

## COMBINED NATURAL CONVECTION AND RADIATION WITH TEMPERATURE-DEPENDENT PROPERTIES

by

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*This paper investigates the effects of temperature dependence of radiative properties of a medium on radiation and natural convection interaction in a rectangular enclosure. The radiative transfer equation is solved using the discrete ordinates method, and the momentum, continuity, and energy equations are solved by the finite volume method. Effects of the conduction-to-radiation parameter, Rayleigh number, and optical thickness are discussed. Results show that temperature dependence of radiative properties affects the temperature gradient, and hence the energy transport even in relatively weak radiation condition. On the other hand, temperature dependence of radiative properties has relatively insignificant effects on convection characteristics, even though it does affect the way that energy transfers into the system. As conduction-to-radiation parameter is decreased or Rayleigh number is increased, the effects of temperature dependence of radiative properties become more significant.*

Key words: radiative transfer, natural convection,  
temperature-dependent properties

### Introduction

Radiative transfer is an important topic and has been investigated for the past few decades, usually with the assumption that the medium is gray and homogeneous. However, in a few areas such as atmospheric applications, inhomogeneity may greatly affect the transport process. Kobayashi [1] used Fourier series decomposition method to solve the radiative transfer equation (RTE). The radiative properties were varied with terrain characteristics, such as cloud, air, and ground, and the atmospheric reflection and effective optical thickness were discussed. Galinsky [2] also focused on the characteristic of the atmosphere, and he modified the discrete ordinates method with Newton's iterative scheme.

Yucel *et al.* [3] set up a square enclosure surrounded by rigid black walls and discussed the effect of natural convection with radiation. Tan and Howell [4] used the product-integral method and finite difference method to analyze a natural convection combined radiation in a 2-D square medium. Colomer *et al.* [5] considered the convection and radiation effect in a 3-D differentially heated cavity. The local and mean heat fluxes as a function of the Rayleigh number were studied, for both transparent and participating media with different optical thicknesses.

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Sangapatnam *et al.* [6] discussed radiation and mass transfer effects on a MHD free convection problem. The Rosseland approximation were used to handle thermal radiation. Kolsi *et al.* [7] studied the effect of the radiation on a transverse spiraling flow in a 3-D vertical cavity. A gray, emitting-absorbing and isotropic scattering fluid was assumed in the cavity.

Tsai and Ozisiki [8] found that the inhomogeneity of a medium greatly affects radiative transfer in a 1-D domain by using the discrete ordinates method. Li *et al.* [9] extended their analysis to 2-D cylindrical domain filled with inhomogeneous medium. Farmer and Howell [10] investigated an anisotropic scattering, non-gray, and inhomogeneous medium in a 3-D rectangular cavity using the Monte-Carlo method. They found the maximum radiative heat flux occurred in the central part near the cold wall. Ruan and Tan [11] also used the Monte-Carlo method to study the effects of property inhomogeneity due to location and proposed an average method to improve computation speed. Tseng *et al.* [12] investigated convection and radiation transfer characteristics in a solar hydrogen production reactor.

Ravishankar *et al.* [13] solved an inhomogeneous medium problem using modified differential approximation method and compared results with other algorithms. They also studied the effects of inhomogeneity caused by variation of optical thickness. Muthukumaran *et al.* [14] numerically analyzed a short-pulse laser on a human tissue phantom. Radiative properties of human tissue was assumed to vary with location.

Chu *et al.* [15] used the line-by-line approach and the statistical narrow-band correlated-k model to predict the radiative properties of gases. Five test cases were carried out, for an inhomogeneous problem caused by a mixture of different gaseous species.

The properties of a mixture do change with its composition. Jin [16] assumed that the radiative properties varied with the concentration of particle distribution in air and calculated its radiative transfer in a 2-D domain. Meftah *et al.* [17] considered real gas properties, and they analyzed the interaction of radiation and double diffusive natural convection with air-CO<sub>2</sub> and air-H<sub>2</sub>O mixtures. Moufekkir *et al.* [18] studied combined double-diffusive convection and radiation. In the study, the absorption coefficient is a linear function of local dimensionless concentration. These literatures show that inhomogeneity caused by concentration also affect system thermal characteristics.

