A DYE REMOVAL MODEL WITH A FUZZY INITIAL CONDITION

by

Yan ZHANG [a*], ***Ji-Huan HE*** [a*], ***Shu-Qiang WANG*** [b], ***and Ping WANG*** [a]

[a] National Engineering Laboratory for Modern Silk, College of Textile and Clothing Engineering, Soochow University, Suzhou, China

[b] Shenzhen Institutes of Advanced Technology, Chinese Academy of Sciences, Shenzhen, China

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A fuzzy model for dye removal is suggested to study a transport model of the direct textile industry wastewater, and the variational iteration method is adopted to obtain its analytical solutions. The concentration depends upon not only the parameters in the governing equation, but also the pair of the initial condition.

Key words: *fuzzy equation, variational iteration method, Langmuir isotherm, analytical solution*

Introduction

Textile industries produce huge amounts of polluted effluents that are normally discharged to surface water bodies and groundwater aquifers [1-5]. These wastewaters cause many damages to the ecological system of the receiving surface water and create a lot of disturbance to the groundwater resources [1].

To model adsorption process of the direct textile industry wastewater, the following transport equation is widely adopted [1]:

$$R\frac{\partial C}{\partial t} \quad KS\rho_{\mathrm{d}} \quad 0 \tag{1}$$

where C is the equilibrium concentration of the solution, S [mgg^{-1}] – the quantity of mass sorbed on the solid surface, R – the retardation factor, K – the delay constant, and ρ_{d} [1/1000 mgmm^{-3}] – the bulk density of the medium.

Langmuir isotherm reveals the relationship between C and S, which reads [1]:

$$S \quad \frac{Q_0 K_{\mathrm{L}} C}{1 \quad K_{\mathrm{L}} C} \tag{2}$$

where Q_0 is the maximum adsorption capacity, and K_{L} – the Langmuir constant.

Combining eqs. (1) and (2) together, we obtain the following non-linear equation:

$$R\frac{\partial C}{\partial t} \quad \frac{K\rho_{\mathrm{d}} Q_0 K_{\mathrm{L}} C}{1 \quad K_{\mathrm{L}} C} \quad 0 \tag{3}$$

with the following initial condition:

$$C(0) \quad C_0 \tag{4}$$

* Corresponding authors; e-mail: yanzhang86@suda.edu.cn, hejihuan@suda.edu.cn

In practical applications, the initial condition is not deterministic, but fuzzy. In this paper we will consider a fuzzy differential model with a fuzzy initial condition, and apply the variational iteration method to solve fuzzy differential model analytically.

Fuzzy differential model for dye removal

Generally speaking, all governing equations in engineering applications are fuzzy differential equations, because we cannot exactly give the initial conditions. When we say the initial concentration is eq. (4), it actually means that the average initial concentration is C_0. The actual initial condition satisfies the following inequality:

$$\underline{C}_0 < C(0) < \overline{C}_0 \tag{5}$$

The solution of a non-linear equation strongly depends upon the initial condition; a small divergence of the initial condition will result in a remarkable error.

A fuzzy number, C, is a pair $(\underline{C}, \overline{C})$ of functions $\underline{C}(r)$, $\overline{C}(r)$, $0 \le r \le 1$, satisfying the following requirements: $\underline{C}(r)$ is a bounded monotonic increasing left continuous function, $\overline{C}(r)$ is a bounded monotonic decreasing left continuous function, and $\underline{C}(r) < \overline{C}(r)$, $0 \le r \le 1$. Instead of eq. (4), the fuzzy initial condition can be written:

$$C(0) = (\underline{C}_0, \overline{C}_0) \tag{6}$$

where

$$\overline{C}_0(r) = mr + n \tag{7}$$

$$\underline{C}_0(r) = p - qr \tag{8}$$

where m, n, p, and q are known parameters, r is an uncertainty parameter, $0 \le r \le 1$, and $m + q = -n + p$.

Fuzzy variational iteration method

The variational iteration method was first proposed at the end of last century and fully developed in 2006 and 2007, and it has been extensively worked out by numerous authors who successfully applied the method to various kinds of non-linear problems [6-9], and it becomes an effective mathematical tool to fuzzy differential equations [10-15].

We re-write eq. (3) in the form:

$$\frac{\partial C}{\partial t} + aC - bC\frac{\partial C}{\partial t} = 0, \quad C(0) = C_0 \tag{9}$$

where

$$a = \frac{K\rho_{\mathrm{d}}Q_0K_{\mathrm{L}}}{R}, \quad b = K_{\mathrm{L}}$$

Using the variational iteration method, we can construct the following iteration algorithm:

$$\underline{C}_{n+1}(t) = \underline{C}_n(t) - \int_0^t \mathrm{e}^{a(s-t)}\left[\frac{\mathrm{d}\underline{C}_n(s)}{\mathrm{d}s} + a\underline{C}_n(s) - b\underline{C}_n(s)\frac{\mathrm{d}\underline{C}_n(s)}{\mathrm{d}s}\right]\mathrm{d}s \tag{10}$$

