The present study aims to evaluate the mixed convection flow and heat transfer of functionalized DWCNT/water nanofluids with variable properties in a cavity having hot baffles. The investigation is performed at different nanoparticles volume fraction including 0, 0.0002, 0.001, 0.002, and 0.004, Richardson numbers ranging from 0.01 to 100, inclination angles ranging from 0 to 60° and at constant Grashof number of $10^4$. The results presented as streamlines and isotherms plot and Nusselt number diagrams. According to the finding with increasing nanoparticles volume fraction and distance between the left hot baffles of nanoparticles average Nusselt number enhances for all considered Richardson numbers and cavity inclination angles. Also with increasing Richardson number, the rate of changes of average Nusselt number increase with increasing distance between the left hot baffles. For example, at Richardson number of 0.01, by increasing $L_1$ from 0.4 to 0.6, the average Nusselt number increases 7%; while for similar situation at Richardson number of 0.1, 1.0, and 10, the average Nusselt number increases, respectively, 17%, 24%, and 26%. At all Richardson numbers, the maximum value of average Nusselt number is achieved for a minimum length of left baffles.

Key words: nanofluid, square cavity, baffle, functionalized DWCNT-water, mixed convection

Introduction

Convection heat transfer with nanofluids is always encountered in many engineering applications such as solar collectors, heat exchangers, and cooling of electrical components [1-3]. In these applications, engineers are always looking for techniques to enhance the heat transfer performance using solid nanoparticles (less than 100 nm) in the common fluid (e.g. water, ethylene glycol, and oil). Due to the fact that the experimental investigation of heat transfer is costly and time-consuming process, numerical simulation has attracted the interest of researchers in recent decades to make available close comprehension to the heat transfer phenomenon. Since, applications of mixed convection heat transfer in cavities is very important, mixed convection in lid-driven cavities filled with nanofluids is one of the most interesting topics usually considered by researchers.

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Mixed convection in a square lid-driven cavity with copper-water nanofluid was executed numerically by Talebi et al. [4]. In their study, the top and bottom horizontal walls are insulated while the vertical walls are maintained at constant but different temperatures. The predicted results indicated that at fixed Reynolds number, the solid concentration of nanoparticles affects the flow pattern and thermal behavior particularly for a higher Rayleigh number. Also, the results of this paper indicate the Nusselt number increase with increasing nanoparticles volume fraction. Mansour et al. [5] examined numerically mixed convection in a square lid-driven cavity partially heated from below and filled with water-based nanofluid containing various nanoparticles volume fractions of Cu, Ag, Al$_2$O$_3$, and TiO$_2$. The effects of Reynolds number, nanoparticles volume fraction, different values of the heat source length and different locations of the heat source on the Nusselt number were considered in details. The mixed convection in a lid-driven triangular enclosure filled with an Al$_2$O$_3$-water nanofluid was also examined numerically by Ghasemia and Aminossadati [6]. The results presented a comparison between two different scenarios of upward and downward left sliding walls and showed the addition of Al$_2$O$_3$ nanoparticles enhances the heat transfer rate for all values of Richardson number and for each direction of the sliding wall motion. Abbasian Arani et al. [7] predicted mixed convection flow of Cu-water nanofluid in a lid-driven square cavity with adiabatic horizontal walls and sinusoidal heating on sidewalls. The predicted results revealed that for a constant Grashof number at all Richardson numbers, a clockwise eddy was developed inside the cavity, also the rate of heat transfer increases with decrease in Richardson number and increase of nanoparticles volume fraction. Furthermore the results show for a constant Reynolds number the clockwise eddy is observed up to Ri = 1. For Ri = 10 a multicellular flow pattern is formed inside the cavity. The mixed convection in a square lid-driven cavity with filled with nanofluids was examined by Salari et al. [8]. The effects of the Rayleigh number, Reynolds number, the nanoparticles volume fraction, the dimensions of heaters, and their locations on the streamlines and isotherms contours were investigated in details. The results mentioned above focused on the constant thermophysical properties. But, in real applications, the effects of variable thermophysical properties in lid-driven cavity with nanofluids are important.