More recently, Moradi and Rafiee [19] set-up an 1-D steady-state energy equation to express a fin system with constant moving speed and losing heat. Convection and radiation were considered simultaneously, and a temperature-dependent thermal conductivity was assumed. Moradi *et al.* [20] analyzed a triangular porous fin with temperature-dependent thermal conductivity in a convection-radiation system. They used Rosseland approximation to simplify the radiative heat transfer. Sun *et al.* [21] studied an irregular fin heat transfer with temperature-dependent internal heat generation and thermal properties.

In past studies, the inhomogeneity of a medium was usually assumed to vary with geometry or composition of species. However, the radiative properties may also change with other physical properties like temperature. For a gaseous mixture of CO<sub>2</sub> and N<sub>2</sub>, the absorption of the mixture increases with temperature, although its spectral distribution remains approximately the same [22]. In addition to gases, solid materials such as glass, have absorption coefficient changing with temperature without changing the spectral distribution characteristics [23]. The objective of this work is to investigate the effects of inhomogeneity of radiative properties due to temperature variation on thermal and heat transfer characteristics. A rectangular enclosure will be considered and the discrete ordinates method will be employed. Effects of the conduction-to-radiation parameter, Rayleigh number, and optical thickness will also be discussed.

### Numerical analysis

To study the effects caused by property inhomogeneity, a 2-D square enclosure with a length,  $L$ , is considered, as shown in fig. 1. The left wall is a hot wall at temperature,  $T_h$ , and the right wall is a cold one at temperature,  $T_c$ . The upper and lower sides are assumed to be adiabatic. The emissivities of the four walls are also shown in the figure.

To simplify the problem, some assumptions are employed in this study. The flow is considered to be laminar, incompressible, and steady. The fluid is Newtonian, and viscous dissipation is neglected. The fluid is gray with isotropic scattering, and the properties of the fluid are constant except the density and scattering albedo,  $\omega$ . The density is assumed to be a linear function of temperature. Two cases of inhomogeneity for the scattering albedo are considered in this study:

$$\text{Case A: } \omega = \frac{1}{4} + \frac{3}{4}\theta \quad (1)$$

$$\text{Case B: } \omega = 1 - \frac{3}{4}\theta \quad (2)$$

It is assumed that  $\omega$  is a linear function of temperature increases with temperature in Case A and decreases with temperature in Case B. The variation of inhomogeneity is an important factor in this topic. However, past studies only focused on the inhomogeneity due to spatial variation and neglected the dependence of radiative properties on temperature. This temperature dependence effect is the main focus of this work.

The governing equations include continuity, momentum, and energy equations. The dimensionless form of these equations are summarized, eqs. (3)-(6):

– continuity equation

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (3)$$

– momentum equation

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \text{Pr} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (4)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \text{Pr} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \text{Ra Pr } \theta \quad (5)$$

where  $U$  and  $V$  are the dimensionless velocities in x- and y-directions. The source term in eq. (5) on the right-hand-side is the buoyancy force and Boussinesq approximation is employed.

– energy equation

$$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) - \frac{\theta_0 \tau}{Nr} (\nabla Q_r) \quad (6)$$

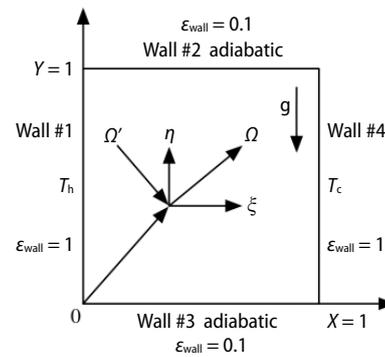


Figure 1. Schematic of computational domain

$$Q_r = \int_{4\pi} I^*(s, \Omega) \Omega d\Omega \tag{7}$$

In the equations,  $\theta$  is the dimensionless temperature,  $\theta_0$  – the dimensionless temperature ratio,  $Nr$  – the conduction-to-radiation parameter, and is an important parameter in the radiative heat transfer problem, and  $Q_r$  – the dimensionless radiative heat flux, which can be evaluated from eq. (7) once the radiative intensity is known, and  $s$  – the position co-ordinate. The radiative intensity can be obtained by solving the RTE, eq. (8):

– RTE

$$\zeta \frac{\partial I^*}{\partial X} + \eta \frac{\partial I^*}{\partial Y} = -\tau I^* + \tau \left[ (1-\omega) I_b^* + \frac{\omega}{4\pi} \int_{4\pi} I^*(s, \Omega') \Phi(\Omega', \Omega) d\Omega' \right] \tag{8}$$

where  $\zeta$  and  $\eta$  are the directional cosines in x- and y-direction, respectively,  $I^*$  – the dimensionless radiative intensity, and  $I_b^*$  – the dimensionless blackbody intensity.

