrow (R_\lambda)_{\text{crit}}$  that guarantee  $P_k \simeq \varepsilon$  with  $\varepsilon \geq 0$  and therefore maintenance of the laminar flow regime in a flat plate boundary layer. Using expressions constructed analytically [7, 20] for the scalar functions  $A$  and  $\psi$ , it is easy to prove from  $2A - \psi = 0$  that breakdown to turbulence can be avoided completely if the anisotropy in the free stream disturbances is sufficiently large,  $\Pi_\infty \geq 0.141$ . The trends in the numerical results of fig. 3 as  $\text{Re} \rightarrow (\text{Re})_{\text{crit}}$  indicate that these tend towards the above-mentioned analytic result very closely and therefore provide support for the theoretical considerations. For vanishing anisotropy in the free stream disturbances,  $\Pi_\infty \rightarrow 0$ , for which all experiments have been carried out from  $2A - \psi = 0$ , we infer the transitional Reynolds number  $(R_\lambda)_T = 13.55$ .

In order to translate  $(R_\lambda)_T$  into  $(\text{Re})_T$ , the relation between the Taylor micro-scale  $\lambda$  and the boundary layer thickness  $\delta$  is required. Using the exact expression  $\lambda = 10^{1/2} x_2$ , which holds, however, only very close to the wall [23], the average value of  $\overline{\lambda}$  across the boundary layer can be related to  $\delta$  as  $\overline{\lambda} \approx 10^{1/2} \delta / 2$ .

## Conclusions

Figure 4 shows that the previous result for  $(Re_\lambda)_T$  together with the suggested approximation for  $\lambda$  predicts the variation of the transition Reynolds number with the relative intensity of the free stream disturbances in fair agreement with the experimental data [24-28] obtained under well-controlled laboratory conditions. The presented results show that it is possible to approach the breakdown of the laminar regime to turbulence from a statistical viewpoint. This approach reveals that the breakdown to small scales is caused by large variation in the anisotropy of the disturbances. The qualitative analysis of the transport equations for the statistical properties of small disturbances allows a prediction of the transition Reynolds number in a boundary layer flow in terms of the relative turbulence intensity in the free stream and its anisotropy.

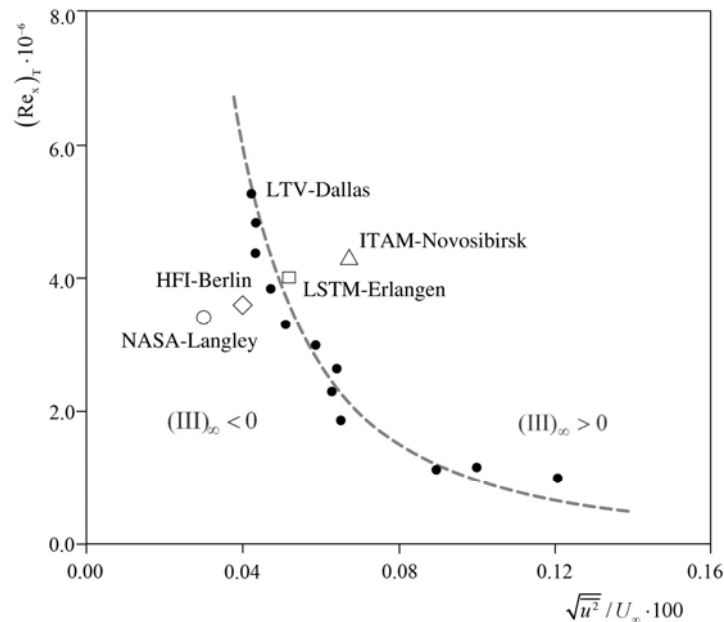


Figure 4. Comparison of experimental data with the prediction of the effect of free-stream disturbances on boundary layer transition;  $(Re_x)_T = xU_\infty/\nu$  is the transition Reynolds number based on distance  $x$  from the leading edge of the plate,  $U_\infty$  corresponds to the velocity of the free stream,  $\nu$  is the kinematic viscosity of the fluid and  $(\overline{u^2})^{0.5}/U_\infty$  represents relative intensity of turbulence in the free stream: • [24],  $\Delta$  [25],  $\circ$  [26],  $\square$  [27],  $\diamond$  [28], ----- prediction of transition and breakdown to turbulence for vanishing anisotropy in the disturbances ( $II = III \rightarrow 0$ )

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## References

- [1] Morkovin, M. V., On the Many Faces of Transition, in: *Viscous Drag Reduction* (Ed. C. S. Wells), Plenum Press, New York, USA, 1969, pp. 1-39
- [2] Lin, C. C., *The Theory of Hydrodynamic Stability*, Cambridge University Press, Cambridge, UK, 1955
- [3] Orszag, S. A., Numerical Studies of Transition in Planar Shear Flows, in: *Laminar-Turbulent Transition* (Eds. E. Eppler, H. Fasel), Springer-Verlag, Berlin, 1980, pp. 153-162