Here, a brief review of recent studies on mixed convection in lid-driven cavities using nanofluids considering variable properties was conducted. Numerical modeling of laminar mixed convection in single and double-lid square cavities filled with an Al$_2$O$_3$-water nanofluid was performed by Chamkha and Abu-Nada [9]. In their study, two viscosity models are used to approximate nanofluid viscosity, namely, the Brinkman model and the Pak and Cho correlation. Sheikhzadeh et al. [10] performed numerical study on mixed convection in a lid-driven square cavity containing Al$_2$O$_3$-water nanofluid focusing variable properties. They estimated the thermal conductivity and viscosity of nanofluid using the experimental models. The results obtained by variable thermal conductivity and viscosity were compared to the results obtained using the Maxwell and Brinkman model. Major differences were found between the heat transfer rates in the cavity for two employed models. Mixed convection of nanofluid consists of CuO and water inside a lid-driven cavity with wavy wall was numerically modeled by Abu-Nada and Chamkha [11]. Computational results revealed that heat transfer enhanced by adding the nanoparticles for all Richardson numbers and bottom wall geometry ratios. Also the presence of nanoparticles causes significant heat transfer augmentation for all values of Richardson numbers and bottom wall geometry ratios.

Modeling of flow and heat transfer in lid-driven cavity with obstacles and heat sources is another topic that has been considered recently [12-16]. The numerical solution of mixed convection flow in a lid-driven square enclosure with a triangular heat source filled with nano-
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Fluid is performed by Kalteh et al. [15]. The fluid in the enclosure was a water-based nanofluid containing Al₂O₃, TiO₂, Ag, or CuO nanoparticles. They presented that adding the nanoparticles in pure fluid leads to a noteworthy heat transfer enhancement. Hemmat Esfe et al. [16] numerically studied mixed convection in an inclined lid-driven cavity equipped to a hot obstacle using Al₂O₃/water nanofluid considering temperature-dependent properties. Aghaei et. al. [17] investigated the numerical study of magnetic field effect on mixed convection and entropy generation of nanofluid in a trapezoidal enclosure. For all of the studied cases, entropy generation due to friction is negligible and the total entropy generation is mainly due to irreversibility associated with heat transfer and variation of the total entropy generation with Hartmann number is similar to that of the average Nusselt number. Mixed convection heat transfer in a CuO–water filled trapezoidal enclosure, effects of various constant and variable properties of the nanofluid was numerically modeled by Arefmanesh et. al. [18]. The results indicated that the differences between the average Nusselt number and the entropy generation obtained using the different considered variable-property models decrease with increasing the nanoparticles volume fraction.

Mixed convection of water-based nanofluids in a rectangular inclined lid-driven cavity partially heated from its left side wall was investigated numerically by Hussein et al. [19]. The results of this study indicate that the presence of nanoparticles in the fluid is found to alter the structure of the fluid flow. Moreover, it is observed that the shape of the circulation vortex is sensitive to the inclination angle and addition of nanofluids. Ahmed et al. [20] investigated numerically the mixed convection from a discrete heat source in enclosures with two adjacent moving walls and filled with micropolar nanofluids. Based on the numerical results, the effects of the dominant parameters such as Richardson number, nanofluid type, length and location of the heat source, nanoparticles volume fractions, moving lid orientations and dimensionless viscosity are examined.

Based on the best knowledge of author there is no study on mixed convection in cavity subjected to functionalized double-walled carbon nanotube-water (DWCNT-water) nanofluid. Furthermore, the effects of changing position of three baffles on control of heat transfer and fluid flow of cavity is considered in the present study.

Analysis

Problem formulation

The 2-D steady-state, laminar, mixed-convection fluid flow and heat transfer within a square enclosure filled with functionalized DWCNT-water nanofluid is simulated numerically using the finite volume method. The schematic of the present study is depicted in fig. 1. The dimensions of the enclosure are denoted by \( H \). The top wall of the cavity moves in its own plane from left to right with a constant speed \( U_0 \). Left heated baffles have equal length at all cases and are kept at high temperature, \( T_h \). Right hot baffle is located at middle of right wall.