The boundary conditions are given in tab. 1.

**Table 1. Boundary conditions**

	Momentum	Energy
Wall #1	$U = V = 0$	$\theta = 1$
Wall #2	$U = V = 0$	$Q_c + Q_r = 0$
Wall #3	$U = V = 0$	$Q_c + Q_r = 0$
Wall #4	$U = V = 0$	$\theta = 1$

The dimensionless conduction heat flux can be calculated from:

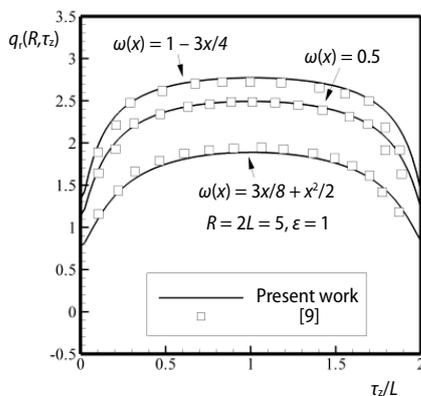
$$Q_c = -\frac{Nr}{\theta_0 \tau} \nabla \theta \tag{9}$$

The overall local Nusselt number on the hot wall is the sum of the local Nusselt numbers for convection and radiation. The former shows the ratio of convection heat flux to conduction heat flux, the latter shows the ratio of radiation heat flux to conduction heat flux, and they can be calculated from eq. (10).

$$Nu = Nu_c + Nu_r = -\frac{\partial \theta}{\partial X} + \frac{\theta_0 \tau}{Nr} Q_r \tag{10}$$

The mean Nusselt number over the hot wall is defined:

$$\overline{Nu} = \frac{1}{L} \int_0^L Nu dY \tag{11}$$



**Figure 2. Code verification results**

In this work, the RTE is solved using the discrete ordinates method [24], and the continuity, momentum, and energy equations are solved by using the SIMPLE algorithm [25]. Both methods are carried out using Fortran, and the convergence criteria in each calculated variable is set as  $10^{-5}$ . To validate the current code, a 2-D radiation in a cylinder with spatially varying albedo [9] problem is repeated. The radiative flux,  $q_r$ , for a hot medium enclosed by cold walls for three albedo distributions are shown in fig. 2 and  $\tau_z$  is the optical co-ordinate in the z-direction. For all cases, the current results are in very good agreement with the published results. This validates the current computer code.

A grid test is performed to verify the independence of results on the grid size. Because changes in temperature and velocity near boundary in the current problem is larger than that in the central part, a sin-enhanced grid distribution is used in this study. Grid numbers from 112×112 to 142×142 are investigated. Table 2 shows that increasing the grid size from 132×132 to 142×142 results in less than 1% monotonic change in the maximum absolute value of stream function and radiative Nusselt number. Thus 142×142 is chosen for this study.

Additionally, a  $S_n$  quadrature scheme test is also performed. As shown in tab. 3,  $S_6$  quadrature scheme is enough for both energy and momentum calculations, and is used in this study.

**Table 2. Grid test results;  $Ra = 10^7$ ,  $Pr = 1$ ,  $\theta_0 = 1.5$ ,  $Nr = 0.005$ ,  $\tau = 1$ , Case B**

Grid	112×112	122×122	132×132	142×142
$ \psi _{\max}$	349.7	353.7	357.4	360.8
$Nu_r$	246.4	246.4	246.5	246.5

**Table 3. Quadrature scheme test results;  $Ra = 10^7$ ,  $Pr = 1$ ,  $\theta_0 = 1.5$ ,  $Nr = 0.005$ ,  $\tau = 1$ , Case B**

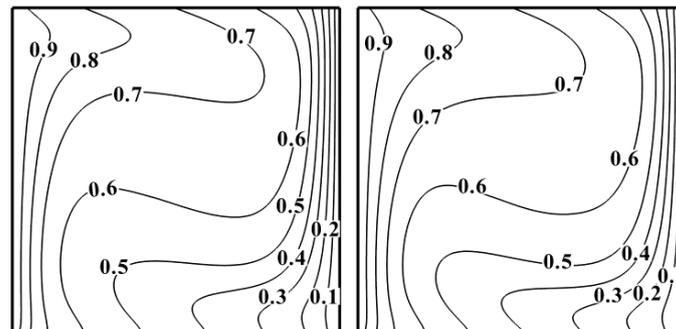
Quadrature scheme	$S_4$	$S_6$	$S_8$
$ \psi _{\max}$	360.9	360.8	360.8
$Nu_r$	246.6	246.5	246.5