- [4] Holmes, P., *et al.*, *Turbulence, Coherent Structures, Dynamical Systems and Symmetry*, Cambridge University Press, Cambridge, UK, 1998
- [5] Jovanović, J., Pashtrapanska, M., On the Criterion for the Determination Transition Onset and Breakdown to Turbulence in Wall-Bounded Flows, *J. Fluids Eng.*, 126 (2004), 4, pp. 626-633
- [6] Lumley, J. L., Computational Modeling of Turbulent Flows, *Adv. Appl. Mech.*, 18 (1978), pp. 123-176
- [7] Jovanović, J., *The Statistical Dynamics of Turbulence*, Springer-Verlag, Berlin, 2004
- [8] Emmons, H. W., The Laminar-Turbulent Transition in a Boundary Layer – Part I, *J. Aero. Sci.*, 18 (1951), 7, pp. 490-498
- [9] Gad-el-Hak, M. A., A Visual Study of the Growth and Entrainment of Turbulent Spots, in: *Laminar-Turbulent Transition* (Eds. E. Eppler, H. Fasel), Springer-Verlag, Berlin, 1980, pp. 297-310
- [10] Cantwell, B., *et al.*, Structure and Entrainment in the Plane of Symmetry of a Turbulent Spot, *J. Fluid Mech.*, 87 (1978), 4, pp. 641-672
- [11] Van Dyke, M., *An Album of Fluid Motion*, Parabolic Press, Stanford, Cal., USA, 1988
- [12] Herbert, T. H., Morkovin, M. V., Dialog on Bridging Some Gaps in Stability and Transition Research, in: *Laminar-Turbulent Transition* (Eds. E. Eppler, H. Fasel), Springer-Verlag, Berlin, 1980, pp. 47-72
- [13] Kim, J., *et al.*, Turbulence Statistics in a Fully Developed Channel Flow at Low Reynolds Numbers, *J. Fluid Mech.*, 177 (1987), Apr., pp. 133-166
- [14] Antonia, R. A., *et al.*, Low Reynolds Number Effects in a Fully Developed Channel Flow, *J. Fluid Mech.*, 236 (1992), 579, pp. 579-605
- [15] Moser, R. D., *et al.*, Direct Numerical Simulation of Turbulent Channel Flow Up to  $Re_\tau = 590$ , *Phys. Fluids*, 11 (1999), 4, pp. 943-945
- [16] Kuroda, A., *et al.*, Direct Numerical Simulation of the Turbulent Plane Couette-Poiseuille Flows: Effect of Mean Shear on the Near Wall Turbulence Structures, *Proceedings*, 9<sup>th</sup> Symposium on Turbulent Shear Flows, Kyoto, Japan, 1993, pp. 8.4.1-8.4.6
- [17] Horiuti, K., *et al.*, Establishment of the DNS Database of Turbulent Transport Phenomena, Report Grants-in-Aid for Scientific Research No. 02302043, University of Tokyo, Tokyo, 1992
- [18] Jovanović, J., Hillerbrand, R., On Peculiar Property of the Velocity Fluctuations in Wall-Bounded Flows, *Thermal Science*, 9 (2005), 1, pp. 3-12
- [19] Jovanović, J., Otić, I., On the Constitutive Relation for the Reynolds Stresses and the Prandtl-Kolmogorov Hypothesis of Effective Viscosity in Axisymmetric Strained Turbulence, *J. Fluids Eng.*, 122 (2000), 1, pp. 48-50
- [20] Jovanović, J., *et al.*, On the Anisotropy of Axisymmetric Strained Turbulence in the Dissipation Range, *J. Fluids Eng.*, 125 (2003), 3, pp. 401-413
- [21] Schlichting, H., *Boundary-Layer Theory*, 6<sup>th</sup> ed., McGraw-Hill, New York, USA, 1968
- [22] Kolmogorov, A. N., On Degeneration of Isotropic Turbulence in an Incompressible Viscous Liquid, *Dokl. Akad. Nauk. SSSR*, 31 (1941), 6, pp. 319-323
- [23] Durst, F., *et al.*, Statistical Interpretation of the Turbulent Dissipation Rate in Wall-Bounded Flows, *J. Fluid Mech.*, 293 (1995), June, pp. 321-347
- [24] Spangler, J. G., Wells, C. S., Effects of Free Stream Disturbances on Boundary-Layer Transition, *AIAA J.*, 6 (1968), 3, pp. 543-545
- [25] Kachanov, Y. S., *et al.*, *The Origin of Turbulence in the Boundary-Layer*, Nauka, Novosibirsk, Russia, 1982
- [26] Saric, W. S., Reynolds, G. A., Experiments on the Stability and Nonlinear Waves in a Boundary Layer, in: *Laminar-Turbulent Transition* (Eds. E. Eppler, H. Fasel), Springer-Verlag, Berlin, 1980, pp. 125-134
- [27] Jovanović, J., *et al.*, Persistence of the Laminar Regime in a Flat Plate Boundary Layer at Very High Reynolds Number, *Thermal Science*, 10 (2006), 2, pp. 63-96
- [28] Bake, S., Herman-Fottinger Institute, Berlin, personal communication, 1999