In order to cast the governing equations into a dimensionless form, the following dimensionless variables are introduced:

![Figure 1. Schematic diagram of cavity with hot baffles](image)
Employing the foregoing dimensionless variable, the continuity, momentum, and energy equations for the nanofluid in a dimensionless form:

\[
 \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

\[
 U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\rho_{nf} \nu_f \text{Re}} \left[ \frac{\partial}{\partial X} \left( \mu_{nf} \frac{\partial U}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \mu_{nf} \frac{\partial U}{\partial Y} \right) \right] + \frac{(\rho \beta)_{nf}}{\rho_{nf} \beta_f} \text{Ri} \theta \sin(\gamma)
\]

\[
 U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\rho_{nf} \nu_f \text{Re}} \left[ \frac{\partial}{\partial X} \left( \mu_{nf} \frac{\partial V}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \mu_{nf} \frac{\partial V}{\partial Y} \right) \right] + \frac{(\rho \beta)_{nf}}{\rho_{nf} \beta_f} \text{Ri} \theta \cos(\gamma)
\]

\[
 U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\text{Re Pr} \alpha_f (\rho_c \rho_p)_{nf}} \left[ \frac{\partial}{\partial X} (k_{nf} \frac{\partial \theta}{\partial X}) + \frac{\partial}{\partial Y} (k_{nf} \frac{\partial \theta}{\partial Y}) \right]
\]

where \( \gamma \) is the cavity inclination angle.

The dimensionless boundary conditions are given:

\[
\begin{align*}
0 < X < H, & \quad Y = 1, \quad U = 0, \quad V = 0, \quad \text{and} \quad \frac{\partial \theta}{\partial n} = 0 \\
0 < Y < H, & \quad X = 0, \quad U = 0, \quad V = 0, \quad \text{and} \quad \frac{\partial \theta}{\partial n} = 0 \\
0 < Y < H, & \quad X = 1, \quad U = 0, \quad V = 0, \quad \text{and} \quad \frac{\partial \theta}{\partial n} = 0 \\
X = 0, & \quad 0 < y < H, \quad U = 0, \quad V = 0 & \left(6\right)
\end{align*}
\]

\[
X = 0, \quad 0 < \frac{H}{4} < y < \frac{H}{2}, \quad \theta = 0 - \frac{H}{4} < y < \frac{H}{2}, \quad \theta = 1 - \frac{H}{2} < y < \frac{3H}{4},
\]

\[
\theta = 0 - \frac{3H}{4} < y < H, \quad \theta = 1
\]

On all baffle, \( \theta = 1 \)

The local Nusselt number is obtained from the relation:

\[
\text{Nu} = -\left( \frac{k_{nf}}{k_f} \right) \frac{\partial \theta}{\partial n} \text{on all hot sections}
\]

The average Nusselt number is given:

\[
\text{Nu}_{\text{avg}} = \frac{1}{L_{\text{on all hot sections}}} \int_{\text{Nu}} \text{d}Y
\]

In order to investigate the effect of the thermal conductivity and viscosity of the nanofluid, experimental results [21] are employed for DWCNT-water nanofluid in the present study:

\[
\frac{k_{nf}}{k_f} = \frac{\varphi}{0.17981 - 0.000369T} + 1.0026
\]
\[
\frac{\mu_{nf}}{\mu} = 1 + 3.575\varphi + 6032.93\varphi^2 - 1153669\varphi^3
\]  
\hspace{1cm} (10)

**Numerical implementation**

The finite volume method and the SIMPLER algorithm are employed to solve the governing equations numerically. The first step of discretizing the governing equations is to generate a finite difference mesh in the computational domain. A control volume is generated around each node of the mesh afterwards. The governing equations are then integrated over each control volume. The diffusion terms are replaced using a second-order central difference scheme, while, a hybrid scheme is employed for the convective terms in order to obtain stable solutions for convection-dominated cases.

**Validation of the code**

In order to validate the numerical procedure, the mixed-convection fluid flow and heat transfer in a square cavity filled with Al\(_2\)O\(_3\)-water nanofluid is simulated using the developed code, and the results are compared with the existing results in the literature. The cavity is filled with Al\(_2\)O\(_3\)-water nanofluid. Table 1 shows comparisons between the average Nusselt number of the hot wall obtained by the present code with the results of Chamkha and Abu-Nada [9]. As it is observed from the table, very good agreements exist between the results of the present simulation.