### Results and discussion

The main purpose of this study is to investigate the effects of inhomogeneity on heat transfer characteristics. Therefore, Prandtl number and the dimensionless temperature ratio,  $\theta_0$ , are fixed to 1 and 1.5. The controlled variables are Rayleigh number, varying from  $10^4$  to  $10^7$ , and conduction-to-radiation parameter,  $Nr$ , varying from 10 to 0.005. In addition, effect of optical thickness is also discussed, with its value varying from 0.1 to 10.

The radiative heat transfer is proportional to the fourth power of temperature. Therefore, it greatly affects the temperature distribution, and also leads to changes in velocity field.

The  $Nr$  is a key dimensionless parameter to describe the strength of radiation, as compared with conduction. As  $Nr$  decreases, radiation effect increases. Figure 3 shows the temperature distribution for  $Nr = 0.05$  for the two cases. In general, the temperature distributions for both cases in this condition are similar. But a further look reveals that the temperature gradient near the hot wall in Case B is larger than that in Case A. Because fluid near the hot wall has a  $\theta$  value close to 1, therefore the absorption coefficient for Case B is much higher than that in Case A. Energy emitted from the hot wall is more easily absorbed by nearby fluid in Case B.



**Figure 3. The temperature distribution for Case A (left) and Case B (right) for  $Nr = 0.05$ ;  $Ra = 10^5$ ,  $Pr = 1$ ,  $\theta_0 = 1.5$ ,  $\tau = 1$**

Figure 4 shows the temperature distribution for  $Nr = 0.005$ . In this condition,  $Nr$  is small enough so that radiation dominates energy transport in the system. The isotherms in Case B appear to be a diffusion-like process. Again this is due to the highly absorbing ability of the fluid near the hot wall. Energy is absorbed and re-emitted by neighboring fluid, and the process is just like diffusion. As temperature decreasing to the right, isotherms are more and more distorted, and convection effect increases slightly. On the other hand for Case A, absorption is

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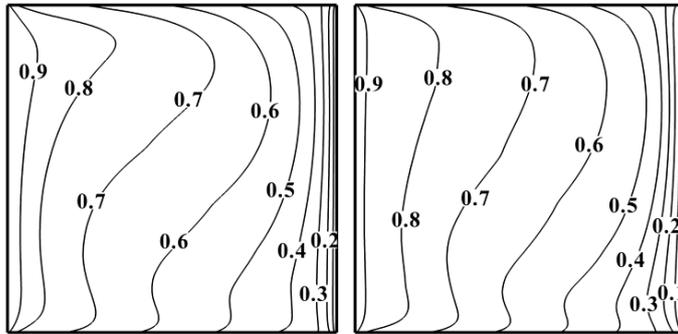


Figure 4. The temperature distribution for Case A (left) and Case B (right) for  $Nr = 0.005$ ;  $Ra = 10^5$ ,  $Pr = 1$ ,  $\theta_0 = 1.5$ ,  $\tau = 1$

thermal energy is more limited to regions close to the hot wall in Case B, so the temperature gradient is larger. Therefore, conductive heat flux is larger in Case B.

In fig. 5(a), it can be noticed that the value of the conductive heat flux is small. For the condition of  $Nr = 0.005$ , radiation dominates the energy transport and conduction accounts for only 10 to 20 percent of heat transfer. The difference between Case A and Case B is large. When  $Nr$  increases to 0.05 and 0.5, as shown in figs. 5(b) and 5(c), the dimensionless conductive heat flux in Case B is still larger than in Case A, but the difference between the two cases becomes smaller. Additionally, it can be found that  $Q_c$  is larger near the bottom side ( $Y = 0$ ) than the top side ( $Y = 1$ ). For the condition of  $Nr = 0.05$ , natural convection is more pronounced. Following the circulation of natural convection, the low-temperature medium is pushed to the bottom side of the hot wall. Temperature gradient is increased and hence the conductive heat flux. For  $Nr = 0.5$ , as shown in fig. 5(c), the difference of dimensionless conductive heat flux in these cases is still noticeable. It shows that even radiation effect is not the dominant mechanism, the inhomogeneity of the medium still affects the system through the influence of thermal conduction.