$$\overline{C}_{n+1}(t) = \overline{C}_n(t) - \int_0^t \mathrm{e}^{a(s-t)}\left[\frac{\mathrm{d}\overline{C}_n(s)}{\mathrm{d}s} + a\overline{C}_n(s) - b\overline{C}_n(s)\frac{\mathrm{d}\overline{C}_n(s)}{\mathrm{d}s}\right]\mathrm{d}s \tag{11}$$

or

$$\underline{C}_{n+1}(t) = \underline{C}_0(t) + b\int_0^t \mathrm{e}^{a(s-t)}\underline{C}_n(s)\frac{\mathrm{d}\underline{C}_n(s)}{\mathrm{d}s}\mathrm{d}s \tag{12}$$

$$\overline{C}_{n\ 1}(t) \quad \overline{C}_0(t) \quad b\,{}_0^t\, \mathrm{e}^{a(s\ \ t)}\overline{C}_n(s)\frac{\mathrm{d}\overline{C}_n(s)}{\mathrm{d}s}\mathrm{d}s \tag{13}$$

Equations (10) and (11) are called the variational iteration algorithm-I, and eqs. (12) and (13) the variational iteration algorithm-II. We begin with:

$$\overline{C} \quad \overline{C}_0\mathrm{e}^{\ \overline{\beta}t}, \quad \underline{C} \quad \underline{C}_0\mathrm{e}^{\ \underline{\beta}t} \tag{14}$$

where $\underline{\beta}$ and $\overline{\beta}$ are unknown constants to be determined later.

By the variational iteration algorithm-II, we have:

$$\underline{C}_1(t) \quad \underline{C}_0\mathrm{e}^{\ \underline{\beta}t} \quad b\underline{\beta}\underline{C}_0^2\,{}_0^t\,\mathrm{e}^{a(s\ \ t)}\mathrm{e}^{\ 2\underline{\beta}s}\mathrm{d}s \quad C_0\mathrm{e}^{\ \underline{\beta}t} \quad \frac{b\underline{\beta}\underline{C}_0^2}{a\ \ 2\underline{\beta}}(\mathrm{e}^{\ 2\underline{\beta}t} \quad \mathrm{e}^{\ at}) \tag{15}$$

From eq. (3), we have an additional initial condition, that is:

$$\underline{C}'(0) \quad \underline{C}_0 \quad \frac{a\underline{C}_0}{1\ \ b\underline{C}_0} \tag{16}$$

Using this relationship, we can identify $\underline{\beta}$ in eq. (15):

$$\underline{C}_1\ (0) \quad \underline{\beta}\underline{C}_0 \quad \frac{b\underline{\beta}\underline{C}_0^2}{a\ \ 2\underline{\beta}}(\ 2\underline{\beta} \quad a) \quad \frac{a\underline{C}_0}{1\ \ b\underline{C}_0} \tag{17}$$

From eq. (10) $\underline{\beta}$ can be solved, which is:

$$\underline{\beta} \quad \frac{(ab\underline{C}_0^2 \quad a\underline{C}_0 \quad 2\underline{C}_0) \quad \sqrt{(ab\underline{C}_0^2 \quad a\underline{C}_0 \quad 2a\underline{C}_0)^2 \quad 8a^2\underline{C}_0\underline{C}_0(1 \quad b\underline{C}_0)}}{4\underline{C}_0(1 \quad b\underline{C}_0)} \tag{18}$$

Therefore, we obtain the following analytical solution:

$$\underline{C}(t) \quad \underline{C}_0\mathrm{e}^{\ \underline{\beta}t} \quad \frac{b\underline{\beta}\underline{C}_0^2}{a\ \ 2\underline{\beta}}(\mathrm{e}^{\ 2\underline{\beta}t} \quad \mathrm{e}^{\ at}) \tag{19}$$

Similarly, we have:

$$\overline{C}(t) \quad \overline{C}_0\mathrm{e}^{\ \overline{\beta}t} \quad \frac{b\overline{\beta}\overline{C}_0^2}{a\ \ 2\overline{\beta}}(\mathrm{e}^{\ 2\overline{\beta}t} \quad \mathrm{e}^{\ at}) \tag{20}$$

where

$$\overline{\beta} \quad \frac{(ab\overline{C}_0^2 \quad a\overline{C}_0 \quad 2\overline{C}_0) \quad \sqrt{(ab\overline{C}_0^2 \quad a\overline{C}_0 \quad 2a\overline{C}_0)^2 \quad 8a^2\overline{C}_0\overline{C}_0(1 \quad b\overline{C}_0)}}{4\overline{C}_0(1 \quad b\overline{C}_0)} \tag{21}$$

Discussion and conclusion

As a practical case, we consider an example $a = 1$, $b = 1$, and $r = 0.1$, the pair for the initial condition read:

$$\underline{C}_0(r) \quad 0.1r \quad 0.1 \tag{22}$$

$$\overline{C}_0(r) \quad 0.1 \quad 0.1r \tag{23}$$

Equations (22) and (23) imply that the mean initial condition is $C(0) = 0$, and the parameter, r, implies uncertainty of the initial condition. The solutions, eqs. (19) and (20) not only depend upon main parameters in eq. (3), but also the pair of the initial condition. The two solutions give the maximal and minimal concentration properties, which have great importance in dye removal.

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