### Table 1. Comparisons between the average Nusselt number of the hot wall with the results of Chamkha and Abu-Nada [9]

<table>
<thead>
<tr>
<th>Ri</th>
<th>(\varphi)</th>
<th>Present work</th>
<th>Chamkha and Abu-Nada [9]</th>
<th>Percent difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>0.02</td>
<td>33.14</td>
<td>32.80</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>36.40</td>
<td>36.90</td>
<td>1.36</td>
</tr>
<tr>
<td>1</td>
<td>0.02</td>
<td>4.76</td>
<td>4.92</td>
<td>3.25</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>4</td>
<td>4.95</td>
<td>2.22</td>
</tr>
</tbody>
</table>

**3.2 Grid independence study**

In order to determine a proper grid for the numerical simulations, a grid independence study is carried out. The obtained average Nusselt number of the hot wall for these grids are presented in tab. 2.

It is observed from this table that the average Nusselt number converges for a grid having 131×131 nodes implying that this grid is sufficiently The convergence criterion in the following numerical simulations for the temperature, pressure, and velocity fields is:

\[
\sum_{i=1}^{M} \sum_{j=1}^{N} \left| \frac{\xi_{i+1,j}^r - \xi_{i,j}^r}{\xi_{i,j}^r} \right| \leq 10^{-7}
\]  
\hspace{1cm} (11)

where \(m\) and \(n\) are the number of grid points in the x- and y-directions, respectively, \(\xi\) is any of the computed field variables, and \(r\) is the number of iterations.

**Results and discussion**

In this work, mixed convection flow and heat transfer of square cavity subjected to DWCNT-water nanofluids with variable properties having three hot baffles was numerically
studied. The study was performed at constant Grashof number of $10^4$, for different Richardson numbers ranging from 0.01 to 100, inclination angles ranging from 0 to 60° and for different nanoparticles volume fractions (0, 0.0002, 0.001, 0.002, and 0.004).

Figure 2 shows the streamlines and isotherms of COOH-functionalized DWCNT-water for different inclination angles and Richardson numbers at nanoparticles volume fraction of 0.004. At inclination angle of 0 (Case 1) and for all Richardson numbers, a great vortex occurs in the upper part of the enclosure near the moving wall. At all Richardson number, streamlines are more compact in the vicinity of moving wall. This behavior of streamlines is due to the fact that this vortex is strongly affected by the velocity of moving wall. Also, for all Richardson numbers the center of the vortex, proportional to the speed of the wall, is leaning to the right.
side of the enclosure. At Richardson number of 0.01, the slope of the streamlines in the upper right corner of the enclosure is higher compared to other Richardson numbers, which is due to higher velocity of moving wall at Richardson number of 0.01. In the Case 1, secondary vortex forms in the lower section of enclosure under the influence of hot baffles and lower hot parts. At central zone of secondary vortex, under the influence of the vortex formed in the upper part of the enclosure, streamlines is more compact. By changing the inclination angle of enclosure, there is not a significant change in the overall behavior of flow. At inclination angles of 30° and 60° (Case 2 and Case 3), for all Richardson numbers, two vortexes are observed. In these cases, because of the buoyancy force components, the upper large vortex is slightly affected by direction of moving wall velocity. By comparing the slope of the streamlines in the upper right
corner, at Richardson number of 0.01 and for different inclination angles of enclosure, this phenomenon is characterized clearly. With increasing the inclination angle, the secondary vortex in the lower section of enclosure tends to the left and its distance from the adiabatic left wall of the enclosure increases. For all cases, isotherms in the vicinity of the lower wall of enclosure are denser which is due to the higher rate of heat transfer in this part of the enclosure. In the lower right side zone, isotherms, with high curvature, are stretched towards the right adiabatic wall which decreased with increasing Richardson number. Greater curvature of the isotherms in the whole of enclosure, at the small Richardson numbers, indicates greater movement of nanofluids at these Richardson numbers. With increasing the Richardson number isotherms tend to a thermal stratification, however, due to the hot baffle at different parts of the enclosure thermal stratification is not well formed.

Maximum amount of stream function $\Psi_{\text{max}}$ as a criterion of the flow power, in various Richardson numbers for nanofluid with nanoparticles volume fraction of $\phi = 0.004$ is presented in tab. 3. In all $\gamma$ with increasing Richardson numbers the value of the Maximum of stream function decrease.