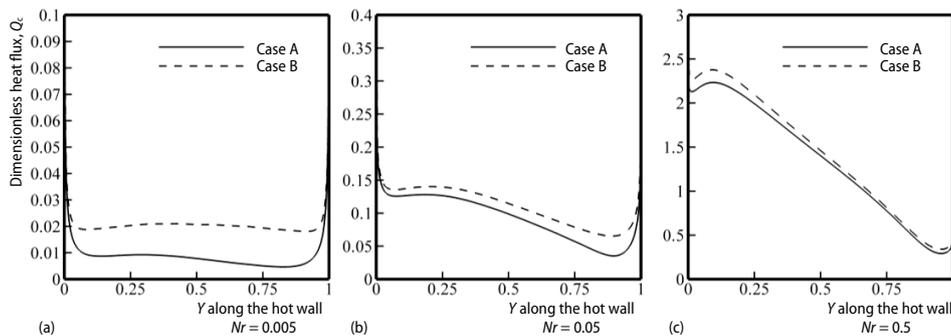


Figure 5. Effects of inhomogeneity of medium on local dimensionless conductive heat flux along the hot wall;  $Ra = 10^5$ ,  $Pr = 1$ ,  $\theta_0 = 1.5$ ,  $\tau = 1$

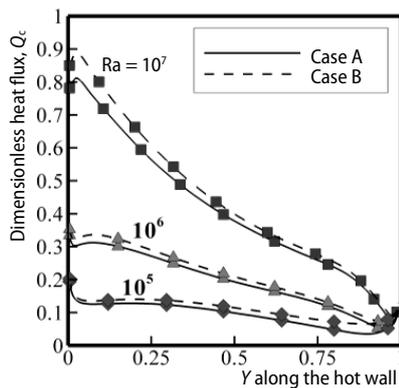
Figure 6 shows the effects of Rayleigh number on dimensionless conductive heat flux for  $Nr = 0.05$ . As Rayleigh number is increased from  $10^4$  to  $10^7$ , the conductive heat flux along the hot wall is increased due to the gradual increase of buoyancy force. For  $Ra = 10^4$ , buoyancy force is not as pronounced, and the difference in conductive heat flux between Cases A and B is obvious. When Rayleigh number is increased to  $10^5$  and  $10^6$ , buoyancy force becomes more pronounced, and the difference caused by the inhomogeneity of medium declines. However, when

weaker than in Case B near the hot wall. Thermal energy can penetrate further away from the hot wall and convection effect is more pronounced.

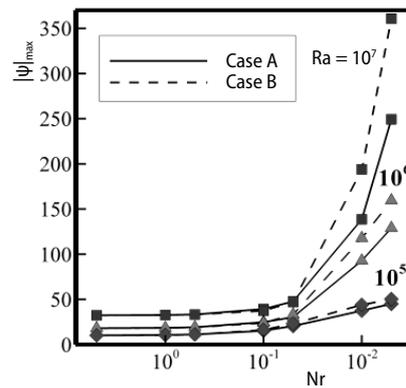
Figures 5 shows the local dimensionless conductive heat flux,  $Q_c$ , along the hot wall for different  $Nr$ . In these figures, it can be seen that dimensionless conductive heat flux in Case B is always larger than in Case A. As explained previously,

Rayleigh number is further increased to  $10^7$ , buoyance force is so strong that large amount of low temperature medium is pushed to the lower side near the hot wall by the strong circulation of natural convection. The local temperature difference is increased, the effect of inhomogeneity is also increased due to the variation of radiative property with temperature. The difference in conductive heat flux between Cases A and B is large, especially at the bottom area along the hot wall.

Figure 7 depicts the effects of inhomogeneity on the maximum absolute value of stream function for different Rayleigh numbers. When  $Nr$  is small, the difference between Cases A and B becomes more significant. With more energy absorbed at the high temperature area in Case B, the buoyance force in the fluid is enhanced. When  $Nr$  is larger than 0.05, the flow is less affected by the inhomogeneity of media. On the other hand, when  $Nr$  is smaller than 0.05, radiation becomes the dominating mechanism of energy transport. More energy is transported into the medium from the hot wall in Case B, and the circulation becomes stronger as evidenced by the higher maximum absolute value of stream function. In addition, the difference in the maximum absolute value of stream function of value of stream function of these two cases increases with the value of Rayleigh number. This is also due to the enhancement of buoyance force in Case B.