Figure 3 demonstrates the variation of average Nusselt number versus nanoparticles volume fraction at different Richardson numbers for different distances, $L_1$, between the left hot baffles.
baffles. For all $L_1$ at all Richardson numbers, with increasing nanoparticles volume fraction average Nusselt number enhances. For all cases at all Richardson numbers, with increasing $L_1$ average Nusselt number increases and better temperature distribution occurs throughout the enclosure. Increase in $L_1$ also provides more space to move nanofluids, which leads to improve heat transfer rate. With increase in Richardson number, the values of average Nusselt number decrease compared with the similar conditions for lower Richardson numbers. When the Richardson number increases, the amounts of changes of average Nusselt number increase with increasing $L_1$. For example, at $Ri = 0.01$, by increasing $L_1$ from 0.4 of height of enclosure to 0.6 of it, the average Nusselt number increases 7%; while for similar situation at Richardson number of 0.1, 1.0, and 10, the average Nusselt number increases, respectively, 17%, 24%, and 26%. Obtained results are similar with previous research in view to baffles locations [22-24] and lid velocity, changing the Richardson number [5, 25, 26].

The variation of average Nusselt number vs. nanoparticles volume fraction at different Richardson numbers for different lengths, $L_4$, of the left hot baffles is illustrated in fig. 4. For all $L_4$ at all Richardson numbers, with increasing nanoparticles volume fraction average Nusselt number enhances. For all cases considered, with increasing Richardson number, the variation of average Nusselt number with the nanoparticles volume fraction is more perceptible. At all Richardson numbers, the maximum value of average Nusselt number is achieved for a minimum length of left baffles. For conditions considered in fig. 3, at low Richardson numbers, the values of average Nusselt number are greater than similar

![Figure 4. The variation of average Nusselt number vs. nanoparticles volume fraction at different Richardson numbers for different lengths ($L_4$) of the left hot baffles](image-url)
situations at high Richardson numbers. Obtained results in this section also are similar with previous research in view to baffles locations [22-24] and lid velocity, changing the Richardson number [5, 25, 26].

In eq. (9) the new correlation is proposed to predict the average Nusselt number for \( \gamma = 0^\circ \) to \( \gamma = 60^\circ \).

\[
Nu = A \text{Ri}^3 + B \text{Ri}^2 + C \text{Ri} + D\varphi + E
\]
\[
A = -1.3, \quad B = 14.53, \quad C = -14.92, \quad D = 14.52, \quad E = 2.82, \quad R^2 = 0.999
\]  

Conclusions

In this study, mixed convection flow and heat transfer of DWCNT-water nanofluid with variable properties in an enclosure equipped with hot baffles was numerically investigated. The study was conducted at Grashof number of \( 10^4 \), for Richardson numbers ranging from 0.01 to 100, inclination angles ranging from 0 to 60° and for different nanoparticles volume fractions (0, 0.0002, 0.001, 0.002, and 0.004). Also, the effect of length and location of hot baffles on fluid flow and heat transfer were investigated. Based on the numerical results, the following conclusions were obtained.

- For all considered Richardson numbers and inclination angles, with increasing nanoparticles volume fraction average Nusselt number enhances.
- For all cases at all Richardson numbers, with increasing distance between the left hot baffles average Nusselt number increases.
- With increasing Richardson number, the amounts of changes of average Nusselt number increase with increasing distance between the left hot baffles. For example, at Richardson number of 0.01, by increasing \( L_1 \) from 0.4 to 0.6, the average Nusselt number increases 7%; while for similar situations at Richardson number of 0.1, 1.0, and 10, the average Nusselt number increases 17%, 24%, and 26%, respectively.
- At all Richardson numbers, the maximum value of average Nusselt number is achieved for a minimum length of left baffles.

Acknowledgment

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Nomenclature

| g | gravitational acceleration, [m/s²] |
| h | heat transfer coefficient, [W/m²K] |
| k | thermal conductivity, [W/mK] |
| L | enclosure length, [m] |
| Nu | Nusselt number, [-] |
| P | dimensionless pressure, [-] |
| Pr | Prandtl number, [-] |
| Ri | Richardson number, [-] |
| T | temperature, [K] |
| U, V | dimensionless velocity components, [-] |
| X, Y | dimensionless Cartesian co-ordinates, [-] |
| Avg | average |
| c | low temperature |
| f | fluid |
| h | high temperature |
| nf | nanofluid |
| p | nanoparticles |

Greek symbols

- \( \varphi \) – nanoparticles volume fraction
- \( \theta \) – inclination angle of cavity, [°]
- \( \psi \) – streamlines

Subscript

- Avg – average
- c – low temperature
- f – fluid
- h – high temperature
- nf – nanofluid
- p – nanoparticles
References