**Figure 6.** Effects of Rayleigh number on local dimensionless conduction heat flux along the hot wall;  $Nr = 0.05$ ,  $Pr = 1$ ,  $\theta_0 = 1.5$ ,  $\tau = 1$



**Figure 7.** Effects of Rayleigh number on stream function for different  $Nr$ ;  $Pr = 1$ ,  $\theta_0 = 1.5$ , and  $\tau = 1$

Optical thickness,  $\tau$ , is a parameter that describes the difficulty that radiation pass through a medium. With larger optical thickness, radiative energy is more likely to be absorbed or scattered by the medium. Figure 8 shows the maximum absolute value of stream function,  $|\Psi|_{\max}$ , vs. optical thickness for different Rayleigh numbers. It can be found that  $|\Psi|_{\max}$  increases with optical thickness. This is because increasing optical thickness makes energy easier to be absorbed or scattered by the medium. The radiative energy is absorbed by the fluid near the hot wall, and natural convection is enhanced. However, the difference in  $|\Psi|_{\max}$  caused by inhomogeneity of the medium is relatively small for the cases considered.

Figure 9 shows mean Nusselt numbers vs. optical thickness for different Rayleigh numbers. Convective Nusselt number increases with Rayleigh number due to stronger buoyancy force. The difference in convective Nusselt number for Cases A and B is quite large. For Case A,  $Nu_c$  decreases monotonically with increasing optical thickness. On the other hand,  $Nu_c$  for Case B first decreases and then increases with increasing optical thickness. The difference caused by inhomogeneity is small when optical thickness is small because the dif-

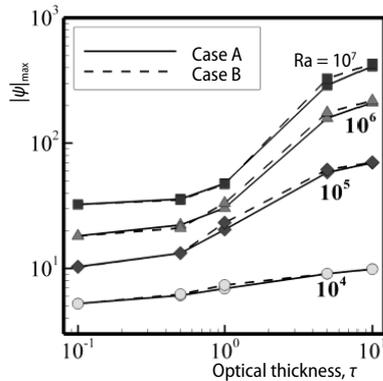


Figure 8. Effects of Rayleigh number on stream function for different optical thickness;  $Pr = 1$ ,  $\theta_0 = 1.5$ , and  $Nr = 0.05$

large and maintains a vortex shape. The cold medium is moved by the flow motion from the right side and is close to the hot wall, and it leads to a higher radiative energy transfer near the hot wall. Before the radiation mechanism fully dominates the system, a stronger convection would enhance the radiative energy transportation along the hot wall, and it also leads to the result of increasing of radiative Nusselt number.

ference caused by different radiative transfer mechanism (different albedo value) is small for small  $\tau$ . The radiation effect becomes obvious when  $\tau$  is large. The  $Nu_c$  for Case A continues to decrease. This is because in Case A, albedo increases with temperature, energy is more likely to be transported to cold region by scattering. Therefore, temperature gradient near the hot wall, and hence  $Nu_c$ , decreases. However, for Case B, radiation effect is limited to near the hot wall region when  $\tau$  is large. A large amount of energy concentrates in the high temperature area. The convection effect is enhanced under these conditions, and hence  $Nu_c$  is also increased.

It can also be found in fig. 9 that radiative Nusselt number,  $Nu_r$ , increases with increasing  $\tau$  for all Rayleigh numbers. In the case of strong buoyance force, the temperature distribution is not yet controlled by the radiation mechanism even the optical thickness of the system is

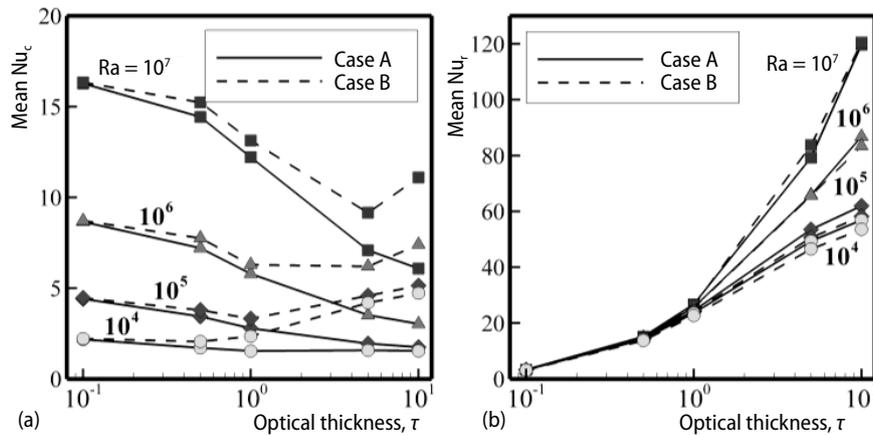


Figure 9. Effects of Rayleigh number on mean convective Nusselt number (a) and radiative Nusselt number (b) of the hot wall for different optical thickness;  $Pr = 1$ ,  $\theta_0 = 1.5$ , and  $Nr = 0.05$

## Conclusions

Effects of radiation on the heat transfer and flow characteristics of an inhomogeneous medium are investigated numerically. Two types of inhomogeneity are considered, and effects of conduction-to-radiation parameter, Rayleigh number, and optical thickness are investigated.

The results show that temperature-dependent properties of the media affect the energy transportation near the hot wall through the change of temperature gradient. For Case B, where the albedo decreases with increasing temperature, radiative transport is more limited to high

temperature area. For Case A, where the albedo increases with temperature, radiative energy is more likely to be transported to cold regions, and the high temperature region extends further from the hot wall. This leads to smaller temperature gradients along the hot wall.

The difference between Cases A and B is more significant for small  $Nr$ , large Rayleigh number, and large  $\tau$ . A smaller  $Nr$  means more energy transfer into system by radiation, more energy absorbed at the high temperature area in Case B, stronger buoyance force occurred, and it leads to larger difference from Case A. Convective Nusselt number of hot wall is obviously different between the two cases when  $\tau$  is large and  $Nr$  is small. In Case A, energy is more likely to be transported to low temperature area by scattering. Thus buoyancy force is reduced and  $Nu_c$  is decreased. However, for Case B, radiation effect is limited to high temperature area near the hot wall. A large amount of energy concentrates, the convection effect is enhanced, and  $Nu_c$  is increased when  $\tau$  is large.

### Nomenclature

$I^*$	–dimensionless radiative intensity ( $= I/4\sigma T_c^4$ ), [–]	$X, Y$	–dimensionless co-ordinates ( $= x/L$ ), ( $= y/L$ ), [–]
$I_b^*$	–dimensionless blackbody radiative intensity, [–]	$x, y$	–cartesian co-ordinates, [m]
$Nr$	–conduction-to-radiation parameter ( $= k\beta/4\sigma T_c^3$ ), [–]	<i>Greek symbols</i>	
$Nu$	–Nusselt number, [–]	$\alpha_f$	–thermal diffusivity, [m <sup>2</sup> s <sup>-1</sup> ]
$Nu_c$	–convective Nusselt number, [–]	$\beta_f$	–volume expansion coefficient, [K <sup>-1</sup> ]
$Nu_r$	–radiative Nusselt number, [–]	$\varepsilon_{\text{wall}}$	–wall emissivity, [–]
$P$	–dimensionless pressure, [–]	$\eta$	–directional cosine in y-direction, [–]
$Pr$	–Prandtl number, ( $= \nu_f/\alpha_f$ ), [–]	$\theta$	–dimensionless temperature [ $= (T - T_c)/(T_h - T_c)$ ], [–]
$Q_c$	–dimensionless conductive heat flux, [–]	$\theta_0$	–dimensionless temperature ratio [ $= T_c/(T_h - T_c)$ ], [–]
$Q_r$	–dimensionless radiative heat flux, [–]	$\nu_f$	–kinematic viscosity, [m <sup>2</sup> s <sup>-1</sup> ]
$Ra$	–Rayleigh number, [ $= g\beta_f(T_h - T_c)L^3/\nu_f\alpha_f$ ], [–]	$\zeta$	–directional cosine in x-direction, [–]
$s$	–position co-ordinate	$\tau$	–optical thickness, [–]
$T_c$	–cold wall temperature, [K]	$\psi$	–dimensionless stream function, [–]
$T_h$	–hot wall temperature, [K]	$\Omega$	–solid angle, [sr]
$U, V$	–dimensionless velocity ( $= uL/\alpha_f$ ), ( $= vL/\alpha_f$ ), [–]	$\Omega'$	–incident solid angle, [sr]
$u, v$	–velocity, [ms <sup>-1</sup> ]	$\omega$	–scattering albedo, [–]

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